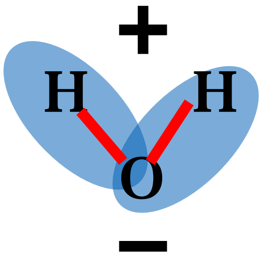
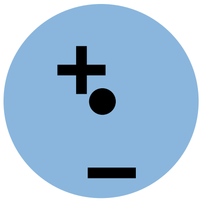
The Polarization Field

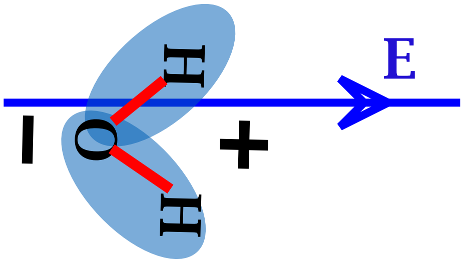
All matter is atomic.
Atoms (or molecules) contain electrically charged particles.
That leads to *polarization.*

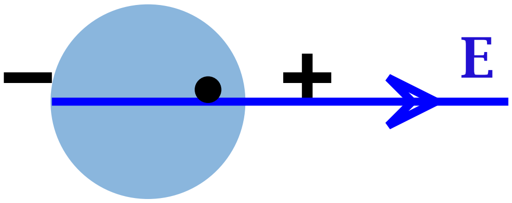




polar molecule; atom;

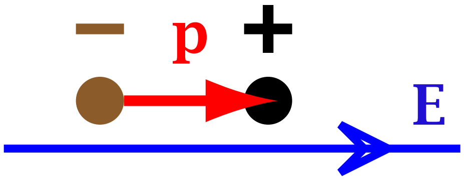
aligned dipole induced dipole





Dipole Moment $\mathbf{p} = e\,\delta$

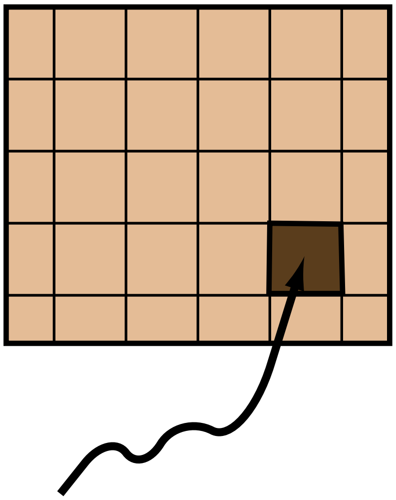
The dipole moment aligns with the local field,



$$[\mathbf{p}]_{\mathrm{av}} = \alpha\,\mathbf{E}(\mathbf{x})$$

The Polarization Field

Consider a sample of dielectric matter.
Subdivide the sample into many ***small*** subvolumes.
But atoms are so tiny that each subvolume contains ***many*** atoms.



subvolume at $\boldsymbol{x}$

$$\mathbf{P}(\mathbf{x}) = \frac{1}{\delta V} \sum_{i=1}^{\delta N} \boldsymbol{p_i} = \frac{\delta N}{\delta V} [\mathbf{p}]_{av}$$

$\mathbf{P}(\mathbf{x})$ = “ moment density ”

The critical theorem for dielectrics

$$\mathrm{div}\, \mathbf{P} = -\rho_{\mathrm{bound}}(\mathbf{x})$$

Free charge and Bound charge

$$\rho(\mathbf{x}) = \rho_{\mathrm{free}}(\mathbf{x}) + \rho_{\mathrm{bound}}(\mathbf{x})$$

external charge in the electrostatic system

charge in the atoms of the dielectric

The fundamental equation

$$\boxed{\mathrm{div}\, \mathbf{E} = \rho / \varepsilon_0}$$

But it’s easier if we separate free charge and bound charge.

Theorem

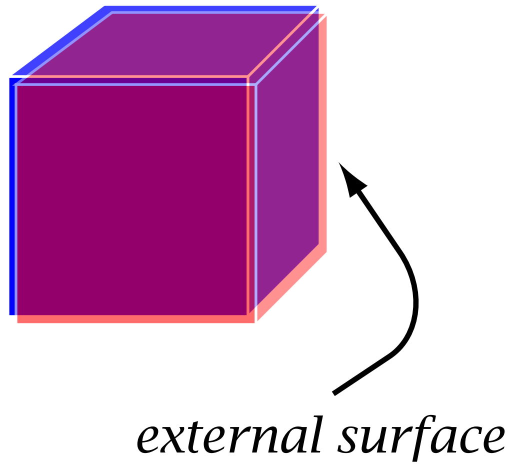
$$\rho_b(\mathbf{x}) = -\operatorname{div}\mathbf{P} \qquad \text{(\textit{inside the dielectric})}$$

$$\sigma_b(\mathbf{x}) = \mathbf{n}\cdot\mathbf{P} \qquad \text{(\textit{on the surface of the dielectric})}$$

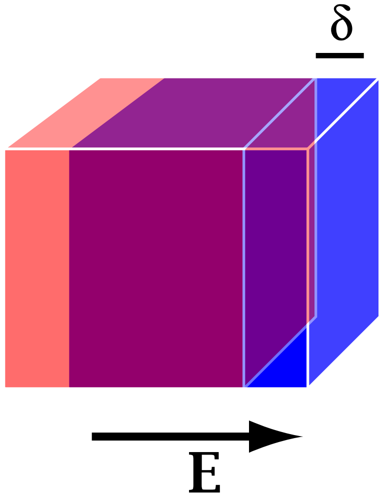
Proof

surface charge

unpolarized



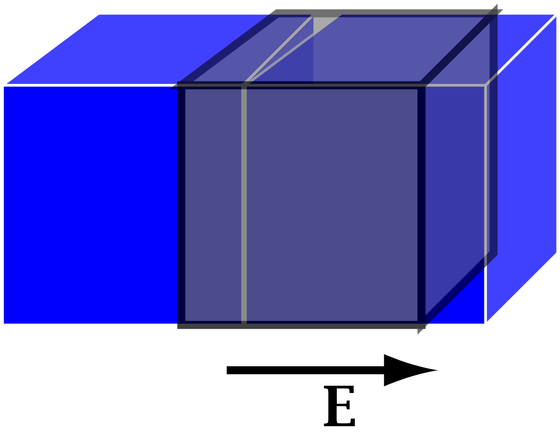
polarized;
moment = $p = e\,\delta$



charge = $N\,e = n\,(A\,\delta)\,e$
charge/A = $n\,p = P_x$
Q. E. D.

bound charge density

a small internal subvolume



charge in the subvolume

$$= -\,n_{(\text{right})}\,(A\delta)_{\text{right}}\,e + n_{(\text{left})}\,(A\delta)_{\text{left}}\,e$$

$$= -\int \mathbf{P}\,.\,d\mathbf{A}$$

$$= -\int \operatorname{div}\mathbf{P}\ \ dV$$

(by Gauss's theorem)

so

$$\rho_b = -\operatorname{div}\mathbf{P}$$

Q. E. D.