1. **Mizar, the first binary star deduced from the spectrum.** Even though Mizar appears to be a single star, Pickering's spectrum showed it to be a binary star. You will need to refer to the figure in the in-class exercise of Nov. 9th. The speed of light is $3 \times 10^5$ km/s.
   a. (1 pt.) Draw the orbit of the binary star. Place the two stars on 1 October. Be certain to draw the location of the earth. (2 pts.) Why is there only a single spectral line on 1 and 23 October and two spectral lines on other days?
   b. (2 pts.) Why is the Hβ line of hydrogen not at its laboratory wavelength even on 1 October?
   c. (1 pt.) Find the speed of the center of the binary star system.
   d. (1 pt.) Find the orbital speed of one of the stars.

2. **Material at radius R rotates about the center of a galaxy at speed v.** The mass $M(R)$ of the galaxy within radius $R$ is $M(R) = 233 v^2 R M_{\odot}$ for $R$ expressed in pc and $v$ expressed in km/s.
   a. (2 pts.) What is the mass within 16 kpc of the galaxy?
   b. (2 pts.) What is the mass within 3 kpc of the center of the galaxy? The mass found in part (a) is larger than that in part (b). The extra mass is located in a spherical shell between radius 3 and 16 kpc.

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Fig. 3.—*Upper*, major axis heliocentric velocities on plane of sky, as a function of distance from the nucleus. *Lower*, minor axis velocities as a function of distance from the nucleus; note change in scale from upper plot. The steep velocity gradient in nuclear region along minor axis is prominent.

c. (2 pts.) What would be the mass within 160 kpc of the galaxy, if the rotation velocity is constant out to that distance?

d. (2 pts.) If, on the other hand, all of the mass is contained within 9 kpc, how fast would a satellite in a circular orbit at 180 kpc move?

3. **Dark matter in spiral galaxies.** In question 1, you made many calculations; this question asks you to think about the results of questions 1 and to draw some conclusions.

   a. (5 pts.) If dark matter did not exist, how would Figure 3 of the paper by Rubin, Thonnard, & Ford be different?

   b. (5 pts.) What is the evidence for dark matter?