Physics 471
More in-class Discussion Questions

I am grateful to Michael Dubson of the University of Colorado for the vast majority of these questions.

QM1-52. Consider three functions $f(x)$, $g(y)$, and $h(z)$. $f(x)$ is a function of x only, $g(y)$ is a function of y only, and $h(z)$ is a function of z only. They obey the equation $f(x) + g(y) + h(z) = C = \text{constant}$. What can you say about f, g, and h?

A) f, g, and h must all be constants.
B) One of f, g, and h, must be a constant. The other two can be functions of their respective variables.
C) Two of f, g, and h must be constants. The remaining function can be a function of its variable.

Answer A.

QM1-53. For the particle in a 3D box, is the state $(n_x, n_y, n_z) = (1, 0, 1)$ allowed? A) Yes B) No

Answer: B. The value $n_y=0$ will result in $\Psi(x, y, z) = 0$.

QM1-54. The ground state energy of the particle in a 3D box is $(1^2 + 1^2 + 1^2)\dfrac{\hbar^2\pi^2}{2ma^2} = 1\varepsilon$. What is the energy of the 1st excited state?

A) 2ε B) 3ε C) 4ε D) 5ε E) 6ε

Answer: A. (If ε were defined in the logical way, the answer would be E. But check the definition.)

QM1-55. What is the degeneracy of the state $(n_x, n_y, n_z) = (1, 2, 3)$?

A) 1 B) 3 C) 4 D) 6 E) 9

Answer: D.

QM1-56. Is the 3D wavefunction

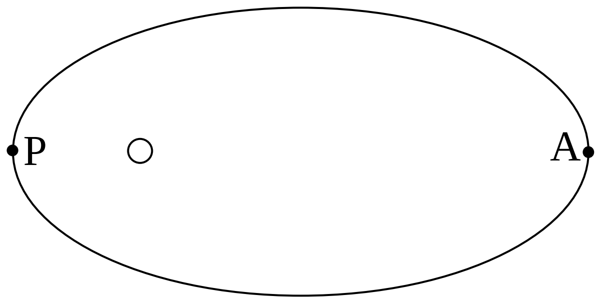
$$\psi(x, y, z) = \left(\frac{2}{a}\right)^{3/2} \sin\left(\frac{n_x \pi x}{a}\right) \sin\left(\frac{n_y \pi y}{a}\right) \sin\left(\frac{n_z \pi z}{a}\right)$$

an eigenfunction of $\hat{H}_x = \dfrac{-\hbar^2}{2m}\dfrac{\partial^2}{\partial x^2}$?

A) Yes B) No

Answer: A.

QM1-57. A planet is in elliptical orbit about the sun.



The torque $\vec{\tau} = \vec{r} \times \vec{F}$ on the planet about the sun is
A) zero always. B) Non-zero always
C) zero at some points, non-zero at others

Answer: A. The force vector is antiparallel to the position vector.

QM1-58. The magnitude of the angular momentum of the planet about the sun $\vec{L} = \vec{r} \times \vec{p}$ is

A) greatest at perihelion (point P)
B) greatest at aphelion (point A)
C) constant everywhere in the orbit

Answer: C. The angular momentum is conserved because there is no torque.

QM1-59. The commutator $[\hat{y}\,\hat{p}_z, \hat{x}\,\hat{p}_z]$ is

A) zero B) none-zero C) sometimes zero, sometimes non-zero

Answer: A.

QM1-60. The commutator $[L_z^{\,2}, L_z]$ is

A) zero B) none-zero C) sometimes zero, sometimes non-zero

Answer: A.

QM1-61. In Cartesian coordinates, the volume element is dx dy dz . In spherical coordinates, the volume element is

A) $r^2 \sin\theta \cos\varphi \, dr \, d\theta \, d\varphi$

B) $\sin\theta \cos\varphi \, dr \, d\theta \, d\varphi$

C) $r^2 \cos\theta \sin\varphi \, dr \, d\theta \, d\varphi$

D) $r \sin\theta \cos\varphi \, dr \, d\theta \, d\varphi$

E) None of these

Answer: E. It looks like A without the $\cos(\varphi)$ term.

QM1-62. In Cartesian coordinates the normalization condition is $\int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} dy \int_{-\infty}^{+\infty} dz \, |\Psi|^2 = 1$. In spherical coordinates, the normalization integral has limits of integration:

A) $\int_{0}^{+\infty} dr \int_{0}^{2\pi} d\theta \int_{0}^{\pi} d\varphi \ldots$

B) $\int_{-\infty}^{+\infty} dr \int_{0}^{2\pi} d\theta \int_{0}^{\pi} d\varphi \ldots$

C) $\int_{0}^{+\infty} dr \int_{0}^{2\pi} d\theta \int_{0}^{2\pi} d\varphi \ldots$

D) $\int_{-\infty}^{+\infty} dr \int_{0}^{\pi} d\theta \int_{0}^{\pi} d\varphi \ldots$

E) None of these

Answer: E. A is the closest, but the π and 2π should be exchanged.

QM1-63. Recall that an operator $\hat{Q}$ is Hermitian if $\langle f \mid \hat{Q} g \rangle = \langle \hat{Q} f \mid g \rangle$ for all normalizable functions f and g. The operator $\hat{L}_z$ is Hermitian, since it corresponds to an observable. Is the operator $i\hat{L}_z$ Hermitian?

A) Yes B) No

Answer: B. The Hermitian conjugate of i is –i.

QM1-64. $[L^2, L_+] = [L^2, L_x + i L_y]$ Does this commutator equal zero?

A) Yes, $[L^2, L_+] = 0$ B) No $[L^2, L_+] \neq 0$

Answer: A. The raising operator increases the eigenvalue of L_z, but it keeps the state in the same L^2 ladder.

QM1-65. The operator for (angular momentum)^2 is
$L^2 = L_x{}^2 + L_y{}^2 + L_z{}^2$.

Is it true that $\langle L^2 \rangle = \langle L_x{}^2 \rangle + \langle L_y{}^2 \rangle + \langle L_z{}^2 \rangle$?

A) Yes, always B) No, never
C) Sometimes yes, sometimes no, depending on the state function Ψ used to compute the expectation value.

Answer: A.

QM1-66. In spherical coordinates,

$$\nabla f = \hat{r}\frac{\partial f}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial f}{\partial \theta} + \hat{\varphi}\frac{1}{r\sin\theta}\frac{\partial f}{\partial \varphi}$$

, and in QM, the angular momentum operator is $\hat{L} = \frac{\hbar}{i}\vec{r}\times\nabla = \frac{\hbar}{i} r\hat{r}\times\nabla$,

the $\hat{r}$ component of $\hat{L}$ is ?

A) 0

B) non-zero but dependent on θ, φ only (independent of r)

C) non-zero but dependent on r, θ, and φ

Answer: A. $\hat{r}\times\vec{A}$ has no $\hat{r}$ component, for any $\vec{A}$.

QM1-67. In QM, the operator $L^2 = \hat{L}\cdot\hat{L}$

A) depends on θ, and φ only (independent of r)

B) depends on r, θ, and φ

C) depends on θ only (independent of r, φ)

Answer: A. There are two ways to see this. First, you can look at the differential form of the L^2 operator. Second, you can realize that any wavefunction of the form $\Psi(r,\theta,\phi) = R(r)Y_l^m(\theta,\phi)$ is an eigenstate of L_2 with eigenvalue $\hbar^2 l(l+1)$, for any arbitrary *R(r)*. So L^2 can't depend on *r*.

QM1-68. Ignoring spin, what is the angular momentum of the ground state of an electron in a hydrogen atom, in units of h-bar?

A) 0 B) 1/2 C) 1 D) 3/2 E) I don't know

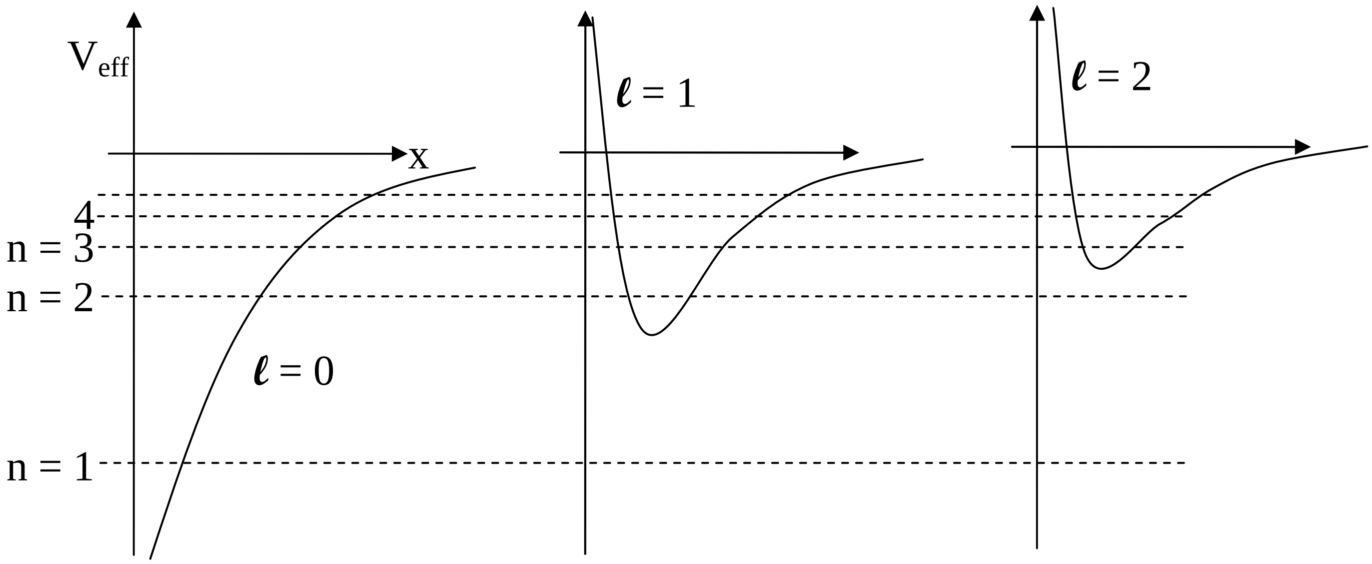
Answer: A. The ground state is spherically symmetric.

QM1-69. In classical mechanics, the translational KE of a particle is $\frac{p^2}{2m}$. What is the formula for rotational KE (where I is moment-of-inertia)?

A) $\frac{1}{2} I L^2$ B) $\frac{L^2}{2I}$ C) $I\omega$ D) $2 I L^2$

Answer: B.

QM1-70. The effective potential is shown for $\ell = 0$, 1, and 2. The first several allowed energy levels are shown.

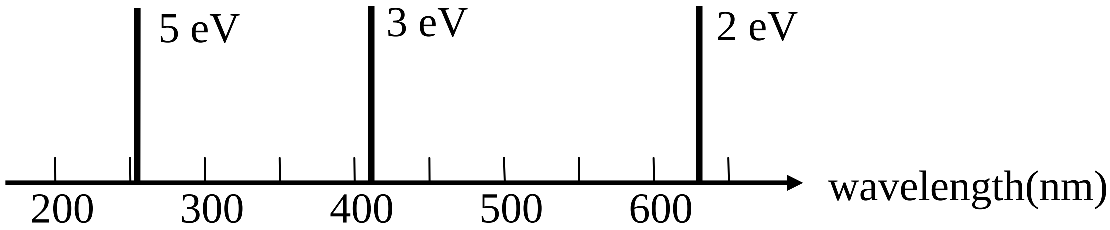


As indicated in the figure, the n = 2, $\ell = 0$ state and the n = 2, $\ell = 1$ state happen to have the same energy (given by $E_{n=2} = E_1/2^2$). Do these states have the same radial wavefunction R(r) ?

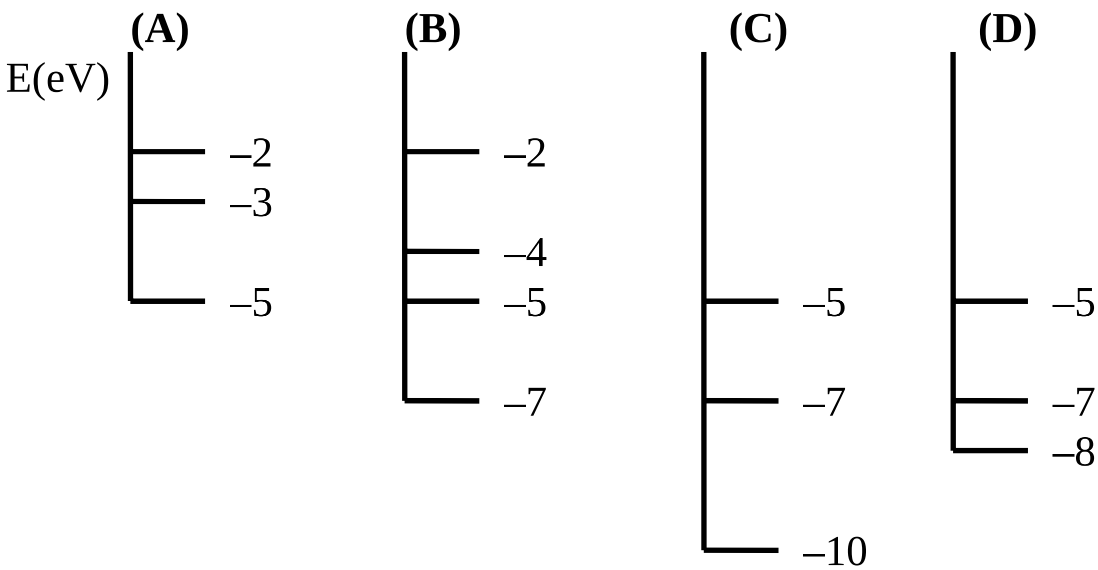
A) Yes B) No

Answer: B. See the table in Griffiths.

QM1-71. The spectrum of "Perkonium" has 3 emission lines



Which energy level structure is consistent with the spectrum?



Answer: C. The spectrum given at the top of the page consists of energy differences between pairs of levels.

QM1-72. If $\exp(+i\,m\,2\pi) = 1$, then it must be true that

A) m = 0, 1, 2, …
B) m = 0, 1/2, 1, 3/2, 2, …
C) m = 0, ±1, ±2, …
D) m = 2πn where n = 0, ±1, ±2, …
E) None of these

Answer: C.

QM1-73. Apart from normalization, the spherical harmonic $Y_\ell^\ell(\theta,\phi) = (\sin\theta)^\ell \exp(i\,\ell\,\phi)$. The zero-angular momentum state Y_0^0 ..

A) has no θ, ϕ dependence: it is a constant
B) depends on θ only; it has no ϕ dependence
C) depends on ϕ only; it has no θ dependence
D) depends on both θ and ϕ

Answer: A.

QM1-74. Normalization $\int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sin\theta \left|Y_0^0\right|^2 = 1$ requires that $Y_0^0 =$

A) 1 B) 4π C) $\frac{1}{4\pi}$ D) $\frac{1}{\sqrt{4\pi}}$

E) None of these

Answer: D.

QM1-75. True (A) or False (B) ?
Any arbitrary physical state of an electron bound in the H-atom potential can always be written as

$$\psi_{n\ell m}(r, \theta, \phi) = R_{n\ell}(r)\, Y_\ell^m(\theta, \phi)$$

with suitable choice of n , ℓ , and m.

Answer: B. Most of you got this one wrong in class. The correct statement many of you were thinking about was, "Any arbitrary physical state of an electron bound in the H-atom can be written as a linear superposition of the energy eigenstates."

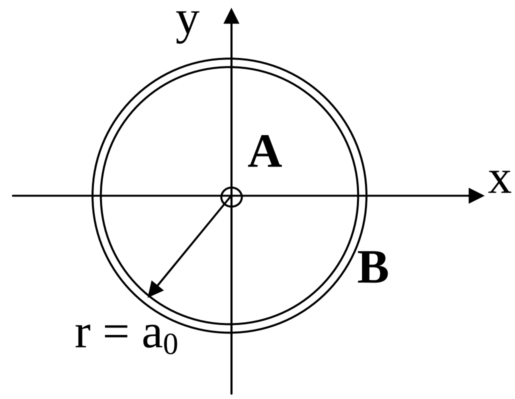
QM1-76. A particle in a 1D Harmonic oscillator is in the state $\Psi(x) = \sum_n c_n u_n(x)$ where $u_n(x)$ is the n^{th} energy eigenstate $\hat{H} u_n = E_n u_n$. A measurement of the energy is made. What is the probability that result of the measurement is the value E_m ?

A) $\langle c_m | \Psi(x) \rangle$ B) $\left| \langle c_m | \Psi(x) \rangle \right|^2$

C) $\left| \langle u_m | \Psi(x) \rangle \right|^2$ D) $\langle u_m | \Psi(x) \rangle$ E) c_m

Answer: C

QM1-77. Consider an electron in the ground state of an H-atom:
The wavefunction is $\psi(r) = A\exp(-r/a_0)$
Where is the electron more likely to be found?
A) Within dr of the origin (r = 0)
B) Within dr of a distance $r = a_0$ from the origin?



Answer: B. Remember, the volume element contains the factor r^2.

QM1-78. Consider the object formed by placing a ket to the left of a bra like so: $|f\rangle\langle g|$. This thing is best described as...

A) nonsense. This is a meaningless combination.
B) a functional (transforms a function or ket into a number)
C) a function (transforms a number into a number)
D) an operator (transforms a function or ket into another function or ket).
E) None of these.

Answer: D.

QM1-79. Consider the object formed by placing a bra to the left of a operator like so: $\langle g|\hat{Q}$. This thing is best described as...

A) nonsense. This is a meaningless combination.
B) a functional (transforms a function or ket into a number)
C) a function (transforms a number into a number)
D) an operator (transforms a function or ket into another function or ket).
E) None of these.

Answer: B.

QM1-80. Consider the state $|\psi\rangle = c_1|1\rangle + c_2|2\rangle$. What is $\hat{P}_2|\psi\rangle$, where $\hat{P}_2 = |2\rangle\langle 2|$ is the projection operator for the state $|2\rangle$?

A) c_2 B) $|2\rangle$ C) $c_2|2\rangle$ D) $c_2{}^*\langle 2|$ E) 0

Answer: C.

QM1-81. Consider the state $|\psi\rangle = c_1|1\rangle + c_2|2\rangle$.

What is $\hat{P}_{12}|\psi\rangle$, where $\hat{P}_{12} = |1\rangle\langle 1| + |2\rangle\langle 2|$?

A) $|\psi\rangle = c_1|1\rangle + c_2|2\rangle$ B) $|1\rangle + |2\rangle$ C) 0

D) $\langle\psi| = c_1^*\langle 1| + c_2^*\langle 2|$ E) None of these

Answer: A.

QM1-82. If the state $|\psi\rangle = c_1|1\rangle + c_2|2\rangle$, as well as the basis states $|1\rangle$ and $|2\rangle$ are normalized, then the state $\hat{P}_1|\psi\rangle = |1\rangle\langle 1|\psi\rangle = c_1|1\rangle$ is

A) normalized

B) not normalized.

Answer: B (unless c_1=1 and c_2=0.)

QM1-83. Consider two kets and their corresponding column vectors:

$$|\Psi\rangle = \begin{pmatrix} 1 \\ 1 \\ \sqrt{2} \end{pmatrix} \qquad |\phi\rangle = \begin{pmatrix} 1 \\ 1 \\ -\sqrt{2} \end{pmatrix}$$

Are these two state orthogonal? Is $\langle\psi|\phi\rangle = 0$?

A) Yes B) No

Answer: A.

Are these states normalized? A) Yes B) No

Answer: B.

QM1-84. Consider a Hilbert space spanned by three energy eigenstates:
$\hat{H}|n\rangle = E_n|n\rangle, \quad n = 1, 2, 3$. In this space, what is the matrix corresponding to the Hamiltonian?

A) $\begin{pmatrix} E_1 & E_2 & E_3 \\ E_1 & E_2 & E_3 \\ E_1 & E_2 & E_3 \end{pmatrix}$ B) $\begin{pmatrix} E_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_3 \end{pmatrix}$ C) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

D) $\begin{pmatrix} E_1 & E_1 & E_1 \\ E_2 & E_2 & E_2 \\ E_3 & E_3 & E_3 \end{pmatrix}$ E) None of these

Answer: B.