

## Worksheet #5 - PHY102 (Spr. 2003)

### Solving Equations and Differential Equations II Due Monday Feb. 19 2001

Last week you learned about the following functions which are among the most important in Mathematica:

Solve - solves polynomial equations analytically  
NSolve - solves polynomial equations numerically  
FindRoot - solves all equations numerically  
DSolve - solves differential equations analytically  
NDSolve - solves differential equations numerically

Once you know what these routines do and how to use them, you have a very powerful set of tools for solving problems in physics. However the hardest part of physics is to set up the mathematical description of the problem, and that you still need to do that by hand.

#### Problem 1.

(i) A ball is falling vertically through a fluid. Apart from gravity, a drag force  $F_d$  acts on the ball. The drag force opposes the motion and increases in proportion to the speed ( $\vec{F}_d = -k\vec{v}$ , where  $k$  is a *drag coefficient* that depends on the fluid). Find and plot the time dependence of the position and velocity of a 100g ball which is released from rest at  $t = 0$ , in a fluid with drag coefficient  $k = 0.02$ . Choose a time range which shows the terminal velocity of the ball. Don't forget that velocity and position are vectors in general - you can define them as lists in Mathematica and make the equations vector equations. Refer to worksheet 2 for reference.

(ii) Now consider a ball which has an initial velocity  $v_x = 10m/s$  in the horizontal direction. Again find and plot the position and velocity of the particle.

(iii) Consider a cannon on a 500m high hill. Assuming that the cannon fires 10kg cannonballs horizontally with initial velocity 500m/s, find the range of the cannon as a function of the drag coefficient of the air (let the

drag coefficient vary from 0 to 1). This problem is difficult if we assume the drag force affects both the vertical *and* horizontal motion as it must in reality. We will learn later in the semester how to solve it properly using Mathematica. To make it more straightforward (though less physical) for now assume that the drag force acts only on the horizontal motion and there is no drag force acting on the vertical motion.

**Problem 2.**

You lift a box of mass  $30kg$  vertically a height of  $h$  meters. However you decide that you want your little brother to lift the other 30 boxes that must be lifted to the same height. Since he is weaker than you, you kindly put an inclined plane (at angle  $\theta$  to the horizontal) in place to assist him.

(i) If there is no friction, use mathematica to evaluate the integral  $\int \vec{F} \cdot \vec{dr}$  and hence prove that you and he do the same amount of work per box.

(ii) Now consider adding friction  $f = \mu xN$  which increases as the box moves up the plane. Here  $x$  is distance along the inclined plane,  $\mu$  is the friction coefficient and  $N$  is the force normal to the inclined plane. How much additional work does your brother do (per box) due to the friction on the inclined plane? (Solve this problem analytically using Mathematica).