

Worksheet #8 - PHY102 (Spring 2003)

Motion in a potential

Although you first learn about Newton's second law $\vec{F} = m\vec{a}$ and the dynamics that results from it, much of the discussion in the more advanced physics texts is in terms of "potentials" $V(\vec{r})$. A particle undergoes motion "in a potential". Note that V is a *scalar*, while \vec{F} is a vector. It is often easier to work with the potential unless you are forced to work with the force. Actually even motion in a potential is carried out using Newton's second law. However visualizing the potential is very helpful in developing physical insight into the trajectories. It is also useful in understanding thermodynamic processes, which are statistical in nature. Anyway for our purposes, we just need to know how to relate the the force to the potential, and that is via the equation:

$$\vec{F}(x, y, z) = -\left(\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z}\right) \quad (1)$$

Often it is easier to work in polar co-ordinates (r, θ, ϕ) . If we work with *central potentials*, $V(r)$ which do not depend on the angles (θ, ϕ) things are simple,

$$\vec{F}(r) = -\frac{\partial V}{\partial r} \hat{r} \quad \text{-- for central potentials.} \quad (2)$$

Almost all that you do in undergrad. physics (and most of postgrad. physics courses) is with central potentials.

This week we study motion in two different central potentials: The gravitational potential near a mass M :

$$V_G(r) = -\frac{GM}{r}. \quad (3)$$

The "Lennard-Jones" potential between two inert gas atoms:

$$V_{LJ}(r) = \frac{A}{r^{12}} - \frac{B}{r^6}. \quad (4)$$

The constants A and B depend on the inert gas (e.g. they are different for Helium than for Xenon).

Assignment 8. - Hand in by Monday Mar. 19

Problem 1.

- (i) Make a plot of the Lennard-Jones potential.
- (ii) Find the value, r_0 , at which the Lennard-Jones Potential is a minimum. Evaluate $V_{LJ}(r_0)$. What is the physical meaning of $V_{LJ}(r_0)$.
- (iii) By expanding around the minimum of the Lennard-Jones potential (use the “Series” function), show that, at low kinetic energies, two inert gas atoms undergo simple harmonic motion with respect to each other. For what kinetic energies would you expect this to be true (compare the kinetic energy with the “depth” of the potential well).

Problem 2.

- (i) Make a plot of the gravitational potential energy.
- (ii) Write a piece of Mathematica code to study the motion of a comet as it approaches the sun (ignore the planets in this calculation). Sun mass = $1.991 \cdot 10^{30}$ kg, Sun radius = $6.96 \cdot 10^8$ m, Assume that the ratio (mass of comet/mass of sun) \rightarrow zero. For a few initial conditions, plot out the trajectory of the comet as it passes by the sun (e.g. try to find initial conditions that lead to trapping of the comet). Find a set of initial conditions which makes the comet's orbit a circle.