Review and Examples (Spr. 2003)

Due Monday 16th April

These are examples of questions with the degree of difficulty that you will encounter in the lab-exam. Make sure that you can do them! The solutions have already been posted on the web so you can refer to the solutions as you do them. **Try and do them without referring to the solutions** then refer to the solutions to check that you got it right. If not, make sure that you understand what you did wrong.

Use Mathematica to solve all parts of the problems

Vectors

Problem 1.

Show that for any vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ that,

(i) $\vec{a}.(\vec{a} \wedge \vec{b}) = 0$

(ii)$\vec{a}.(\vec{b} \wedge \vec{c}) = (\vec{a} \wedge \vec{b}).\vec{c}$

Lists and Matrices

Problem 2. Display the following matrix in matrixform.

$$matrix = \{\{0.2, -0.4, 0.1\}, \{0.4, 0.2, -0.3\}, \{0.6, -0.3, -0.1\}\}$$

We haven't found the eigenvalues and vectors of symmetric matrices before; however, this illustrates the power of mathematica. Look up Eigenvalue in the help manual. You won't fully appreciate the power of this until you have to solve eigenvalue problems by hand in advanced mechanics and quantum mechanics.

Find the determinant, eigenvalues and eigenvectors of the matrix *matrix*. Show that the sum of the eigenvalues (= the trace of the matrix) and the product of the eigenvalues (= the product of the eigenvalues) are both real even though the eigenvalues themselves are complex.

Function definitions $f[x\_]$

Problem 3. Define the function $f(x) = 0.8x + x^2 - 3.2x^4$. Find the four roots, x_r, of $f(x)$ and check that one of the non-zero roots satisfies $f(x_r) = 0$.

Plotting

Problem 4. Define $x(t) = 10t, y(t) = 10t - 5t^2$. Plot $x(t)$, $y(t)$ and also do a parametric plot of $(x(t), y(t))$. What sort of motion is this ?

Ordinary differential equations

Problem 5. A mass(m= 1.1 kg)/spring(k = 2.2 N/m) system hangs vertically at equilibrium in Earth's gravity. It is displaced from equilirium by a small amount and then oscillates. It experiences a damping of $b\vec{v}$, where

$b = 0.1$. Consider a small amplitude initial displacement of $1cm$ and initial velocity of zero. Solve the linear differential equation for this problem. $(x''(t) + 0.1x'(t) + 2x(t) = 0)$. Plot $x(t)$.

Partial differential equations

Problem 6. Show that $\rho(x,t) = \frac{A}{t^{1/2}}Exp(-ax^2/t)$ solves the diffusion equation

$$\frac{\partial\rho(x,t)}{\partial t} = D\frac{\partial^2\rho(x,t)}{\partial x^2},$$

provided that D is related to a. What is that relationship?