Nuclear Properties

Thornton and Rex, Ch. 12

A pre-history

1896 Radioactivity discovered - Becquerel

□ rays + (Helium)
□ rays - (electrons)
□rays 0 (EM waves)

1902 Transmutation observed
- Rutherford and Soddy

1909 □ rays are Helium nuclei
- Rutherford and Royds

1912 Nucleus is shown to have very small radius (a few x 10^{-15} meters) at the center of the atom (a few x 10^{-11} meters).
- Rutherford (and Geiger and Marsden)

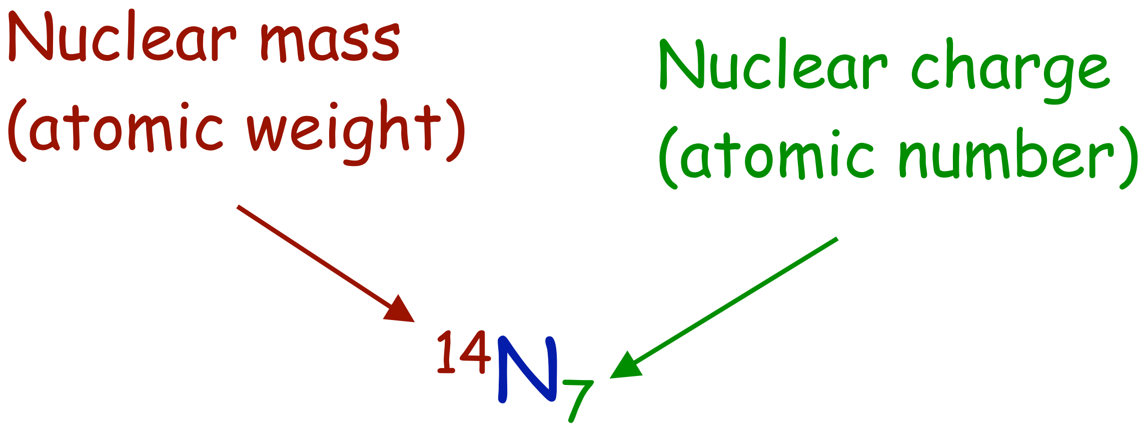
Artificial Transmutation

1919 - Rutherford succeeded in producing the first nuclear reaction in the laboratory.

By colliding Nitrogen with rays,
Rutherford succeeded in creating
Hydrogen and Oxygen.

We can write this reaction as:

$$^{14}N_7 + {}^{4}\square_2 \;\; \square \;\; {}^{1}p_1 + {}^{17}O_8$$



Nuclear mass
(atomic weight)

Nuclear charge
(atomic number)

$$^{14}N_7$$

Both numbers are "conserved".

A shorthand for this reaction:

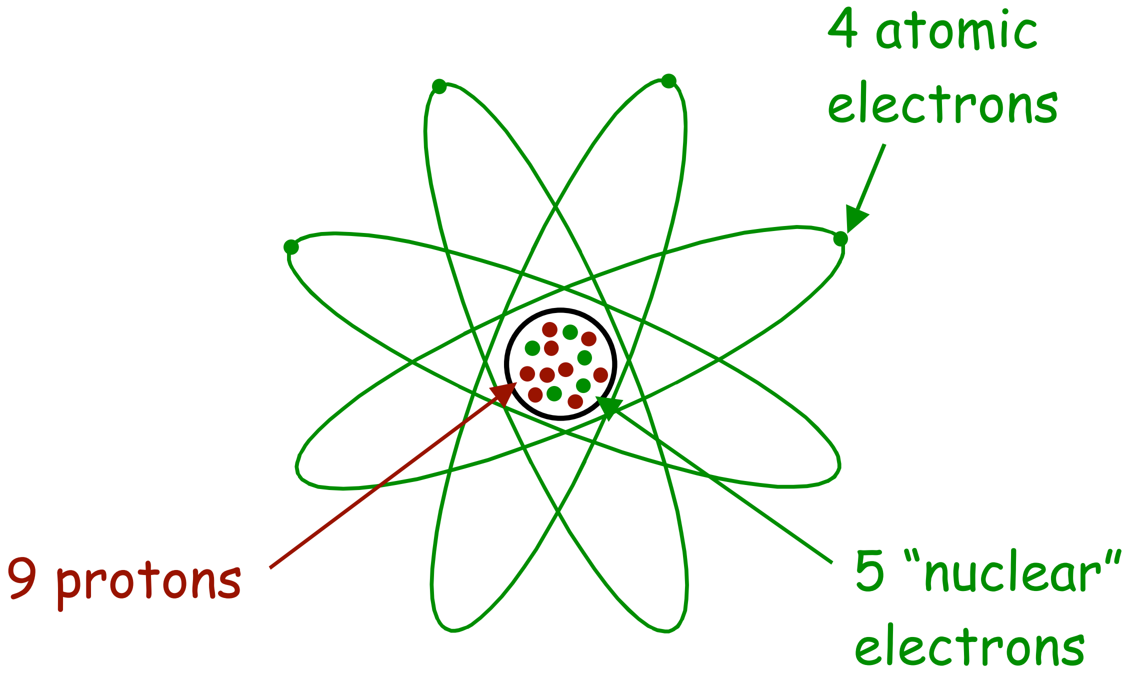
$$^{14}N_7\,(\square,p)\;{}^{17}O_8$$

The Structure of the Nucleus

An early theory:
nucleus composed of protons and electrons:
Protons to get mass right plus enough
electrons to get charge right.

Example: Beryllium has atomic weight 9 and atomic number 4.

□ 9 protons and 5 "nuclear" electrons.
(in addition to the 4 electrons orbiting around the nucleus).



Problems with this model:

- Uncertainty principle => nucleus would have too much energy
- Predicts incorrect spin for nuclei

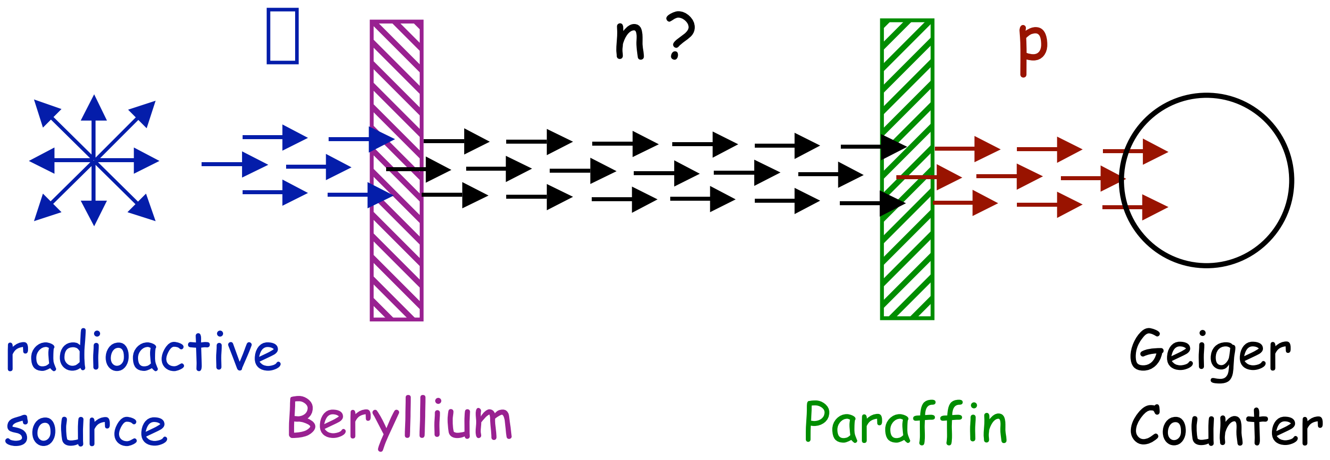
Rutherford suggested a new elementary particle in the nucleus, with roughly the same mass as the proton, and zero charge.

I.e., the "neutron".

The Discovery of the Neutron

1930 - Bothe and Becker discover penetrating new type of radiation while bombarding Beryllium with □ particles.

Curie and Joliot showed that when this new radiation struck a paraffin target (which contains Hydrogen nuclei) it knocked out high energy protons.



James Chadwick - Using measurements of energy and momentum, showed that radiation must be a new (uncharged) particle of about the same mass as a proton.

Chadwick named it the neutron (symbol n).

The reaction for creating it was

$$^{9}Be_{4} + {}^{4}\square_{2} \ \square \ {}^{1}n_{0} + {}^{12}C_{6}$$

or

$$^{9}Be_{4} \ (\square, n) \ {}^{12}C_{6}$$

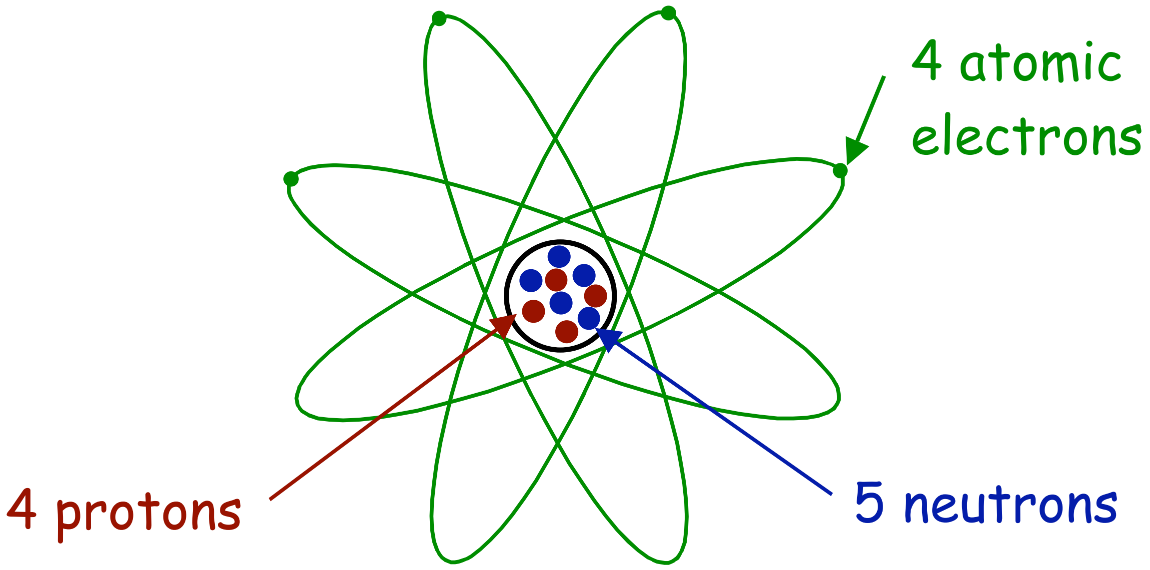
The generic name for a neutron or proton is a nucleon.

The Structure of the Nucleus

The modern picture of the nucleus:
protons and neutrons, but no electrons.

The charge is determined by the number of protons (same as number of electrons). The mass is the sum of the masses of the protons and neutrons.

Example, Beryllium (atomic weight 9,
atomic number 4)
▯ 4 protons and 5 neutrons in the nucleus,
4 electrons orbiting around it.



Properties of Elementary Particles (circa 1932)

particle	symbol	charge (C)	mass (kg)
electron	${}^{0}e_{-1}$	-1.6×10^{-19}	9.1×10^{-31}
proton	${}^{1}p_{1}$	$+1.6\times10^{-19}$	1.7×10^{-27}
neutron	${}^{1}n_{0}$	0	1.7×10^{-27}

We write

A = atomic weight = $N_{\text{protons}} + N_{\text{neutrons}}$

Z = atomic number = $N_{\text{protons}} = N_{\text{electrons}}$

N = neutron number = $N_{\text{neutrons}} = A - Z$

Some examples:

	A	Z	N
$^{1}H_{1}$	1	1	0
$^{4}He_{2}$	4	2	2
$^{12}C_{6}$	12	6	6
$^{56}Fe_{26}$	56	26	30
$^{238}U_{92}$	238	92	146

Isotopes

The "name" of the element (Hydrogen, Iron, Lead, etc.) is determined by the number of electrons = number of protons = Z. It determines the chemical properties of the element.

Some elements exist in forms with different numbers of neutrons. These are called isotopes of the element.

Example: 3 isotopes of Hydrogen:

		A	Z	N
Standard	${}^{1}H_{1}$	1	1	0
Deuterium	${}^{2}H_{1}$	2	1	1
Tritium	${}^{3}H_{1}$	3	1	2

Sizes of Nuclei

Nuclei can usually be approximated by spheres of radius R, where

$$R = r_0 \, A^{1/3}$$

and

$$r_0 = 1.2\times10^{-15}\ \mathrm{m}$$

(Note: Volume R^3 A)

The unit 10^{-15} m = 1 femtometer (fm), often called 1 fermi.

Shapes of Nuclei

One can describe the shape of a nucleus by its "charge distribution".

1950's - measured by Robert Hofstadter at Stanford using 500 MeV electrons ($\lambda_{\text{de Broglie}} \sim 2.5$ fm).

The distribution can be parametrized by

$$\rho(r) = \frac{\rho_0}{1 + e^{(r-R)/a}}$$

The Nuclear Force

The nuclear (or "strong") force is what binds the protons and neutrons together into nuclei.

Properties:

- It is attractive.
- Within the nucleus, it is about 100x stronger than the Electromagnetic force (and about 10^{38}x stronger than gravity).
- It is very short range.
  Outside of the nucleus, the EM force dominates, and the nucleus behaves just like a positive charge.

- It is charge-independent; that is, it acts the same on protons and neutrons.

- It is spin-dependent.

Binding Energy

The nuclear force binds nucleons (protons and neutrons) together. Work must be done to separate them.

Conversely, energy is released when nucleons join together to form a stable nucleus.

The energy difference between a nucleus and its separate constituent nucleons is called the Binding Energy.

$$\text{Binding Energy} = (Z\,m_p + N\,m_n - {}^{A}M_Z)\,c^2$$

Mass of
Bound nucleus

Example: Helium

$M(^1H) = 1.007825\ u$ $(^* = m_p + m_e)$

$m_n = 1.008665\ u$

$M(^4He_2) = 4.002603\ u$ $(^* = m_{\square} + 2m_e)$

Atomic Mass Unit: $u = 931.5\ MeV/c^2$

$$B.\ E. = (2\ M(^1H) + 2\ m_n - M(^4He_2))c^2$$

$$= 0.030377\ u\ c^2$$

$$= 28.30\ MeV$$

*These are actually the atomic masses of Hydrogen and Helium, which include the electron masses. Note that the electron masses cancel out of the formula.

Radioactive Decay

In any nuclear reaction, the following quantities are conserved:

1. Nucleon Number, *A*
2. Charge
3. Energy
4. Momentum

□ Decay

$$^{227}Th_{90} \;\square\; ^{223}Ra_{88} + {}^{4}\square_{2}$$

The masses of the two nuclei on the RHS add up to about 1.1 x 10^{-29} kg less than the mass on the LHS. The difference is made up in Kinetic Energy (using $E=mc^2$).

Most of the energy (about 6 MeV) is taken by the □ particle.

□ Decay

$^{24}\mathrm{Na}_{11}$ □ $^{24}\mathrm{Mg}_{12} + {}^{0}e_{-1}$

The electron takes away 1 unit of electric charge, increasing Z of the nucleus by 1. Atomic mass A is unchanged.

(It's as if a neutron changes into a proton and an electron.)

Positive □ Decay

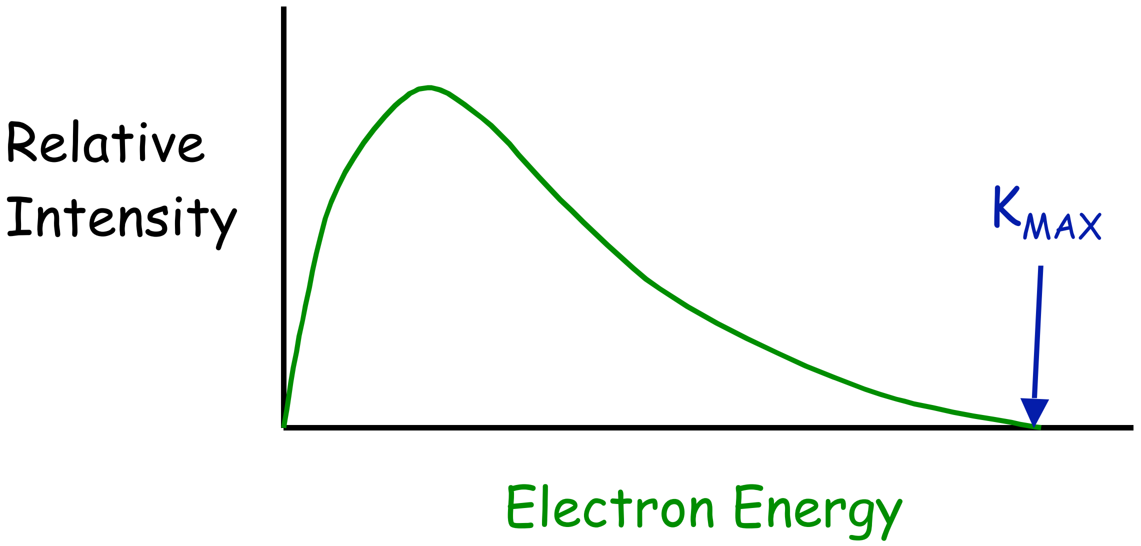
The anti-particle to the electron (the positron, predicted by the Dirac Equation) can also take part in □ decay.

$^{13}\mathrm{N}_{7}$ □ $^{13}C_{6} + {}^{0}e_{+1}$

(It's as if a proton changes into a neutron and a positron.)

For □ decay (positive or negative), one would expect the electron to come out with a single energy, just as for □ decay.

However, experiment showed a continuous spectrum of energies:



In 1930 Wolfgang Pauli solved this by suggesting that the electron energy was shared with a new particle, the neutrino.

The neutrino is chargeless and (almost) massless.

1956 - neutrino finally detected.

Difficult to detect because it does not interact through EM or strong force, only through the weak force.

1998 - evidence of a nonzero mass confirmed.

Its mass is of order 10^{-3} eV/c^2 or less (compared to an electron mass of $m_e = 5.11 \times 10^3$ eV/c^2).

We can now write the □ decays as:

$$^{24}Na_{11} \; \square \; ^{24}Mg_{12} + {}^{0}e_{-1} + \square$$

$$^{13}N_{7} \; \square \; ^{13}C_{6} + {}^{0}e_{+1} + \square$$

Electron Capture

$^{55}Fe_{26} + {}^{0}e_{-1}$ □ $^{55}Mn_{25}$ + □

For higher-Z nuclides, it is possible for electron capture to occur.

The effect is the same as for positive □ decay: a proton is converted to a neutron.

When electron capture occurs, the hole left by the captured inner electron will be filled by an outer electron which drops down, while emitting an X-ray of characteristic wavelength.

□Decay

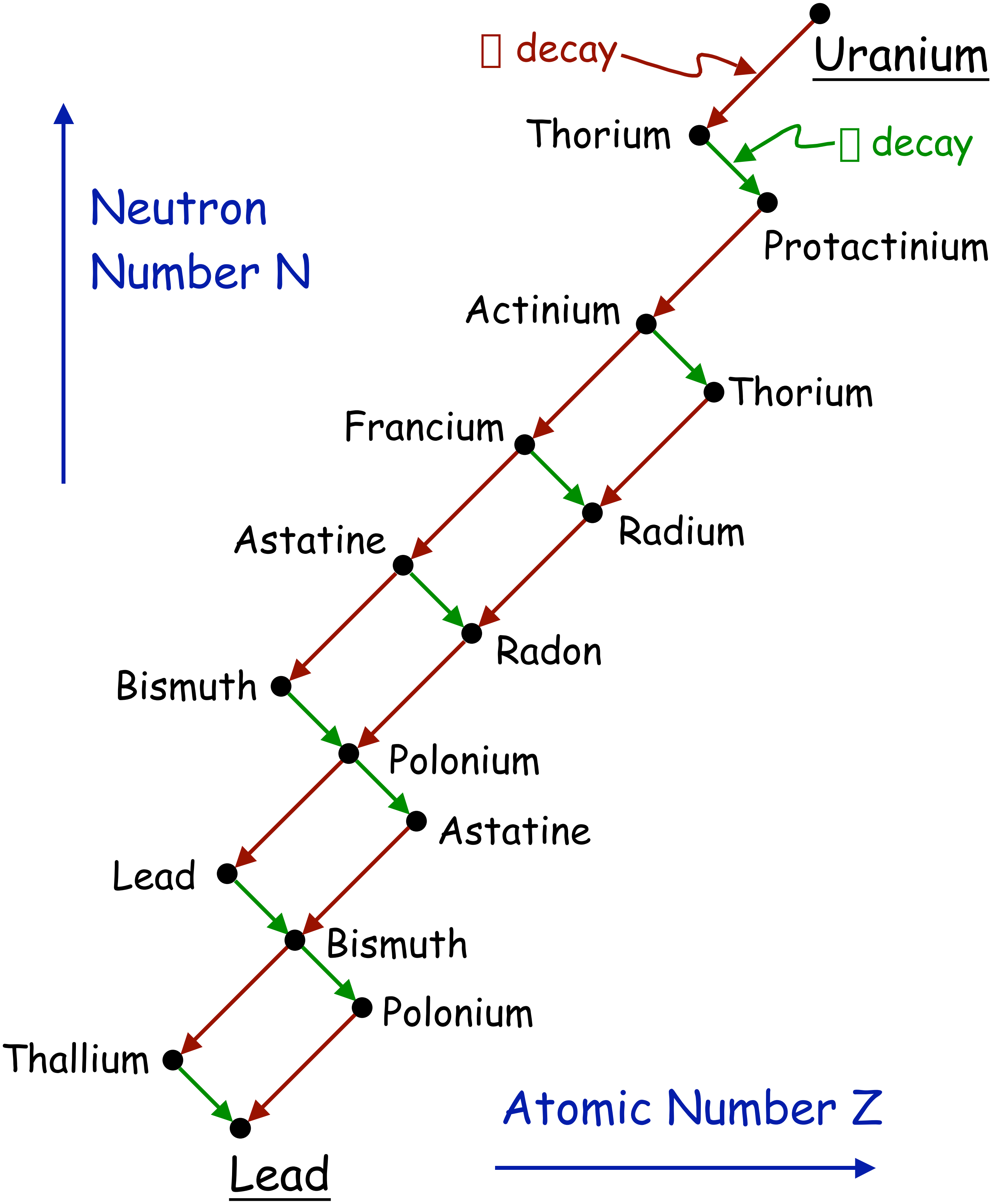
Just like an atom, a nucleus can also have excited states. Often, an □ or □ decay of a radioactive nucleus will leave the daughter nucleus in an excited state.

The excited nucleus will then decay to the ground state with the emission of a high energy photon (□ray).

$$^{225}\mathrm{Th}^{*}_{90} \;\square\; {}^{225}\mathrm{Th}_{90} + \square$$

Obviously, this leaves *A* and *Z* unchanged.

A radioactive decay "chain" (Segre chart)



Decay times

Radioactive decay is a probabilistic event.

For large numbers of nuclei, the number that decay in a short time will be proportional to the total number N and the time □t:

$$□N = - □ N □t$$

The solution to this differential equation is

$$\boxed{N(t) = N(0)\, e^{-□t}}$$

Activity

The Activity of a radioactive substance is defined as the number of decays per unit time:

Activity: $R = - dN/dt = \lambda N(t)$

The SI unit is the Becquerel:

1 Bq = 1 decay/second

An older, but still used, unit is the Curie:

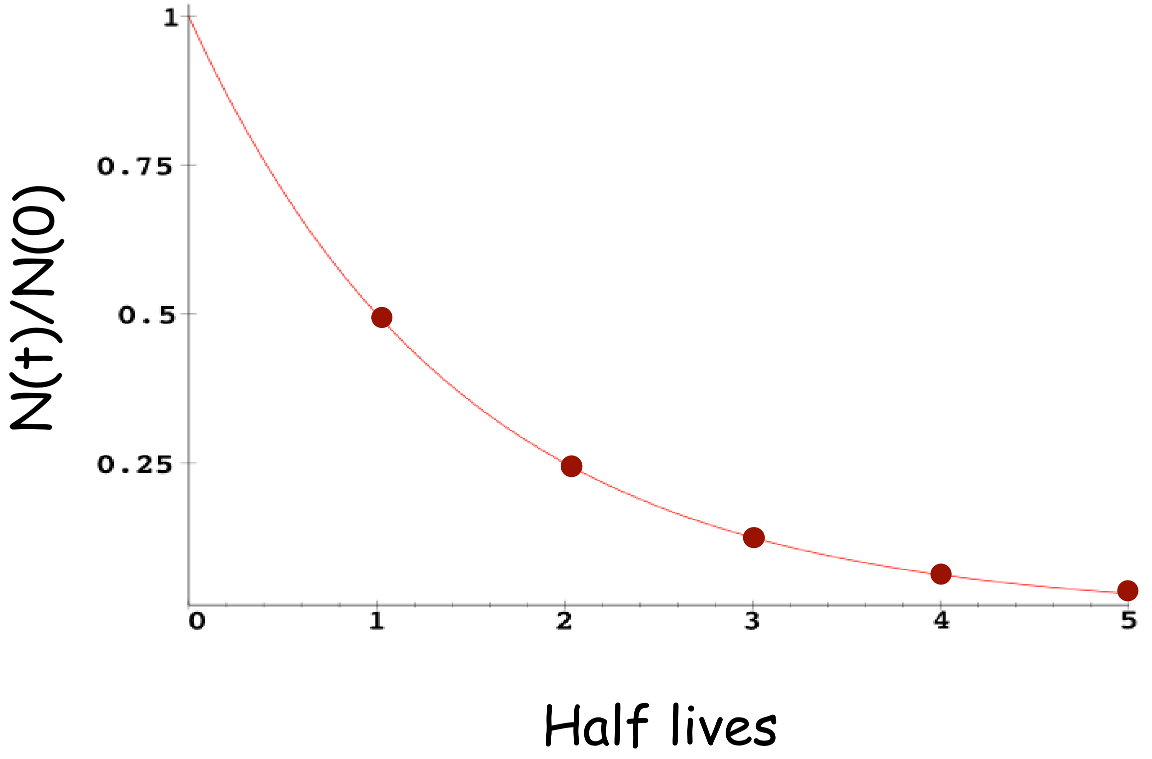
1 Ci = 3.7×10^{10} decays/second

The activity falls also falls off exponentially:

$R(t) = R(0)\, e^{-\lambda t}$

λ is the decay constant.

Half-life



The half-life ($t_{1/2}$) is the time for the number of radioactive nuclei to drop by a factor of 2. It is easy to show

$$t_{1/2} = \ln(2)/\square = 0.693/\square$$

Carbon Dating

Radioactive ^{14}C is continually produced in the atmosphere by bombardment of ^{14}N by neutrons produced by cosmic rays:

$$n + {}^{14}N \rightarrow {}^{14}C + p$$

There is a natural equilibrium ration of ^{14}C to ^{12}C of $R_0 = 1.2 \times 10^{-12}$. This ratio occurs in the carbon in the CO_2 taken up by living organisms.

But when a living organism dies, the ^{14}C decays and the ratio of $^{14}C/^{12}C$ decreases, allowing us to date the time of death of the organism by

$$R(t) = R_0 e^{-\lambda t}$$

Nuclear Reactions

Thornton and Rex, Ch. 13

Reaction Kinematics

Consider a general reaction,

$A\,(x\,,y)\,B$ or $A + x \rightarrow y + B$

with target A at rest.

Ex. ${}^{9}Be_{4} + {}^{4}\alpha_{2} \rightarrow {}^{1}n_{0} + {}^{12}C_{6}$

or equivalently,

$${}^{9}Be_{4}\,(\alpha,n)\,{}^{12}C_{6}$$

Conservation of energy gives:

$$M_A c^2 + m_x c^2 + K_x = m_y c^2 + K_y + M_B c^2 + K_B$$

The difference between final and initial kinetic energies is called the Q-value.

$$\begin{aligned} Q &= K_y + K_B - K_x \\ &= M_A c^2 + m_x c^2 - (m_y c^2 + M_B c^2\,) \end{aligned}$$

If Energy is released, $Q > 0$

▯ Exothermic

If Energy is converted to mass, $Q < 0$

▯ Endothermic

Two of the most important exothermic reactions are Fission and Fusion.

Neutron Activation

Neutrons - uncharged, can penetrate close to the nucleus, can induce reactions (Neutron Activation).

1930's - Enrico Fermi bombarded elements from Hydrogen to Uranium with neutrons. On Uranium, Fermi was unable to identify the final products of the reactions. He expected elements heavier than Uranium, such as in the expected process:

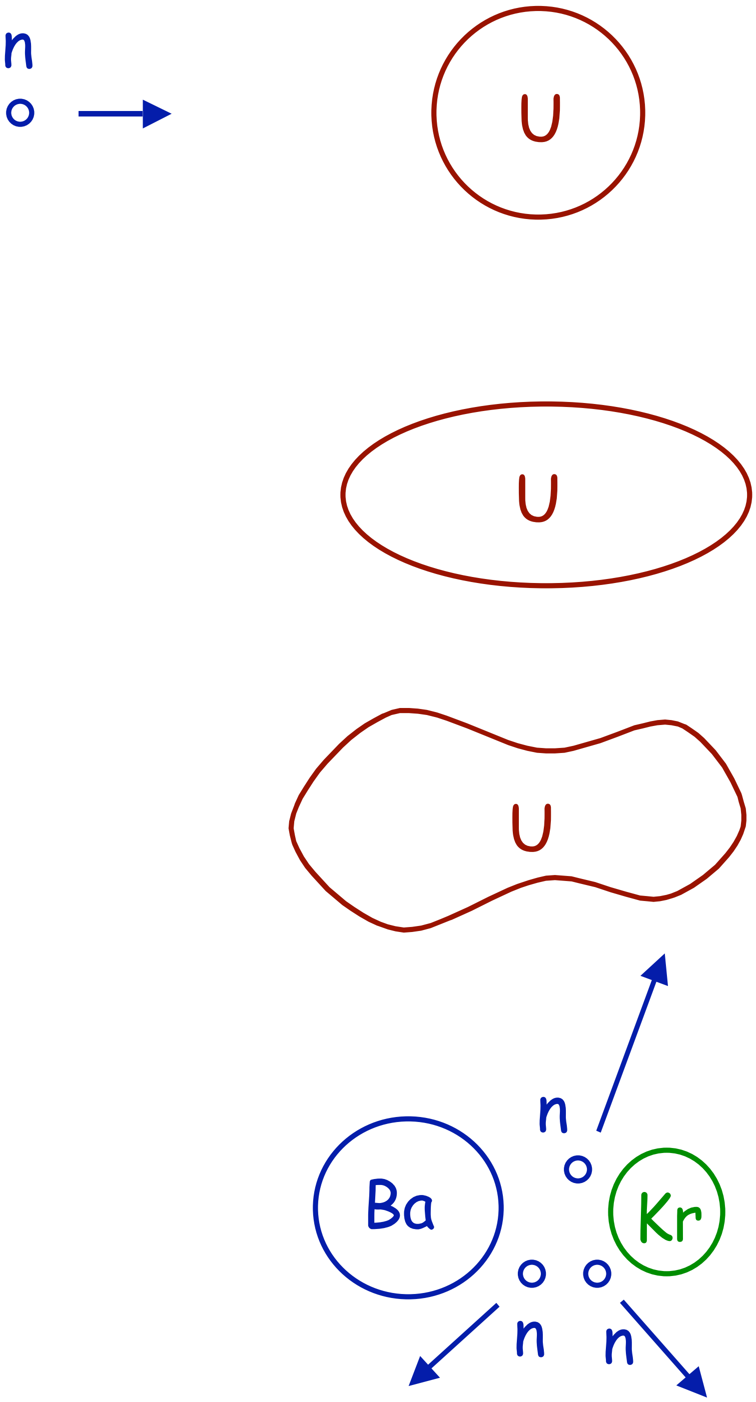
$$^{238}U_{92} + {}^{1}n_{0} \;\square\; {}^{239}U_{92} \;\square\; {}^{239}Np_{93} + {}^{0}e_{-1} + \square$$

Otto Hahn and the chemist Fritz Strassmann eventually identified one of the product nuclei as Barium ($^{141}Ba_{56}$), an element much lighter than Uranium.

1938 - Meitner and Frisch gave an interpretation of the reaction and coined the term Nuclear Fission.

They used a model (due to Bohr and others) in which the nucleus acts like a liquid drop with surface tension.

Fission



Comments on Fission

- The observed reaction is one of many possible reactions:

$$^{235}U_{92} + {}^{1}n_{0} \rightarrow {}^{141}Ba_{56} + {}^{92}Kr_{36} + 3\ {}^{1}n_{0}$$

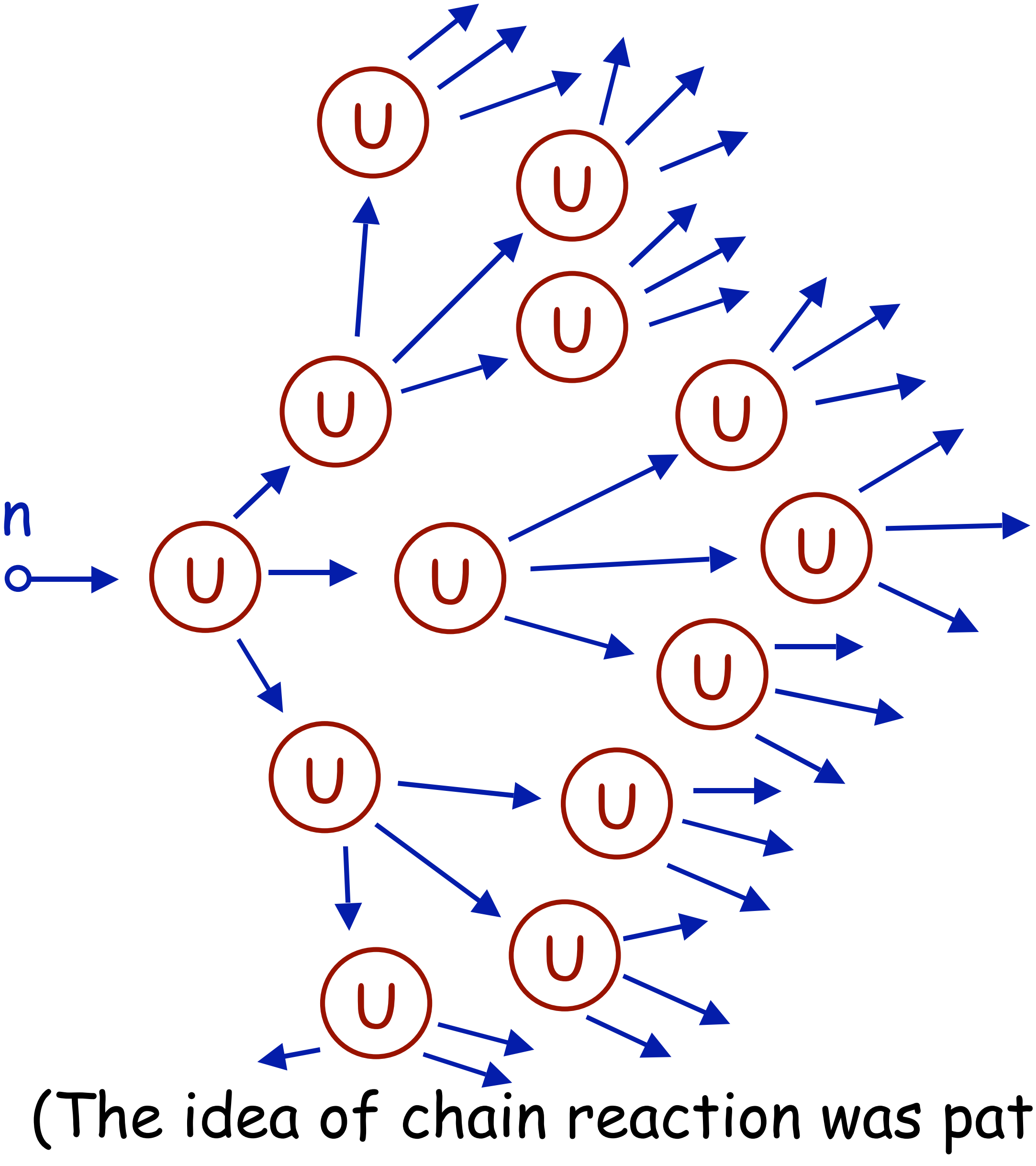
- $^{235}U_{92}$ undergoes fission. $^{238}U_{92}$ does not. Uranium ore contains 99.3% U-238 and only 0.7% U-235.

- Fission occurs more easily if the neutron is slow (allowing more time for the reaction to occur.)

- Q > 0, so energy released (~200 MeV).
(Cf. Binding Energy vs. Atomic Weight)

- Since heavier nuclei are more neutron rich, the fission process results in the release of extra neutrons.
(Cf. plot of N vs. Z)

- Under the right conditions, the extra neutrons could cause more U-235 to fission. This would release even more neutrons, etc., resulting in a chain reaction.



(The idea of chain reaction was patented by Leo Szilard in 1933, before he had any idea what nuclei might participate!)

Two technical problems that had to be solved in order to achieve a chain reaction in Uranium:

1) Fission occurs if U-235 captures a slow neutron, but the neutrons emitted in fission are fast.

   The device must contain a substance which slows the neutrons down. A "moderator" is an element whose nuclei don't absorb neutrons and which are relatively light so that in collisions with neutrons they will absorb energy and thus slow the neutrons down. Typical moderators are water, heavy water (D_2O), or Carbon (graphite).

2) The sample must be enriched with enough U-235 relative to U-238.

An additional technical problem must be overcome to achieve <u>controlled nuclear fission</u>, as for power generation.

A variable amount of an additional material that is highly efficient in capturing neutrons must be inserted in the Uranium. => Cadmium rods

The first self-sustaining nuclear reactor, using graphite as moderator, was built under the stands of the football stadium at the University of Chicago by Enrico Fermi on December 2, 1942.

Fusion

Combining smaller nuclei up to Fe increases stability, and releases energy. Also note sharp peak at ^{4}He.
(Cf. Binding Energy vs. Atomic Weight)

□ Source of energy in the sun.
The predominant process is the pp chain:

1) $^{1}H + {}^{1}H \;\square\; {}^{2}H + e^{+} + \square$

2) $^{2}H + {}^{1}H \;\square\; {}^{3}He + \square$

3) $^{3}He + {}^{3}He \;\square\; {}^{4}He + {}^{1}H + {}^{1}H$

4) $e^{+} + e^{-} \;\square\; \square + \square$

- Net result is conversion of 4 protons and 2 electrons into a ^{4}He nucleus and Q=26.7 MeV of energy.
  (reactions 1,2, and 4 occur twice)

- Most of the energy is in KE of products and in the energy of the photons, and a small amount in the neutrino.

- Photons take millions of years to escape the center of the sun, the neutrino escapes immediately.

- The slowest reaction in the chain is step 1, which keeps the sun burning for a long time (~10 billion years). After that it will collapse further, heat up and begin to build heavier elements, up to Fe.

- All elements on earth (up to Fe) were originally built in earlier generations of stars.

- There are other chains, including the CNO cycle, which uses Carbon to catalyse Fusion. It is more important in massive stars.

Fusion on earth

A hope for virtually unlimited source of clean energy in Fusion reactions, such as:

$$^{2}H + {}^{3}H \rightarrow {}^{4}He + {}^{1}n$$

which releases 17.6 MeV of energy.

The difficulty of controlled fusion is the large energies required by the nuclei to overcome the Coulomb force and react.

The main effort has been to use a hot ionized gas called a plasma. The nuclei are kept at high energy and compressed together by large electric and magnetic fields. The magnets are usually in the form of a toroid (donut), called a Tokamak.

At present no reactor has yet produced more energy from fusion than was required to produce and contain the plasma.

Another scheme is to use powerful lasers to irradiate tiny pellets of deuterium (^{2}H) and tritium (^{3}H) from all sides, resulting in their vaporization and compression, again producing fusion.

Estimates for commercial exploitation of fusion reactors range from 30 to 50 years.

Accelerators

Other nuclear reactions are studied by collisions of fast moving particles on a nuclear target.

Accelerators use electric and magnetic fields to give energy to the charged particles.

One type of accelerator is the cyclotron, which is used at the National Superconducting Cyclotron Laboratory (NSCL) here at MSU. Giant magnets contain the ion beams in circles, while electric fields are applied in phase to accelerate the beam.

NSCL coupled cyclotron

- Ions stripped from heavy nuclei
- K500 accelerates up to 20 MeV/nucleon (B=3-5 T)
- Carbon foil strips more ions (2.5 x charge)
- K1200 accelerates up to 200 MeV/nucleon
- Beam hits metal target, making unstable isotopes
- A1900 fragment separator selects particular isotopes
- Isotopes hit target and are studied in various detectors.

- Properties of unstable isotopes are studied. What is origin of elements beyond Iron? (thought to be made in supernova explosion through neutron capture processes).

- General properties of nuclei, such as:

Compound Nucleus:
(excited nuclear resonance in reaction)

$p + {}^{15}N \;\square\; {}^{16}O^{*} \;\square\; p + {}^{15}N$