PHY251 Fall 2008 Practical Lab #2
The Pendulum

Objectives

- To investigate the functional dependence of the period (τ) of a pendulum on its length (L). The Greek letter tau (τ) is typically used to denote a time period or time interval.
- Use a pendulum to measure the acceleration due to gravity (g).

Apparatus

Point masses and string, a digital timer, period gate, and meter stick will be used.

Theory

Where there exists a constant net force (F), Newton's Law F = ma tells us that the acceleration (a) is a constant and therefore the position of the object can be written as

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

This is the formula we tested in the Free Fall I lab.

In the analysis of the motion of a pendulum we should realize that

1) The motion is part of a circle so angular acceleration (α) is a useful variable
2) The angular acceleration will not be a constant throughout the motion

Consider the pendulum shown in Figure 1. The acceleration of the bob **tangent** to the arc "drawn" by the pendulum as it swings, a_t, is determined by $\mathbf{F_t}$, the force tangent to the arc. Since the tension in the string (T) always acts along the **radius**, it doesn't contribute to $\mathbf{F_t}$. Decomposing the gravitational force (mg) into components perpendicular and parallel to the string as shown in the diagram below, we find that

$$F_t = mg\sin\theta$$

Therefore the acceleration tangent to the circle is given by:

$$a_t = \frac{F_t}{m} = g\sin\theta$$

The angular acceleration α is then found by the relationship for circular motion

$$\alpha = -\frac{a_t}{r} = -\frac{g}{L}\sin\theta$$

Thus, as we have suggested, <u>the angular acceleration α is not a constant</u> but varies as the sine of the displacement angle of the pendulum.

For small angles (about θ < 0.5 radian) angular accelerations can be shown (with a little calculus which we will skip) to lead to an oscillation of the angle θ by

$$\theta = \theta_0 \cos \frac{2\pi \quad t}{\tau}$$

where θ_0 is the angle at time t = 0 (when we release the pendulum), and τ is the period of the motion. The period is the time it takes to complete one full cycle of the motion.

The period of a simple pendulum is given by:

$$\tau = 2\pi\sqrt{\frac{L}{g}} \qquad \text{or} \qquad \tau = \frac{2\pi}{\sqrt{g}}\sqrt{L}$$

This equation has the same form as the equation of a straight line y = mx + b, with an intercept of zero (i.e. b = 0). Notice in this equation, the period (τ) corresponds to y and $\sqrt{L}$ corresponds to x.

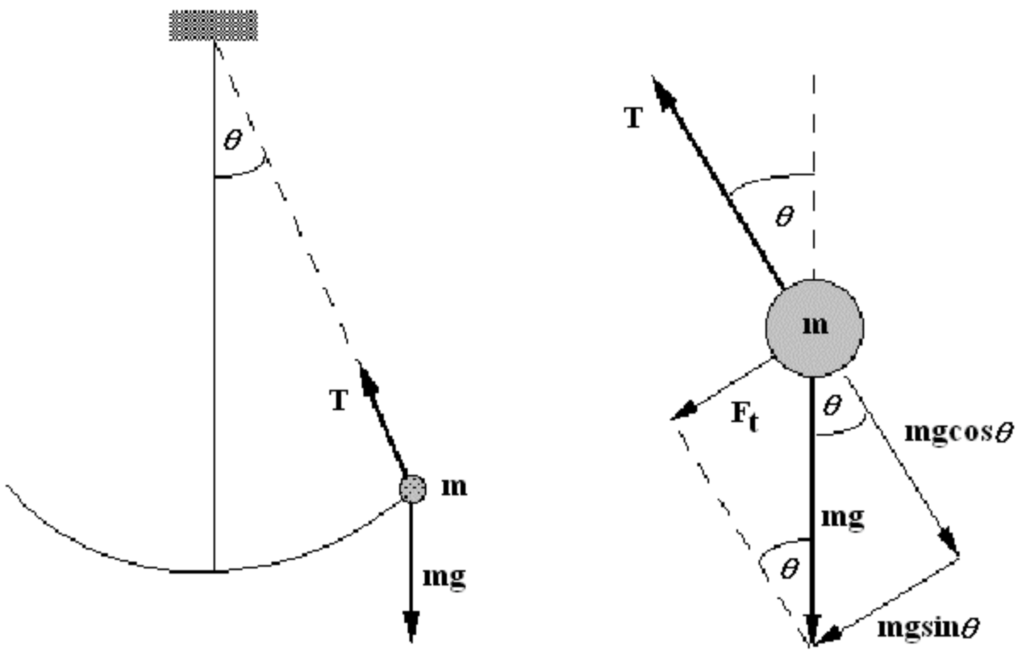

Figure 1

Procedure

The parameter of the system you will vary is the length of the pendulum (L) from the support to the center of mass of the "point mass". The quantity you measure is the period τ.

You will use the PEND setting of the gate and measure the period several times for the lengths of the pendulum specified on your lab worksheet (you will get the worksheet when you do this practical lab). The PEND setting of the gate uses light and a photodetector in the following way: when the light beam is interrupted the first time the timer is started, the timer continues to count when the beam is interrupted a second time but stops on the third interruption of the beam. This measures the time taken for one complete oscillation of the pendulum, or in other words the period.

1. Adjust the length of the pendulum to the first given length and assign a reasonable uncertainty to this length.
2. With the timer in PEND mode, release the pendulum from a small starting angle (i.e. less than 30 degrees from the vertical) and measure the period of the pendulum. Enter this measured value in your spreadsheet.
3. Use Excel to calculate $\sqrt{L}$ and its uncertainty $\delta\left(\sqrt{L}\right) = \frac{\delta L}{2\sqrt{L}}$. The Excel formula for the square root is "=SQRT(CELL#)".
4. Repeat steps 1 through 3 for the other specified lengths.
5. Transfer your data into Kaleidagraph and construct a graph of τ vs $\sqrt{L}$. Include horizontal error bars, the equation of the best fit line and the uncertainties in the slope and intercept of your best fit line.

Questions

1) What is the expected value of the intercept? Does the intercept of your graph agree with this expected value? Justify your response.
2) Use the slope of the graph of τ vs. $\sqrt{L}$ to calculate g and its uncertainty. $\delta g = 2g\frac{\delta(slope)}{slope}$ Show your work.
3) Is your value of g consistent with 980 cm/sec^2? Justify.
4) If the mass at the end of the string is doubled, what will happen to the period of the pendulum? Explain your response.

<u>CHECKLIST</u>

1) the spreadsheet with your data and formula view of your spreadsheet
2) graph with best-fit line and equation of best-fit line and uncertainties
3) answers to the questions
4) other than specified in the questions, NO sample calculations are required

USING UNCERTAINTIES TO COMPARE DATA AND EXPECTATIONS

One important question is whether your results agree with what is expected. Let's denote the result by *r* and the expected value by *e*. The ideal situation would be $r = e$ or $r - e = 0$. We often use Δ (pronounced "Delta") to denote the difference between two quantities:

$$\boldsymbol{\Delta = r - e} \qquad \textbf{(1)}$$

The standard form for comparison is always *result - expected*, so that your difference Δ will be negative if your value is lower than expected, and positive if it is higher than expected.

This comparison must take into account the uncertainty in the observation, and perhaps, in the expected value as well. The data value is $r \pm \delta r$ and the expected value is $e \pm \delta e$. Using the addition/subtraction rule for uncertainties, the uncertainty in $\Delta = r - e$ is just

$$\boxed{\delta\Delta = \delta r + \delta e} \qquad (2)$$

Our comparison becomes, "is zero within the uncertainties of the difference Δ?" Which is the same thing as asking if

$$\boxed{|\Delta| \leq \delta\Delta} \qquad (3)$$

Equation (2) and (3) express in algebra the statement "*r* and *e* are compatible if their error bars touch or overlap." The combined length of the error bars is given by (2). $|\Delta|$ is magnitude of the separation of *r* and *e* . The error bars will overlap (or touch) if r and *e* are separated by less than (or equal to) the combined length of their error bars, which is what (3) says.