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Observation of Zeeman splitting in universal conductance fluctuations

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We provide experimental evidence for a transition from a random-matrix ensemble with broken time-reversal symmetry to an ensemble with partially broken spin-rotation symmetry. The transition from a Gaussian unitary ensemble to the ensemble in which the Zeeman degeneracy of electrons is lifted is accompanied by a factor of 2 change in $1/f$ noise fluctuations measured in a mesoscopic Li wire at low temperature. The magnetic-field scale characterizing the Zeeman crossover is determined by $k_B T$, rather than by the Thouless energy.

The effect of quantum interference on electron transport in disordered metals has been studied extensively in recent years both theoretically and experimentally.¹ At low temperature the electrons diffuse coherently for a distance L_ϕ , the phase-breaking length, which can become much longer than the elastic mean free path. The resulting quantum interference among diffusive paths gives rise to universal conductance fluctuations (UCF), in which the conductance fluctuates as a function of a control parameter by a universal amplitude $\delta G \sim e^2/h$ at zero temperature, independent of the magnitude of the conductance itself.^{2,3} The universality was explained by Al'tshuler² and by Lee and co-workers³ using a diagrammatic perturbation theory. An alternative view, due to Imry,⁴ is based on the transfer matrix T , which relates the electron fluxes at the right and left sides of a disordered conductor, and so determines the conductance. The statistics^{4,5} of the transfer matrix T has the properties characteristic of the random matrix ensembles introduced by Wigner and Dyson⁶ in nuclear physics: the Gaussian orthogonal ensemble (GOE) for a system with time-reversal and spin symmetry, the Gaussian unitary ensemble (GUE) when time-reversal symmetry is broken, and the Gaussian symplectic ensemble (GSE) in the limit of strong spin-orbit scattering. This random matrix approach provides the universal dependence of the amplitude of the conductance fluctuations as follows:⁷

$$(\delta G)^2 \approx \frac{1}{4} \left(\frac{e^2}{h} \right)^2 \frac{k s^2}{\beta}, \quad (1)$$

where k is the number of independent eigenvalue sequences, s is the eigenvalue degeneracy, and $\beta=1, 2$, or 4 for the GOE, GUE, or GSE, respectively.

Experimental observations of the crossover between some limits of Eq. (1) have been reported,⁸⁻¹³ most clearly in disordered metals with strong spin-orbit (SO) scattering (GSE).⁸ In this paper, we report measurements on a metal in the regime of negligible spin-orbit scattering, which has the feature of maximum symmetry (GOE) in the absence of a magnetic field. Application of a magnetic field such that a flux quantum penetrates through a phase-coherent region, causes a transition from GOE to GUE, and hence a reduction of $(\delta G)^2$ by exactly a factor of 2. Application of a much larger field breaks the spin degeneracy of the conduction electrons and creates two independent eigenvalue sequences (s changes from 2 to 1 and k changes from 1 to 2), thereby causing a second factor of 2 reduction. Our data show good agreement with the theory for the complete crossover function, both for the GOE to GUE transition and for the splitting of the Zeeman degeneracy. Our results show that the magnetic-field scale for the Zeeman crossover is determined by the sample temperature, rather than, as suggested previously,^{13,14} by the Thouless energy.

Measurement of $(\delta G)^2$ in a single sample requires that we

vary some parameter to obtain different values of G in the ensemble. The usual method of measuring the static variation of G versus magnetic field (the “magnetofingerprint”) is inadequate because we wish to obtain $(\delta G)^2$ at fixed values of B . One approach, useful for semiconductors, is to measure G versus gate voltage in a metal-oxide-semiconductor field-effect transistor geometry, and then to repeat the measurement many times after thermal cycling to obtain better statistics from several members of the ensemble.¹³ Disordered metals, on the other hand, exhibit dynamic $1/f$ resistance noise due to the slow rearrangement of impurities and scattering centers. This noise persists to very low temperature due to tunneling of defects,⁸ and is enhanced by the long-range interference that gives rise to UCF.^{15,16} The total conductance change (the noise power integrated over the experimental bandwidth) in metals is typically far below the saturation limit of e^2/h per phase-coherent volume; the noise is referred to as “unsaturated.” Measurement of the noise yields excellent statistics at fixed magnetic field; the accuracy of the measurement increases with the measurement time.

We have studied Li metal for minimal SO scattering. Quench-condensed Li wires have been fabricated previously with low SO scattering rates and large values of L_ϕ .¹⁷ Quasi-one-dimensional (1D) samples (defined by $W < L_\phi$ in the diffusive regime) further enhance the noise power via UCF.¹⁶ Samples were patterned with five leads on silicon substrates using electron-beam lithography and a trilayer resist/metal/resist structure. The lateral dimensions of the sample discussed in this paper were $0.44 \times 20 \mu\text{m}^2$ as determined from a scanning electron microscope (SEM) picture before the sample was loaded into the ⁴He cryostat. The Li film was quench-condensed *in situ* at 4.2 K through the metal stencil, then annealed at 35 K. The low-temperature sheet resistance was 0.46Ω . The film thickness was obtained at the conclusion of the experiment by a comparison of the linear temperature dependence of the resistance above 80 K with that of bulk lithium. By this method, we estimated the thickness of the film to be 54 nm. This implies $l_e = 40$ nm and $k_F l_e = 450$ with free-electron values for the electronic constants. The resistance and resistance fluctuations ($1/f$ noise) were measured using a low-frequency ac bridge method.¹⁸ We measured the $1/f$ noise as a function of perpendicular magnetic field at two fixed temperatures, 1.6 and 4.2 K.

The inset to Fig. 1 shows the magnetoresistance at 1.6 K and a fit based on 1D weak-localization theory.¹⁹ Fits to these data provide an upper bound on the SO scattering rate of $\tau_{\text{SO}}^{-1} < 0.2\tau_\phi^{-1}$ at 1.6 K; hence we are always in the low SO scattering limit. (The fit shown in the inset has $\tau_{\text{SO}}^{-1} = 0$.) Figure 1 shows the temperature dependence of L_ϕ assuming no spin-orbit scattering in the magnetoresistance fits. The straight line, $L_\phi^{-2} = (0.096T^2 + 0.20) \mu\text{m}^{-2}$, is a linear least-squares fit to the data. This linear behavior, although unusual for metal films, has been observed previously for Li films over a wide temperature range.¹⁷ We believe that the finite intercept at $T = 0$ arises from the presence of a small amount of spin-dependent scattering (either spin orbit or spin flip).¹⁷ Since the 1D weak-localization correction to the magnetoresistance depends on the width of the sample, it gives another estimate of film width, $W \approx 0.45 \mu\text{m}$, close to the SEM ob-

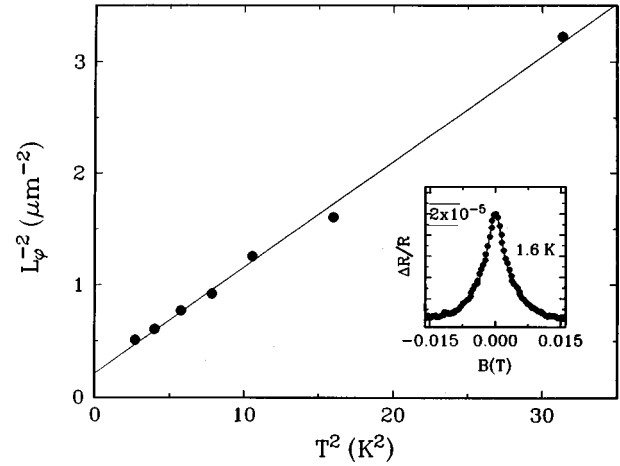


FIG. 1. The electron phase breaking length vs temperature, obtained from the weak-localization fits of low-field magnetoresistance. Inset: magnetoresistance data (●) at 1.6 K and the fit to quasi-1D weak-localization theory with zero spin-orbit scattering.

servation. L_ϕ is 1.4 and $0.75 \mu\text{m}$ at 1.6 and 4.2 K, respectively, hence the sample is quasi-1D ($L_\phi > W$).

Figure 2 illustrates the $1/f$ noise power spectra at 1.6 K for three different values of magnetic field: $B = 0, 1.0,$ and 8.8 T. At least 64 spectra for total noise and background were averaged to reduce the statistical uncertainty at each data point. The relatively large errors at the high-frequency end arise from background subtraction. The straight lines are least-squares fits to a power law. The noise slopes are -1.00 ± 0.03 for all values of magnetic field. The data in Fig. 2 demonstrate clearly that the $1/f$ noise power drops as the magnetic field increases.

Figure 3 shows the relative $1/f$ noise power at 0.1 Hz as a function of the perpendicular magnetic field at temperatures of 1.6 and 4.2 K. The relative noise power at 1.6 K shows a first reduction to $1/2$ with characteristic field $B_{c1} \approx 22$ G,

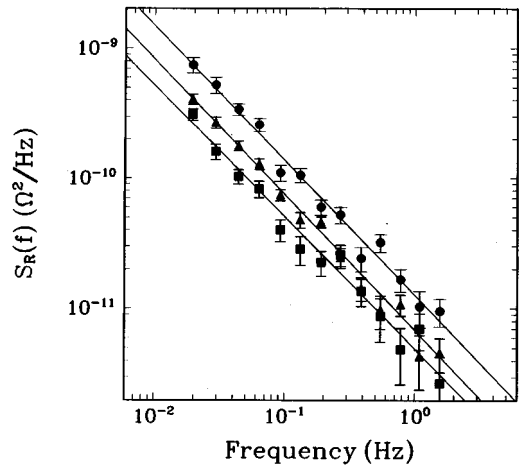


FIG. 2. The resistance noise power spectra, $S_R(f)$, vs frequency at 1.6 K with fits to a power law, at three values of magnetic field, $B = 0$ T (●), 1.0 T (△), 8.8 T (■). The spectral slopes for the three data sets are $-1.03, -1.04,$ and -1.01 , respectively, with an uncertainty of ± 0.03 .

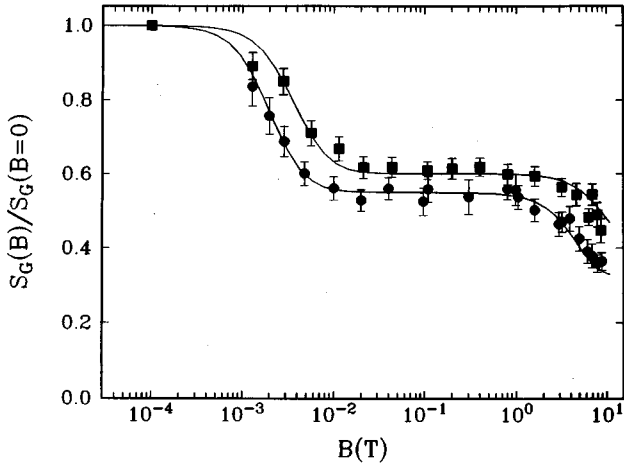


FIG. 3. Conductance noise power, $S_G(0.1 \text{ Hz})$, as a function of magnetic field at 1.6 K (●) and 4.2 K (■). The data are normalized by the noise power at zero field. The solid line is the theoretical expression for the noise crossover function. The theory for the low-field crossover is fit to the data with two free parameters, L_ϕ and c , as discussed in the text. The theory for the high-field crossover has no free parameters.

where B_{c1} is defined as the field where the noise is 3/4 of its zero-field value. B_{c1} is temperature dependent, increasing to 34 G at 4.2 K. B_{c1} corresponds to the penetration of a flux quantum through a phase coherent region, $B_{c1} = A(h/e)/L_\phi W$ for a quasi-1D sample, where A is a numerical constant that depends weakly on the ratio of $L_\phi = \sqrt{D\tau_\phi}$ to $L_T = \sqrt{\hbar D/k_B T}$, where $D = v_F l_e/3$ is the electronic diffusion constant. For this film, $D \approx 7.8 \times 10^{-3} \text{ m}^2/\text{s}$ and L_T is 0.17 μm at 1.6 K.

For a detailed comparison of theory and experiment, we need to derive an expression for the GOE-GUE crossover function. For unsaturated noise in 1D, there are two ways we can obtain such an expression. First, we could modify the numerical procedure given by Stone for the 2D noise crossover function.¹⁴ We have, however, chosen an alternate route based on an approximate analytical expression for the field autocorrelation of conductance, $F(\Delta B) = \langle \delta g(B) \delta g(B + \Delta B) \rangle$ where $g = G/(e^2/h)$, given for the quasi-1D case by Beenakker and van Houten.²⁰ To get the noise crossover function from the field autocorrelation, we follow the procedure outlined by Al'tshuler and Spivak¹⁵ for the case of unsaturated noise. The result for the cooperon contribution to the noise is

$$\delta g'^2(B) = CL_\phi^5(B) \left(1 + \frac{9}{2\pi} \frac{L_\phi^2(B)}{L_T^2} \right)^{-1} \left[1 - \frac{3}{\pi} \frac{L_\phi^2(B)}{L_T^2} \times \left(1 + \frac{9}{2\pi} \frac{L_\phi^2(B)}{L_T^2} \right)^{-1} \right], \quad (2)$$

where the effect of magnetic field is incorporated into $L_\phi(B)$, viz.,

$$L_\phi^{-2}(B) = \frac{1}{L_\phi^2(0)} + \frac{1}{3} \left(\frac{\pi WB}{h/2e} \right)^2.$$

The total noise²¹ is given by cooperon and diffuson contributions that are equal at zero field, so the crossover function for the relative noise power at low magnetic field is

$$\nu(B) = \frac{1}{2} \left(1 + \frac{\delta g'^2(B)}{\delta g'^2(0)} \right). \quad (3)$$

For pure UCF fluctuations, one would expect Eq. (3) to describe the low-field data in Fig. 3, with L_ϕ as the only free parameter. We observe, however, that the noise does not quite drop a full factor of 2. A similar behavior is observed in Ag films,²² and apparently arises from a small contribution of local-interference-type noise²³ that is magnetic field independent. We therefore include a second parameter to account for this small effect. The function fitted to the data is

$$\frac{S_G(B)}{S_G(0)} = c + (1 - c)\nu(B). \quad (4)$$

The solid line in Fig. 3 shows the fit to the data for $B < 0.1 \text{ T}$. The fit yields $c = 0.1$ and 0.2 and $L_\phi = 1.0$ and $0.65 \mu\text{m}$ at 1.6 and 4.2 K, respectively. The latter are remarkably close to the values 1.3 and $0.75 \mu\text{m}$ obtained from magnetoresistance measurements.²⁴

As the magnetic field increases above 1 T, the noise power begins a second decrease. At 1.6 K the characteristic field scale for the second drop is $B_{c2} \approx 4 \text{ T}$, with saturation at about 7–8 T. Similarly, the noise at 4.2 K shows a gradual reduction with B_{c2} near 8.5 T. At this high field where the Cooper channel is completely suppressed, the magnetic field couples to the diffuson channel due to the Zeeman splitting ($E_Z = g\mu_B B$) of the electron spin state. Because the singlet and the $M_Z = 0$ triplet states are not sensitive to the Zeeman splitting, this second reduction arises from suppression of the contribution of the diffusons with $M_Z = \pm 1$. The Zeeman effect can be incorporated into UCF theory following the work of Stone for the 2D case;¹⁴ the result for 1D has also been obtained independently by Feng.²⁵ The solid line at high field in Fig. 3 is our numerical evaluation of the noise crossover function for the Zeeman effect, valid for $B \gg B_{c1}$. In this computation, we use our experimental values for D , $k_B T$, and L_ϕ and the free-electron value of $g = 2$. We emphasize that there are no free parameters in the theory for the Zeeman splitting; the constant c in Eq. (4) was already determined by the fit to the low-field data.

It is of interest to ask whether the Zeeman crossover occurs when E_Z exceeds the *Thouless energy*, $E_c = \hbar D/L_\phi^2$, or the *temperature*, $k_B T$.¹⁴ At 1.6 K, the value of E_c/k_B in our sample is about 59 mK, about 30 times smaller than the temperature; hence we can clearly distinguish which of these two energies governs the Zeeman crossover. In the former case, $B_{c2} \approx E_c/g\mu_B = 0.04 \text{ T}$, which is clearly incompatible with the data. (The uncertainty in our estimate of D due to uncertainty in the film thickness is not enough to account for the discrepancy.) In the latter case, $B_{c2} \approx k_B T/g\mu_B = 1.5 \text{ T}$, which is not far from the experimental value $B_{c2} \approx 4 \text{ T}$. When the correct numerical prefactor is put in, we indeed find the excellent agreement between theory and experiment shown in Fig. 3. This result holds only in the regime $k_B T \gg E_c$; numerical evaluation of the theoretical crossover function^{14,25} shows that it is always the larger of the two energies that governs the Zeeman crossover.

It is useful to reexamine the results of Debray *et al.*¹³ in light of our current understanding of the Zeeman crossover function. Those authors studied UCF using a static method in a quasi-1D GaAs/Al_xGa_{1-x}As heterostructure, and observed a value of $B_{c2} \sim 0.02$ T, using our definition of B_{c2} as the midway point of the second transition. The sample temperature was $T = 1.3$ K, and the authors estimate $E_c \approx 88$ mK; since $k_B T \gg E_c$, we would expect $B_{c2} \approx k_B T / g \mu_B$, which is much larger than the value observed. An important difference between those measurements and ours is that the GaAs experiments measure the saturated UCF amplitude, rather than unsaturated $1/f$ noise. But our numerical evaluation of the Zeeman crossover function for the saturated case also shows

that B_{c2} is determined by the larger of $k_B T$ and E_c . Since it is unlikely that the g factor in GaAs is ~ 100 times the free-electron value, we remain puzzled by this inconsistency.

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