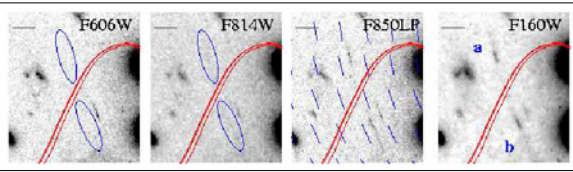
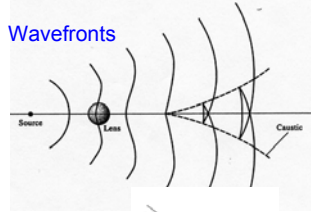


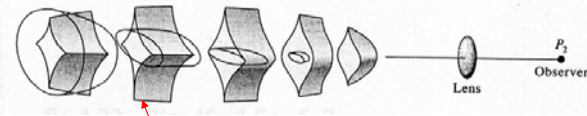
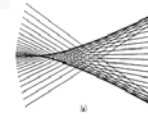
What are "Critical Curves" (the red lines)?



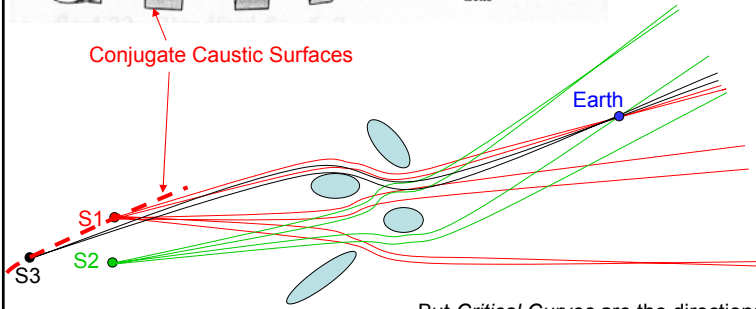
Wavefronts



Rays



Conjugate Caustic Surfaces



But *Critical Curves* are the directions we would need to look from Earth in order to see lensed images of objects which lie on Conjugate Caustic Surfaces.

Closed Box Models

(and friends and relatives)

Metallicity
 $Z = M/G$
 $Z_{\odot} \sim 0.02$

Gas \rightarrow stars \rightarrow enriched gas

S = mass of stars

M = mass of metals (heavy elements) in ISM

G = total mass of gas in ISM

Assume instantaneous recycling from massive stars.

From a new generation of stars:

dS = mass of low mass stars added to S

$p dS$ = mass of heavy elements added to M from massive stars in this generation.

where p = yield.

$dM = p dS - Z dS$

$= -p dG + Z dG$ since $dG = -dS$

But $dZ = d(M/G) = (1/G) dM - (M/G^2) dG$
 $= -p (dG/G)$

$Z(t) = -p \ln [G(t)/G(0)]$ or $G(t) = G(0) e^{-Z(t)/p}$

Also... Leaky box (gas driven out by stars).
 Accreting box models.

G dwarf problem

$$S[Z < Z(t)] = S(t) = G(0) - G(t)$$

$$= G(0) \{ 1 - e^{-Z(t)/p} \}$$

$Z(t)$ = gas metallicity at time t

Compare to case when gas had some arbitrary fraction α of that metallicity:

$$\frac{S[Z < \alpha Z(t)]}{S[Z < Z(t)]} = \frac{1 - X^\alpha}{1 - X}$$

where $X = \frac{G(t)}{G(0)} \sim 0.1 - 0.2$

Predicts broad distribution in metallicity of stars.

$$\rightarrow S[Z < 1/4 Z_{\odot}] = 0.4 S[Z < Z_{\odot}]$$

Very different than what is observed in solar neighborhood:

$$S[Z < 1/4 Z_{\odot}] = 0.02 S[Z < Z_{\odot}]$$

Closed Box Models (and friends and relatives)

Gas → stars → enriched gas

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$$= -p dG + Z dG \quad \text{since } dG = -dS$$

But
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$$= -p (dG/G)$$

$$Z(t) = -p \ln [G(t)/G(0)] \quad \text{or} \quad G(t) = G(0) e^{-Z(t)/p}$$

Also... Leaky box (gas driven out by stars),
 Accreting box models.

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 $Z = M/G$
 $Z_{\odot} \sim 0.02$

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$$S[Z < Z(t)] = S(t) = G(0) - G(t)$$

$$= G(0) \{ 1 - e^{-Z(t)/p} \}$$

$Z(t)$ = gas metallicity at time t

Compare to case when gas had some arbitrary function α of that metallicity:

$$= \frac{1 - X^{\alpha}}{1 - X}$$

$$T = \text{total mass} \quad - 0.2$$

distribution in stars.

$$= 0.4 S[Z < Z_{\odot}]$$

Very different than what is observed in solar neighborhood:

$$S[Z < 1/4 Z_{\odot}] = 0.02 S[Z < Z_{\odot}]$$

Simple Accreting Box Model
 Feed gas in at same rate it turns into stars

$$dG = -dS + dT$$

$$\frac{dS}{dT} = 1 - \frac{dG}{dT}$$

$$\frac{dZ}{dT} = \frac{1}{G} \left[p - Z - p \frac{dG}{dT} \right]$$

Accreting box models.

The Initial Mass Function (IMF)

• $dN = N_{\odot} \xi(M) dM$ = number of stars born with masses in range $M, M+dM$

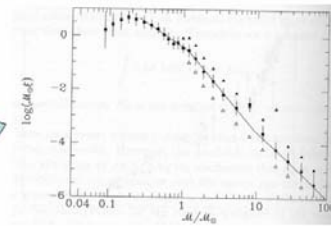
• Salpeter (1955) IMF: $\xi(M) \propto M^{-2.35}$

• Scalo (1986) IMF:

$$\xi(M) \propto M^{-2.45} \quad \text{for } M > 10M_{\odot}$$

$$\xi(M) \propto M^{-3.27} \quad \text{for } 1 < M < 10M_{\odot}$$

$$\xi(M) \propto M^{-1.83} \quad \text{for } 0.2 < M < 1M_{\odot}$$



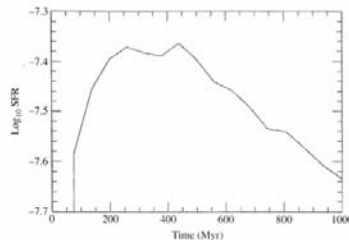
• Others as well.

• Star Formation rate = $\psi(t)$

• Stellar birthrate function

$$B(M, t) = \psi(t) \xi(M) dM dt$$

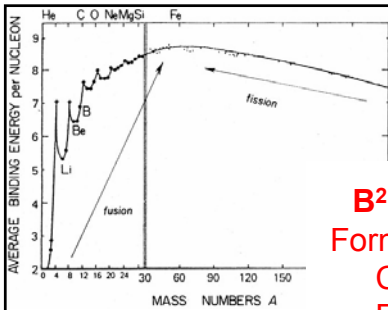
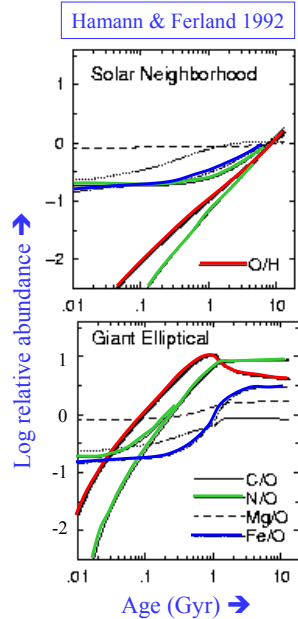
= number of stars born per unit volume with masses in range $M, M+dM$ in time interval $t, t+dt$. [CO eqn. 26.4]



[CO Fig. 26.18]

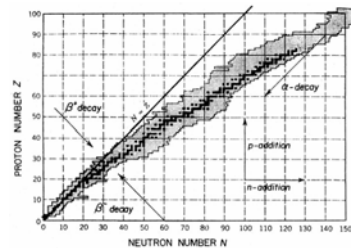
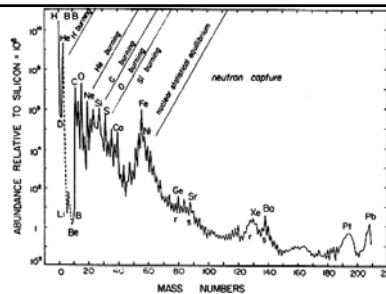
Modeling chemical enrichment

- One zone, accreting box model.
 - Start with pure H, He mix.
 - Further H, He falls in at specified rate.
- Follow evolution of individual elements H, He, C, N, O, Ne, Mg, Si, S, Ar, Ca and Fe.
- Subdivide stellar population into three classes of stars:
 - $< 1M_{\odot}$ nothing recycled
 - $1.0 - 8.0 M_{\odot}$ fraction give Type Ia supernovae
 - $> 8M_{\odot}$ Type Ib, Ic or II supernovae.
- Assume that each class of stars spews specified % of its mass back into ISM in the form of each element at end of a specified lifetime.
- Must provide IMF to specify mix of star masses.
- Two extreme models:
 - “Solar neighborhood”: conventional IMF, slow stellar birthrate, slow infall (15% gas at 10 Gyr).
 - “Giant Elliptical”: flatter IMF, 100x higher birthrate, fast infall (15% gas at 0.5 Gyr).



B²FH (1957) Formation of the Chemical Elements

Burbidge, Burbidge, Fowler & Hoyle.
Reviews of Modern Physics, 29, 547.

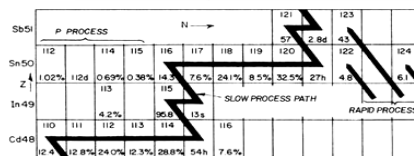


Gradual processes in Interiors of Stars

- H burning ($4H \rightarrow He$)
- α process (C, O, Ne, Mg, Si, S...)
- s process
 - slow neutron capture, relative to beta-decay timescale

Supernovae

- e process
 - nuclear statistical equilibrium
 - iron peak elements
- r process
 - rapid neutron capture



The First Stars

- **Population III**
 - Metallicity $Z = 0$
 - Expected to be very massive →
 - short-lived
 - lots of UV photons → reionization of IGM
 - supernovae/hypernovae → 1st round of chemical enrichment of ISM
 - Gamma ray bursts?
 - No low-mass survivors found yet
- **The Metal-Poorest Pop II stars**
 - Down to $[\text{Fe}/\text{H}] = -5.4$ (Frebel et al. 2005, *Nature*, 434, 871)
 - 3 stars with $[\text{Fe}/\text{H}] < -4.0$ (Norris et al. 2007, *ApJ*, 670, 774)
 - Aim is to trace details of element synthesis from Pop III and very first Pop II stars
 - Heavy-element abundance pattern in observed star may come from single Pop III supernova/hypernova event.
 - Sensitive tracer of mass of Pop III star
 - Abundance patterns change from star to star due to as yet unmixed space distribution of Pop III and/or first Pop II predecessors.