

# Nuclear Properties

Thornton and Rex, Ch. 12

# A pre-history

1896 Radioactivity discovered - Becquerel

□ rays + (Helium)

□ rays - (electrons)

□ rays 0 (EM waves)

1902 Transmutation observed

- Rutherford and Soddy

1909 □ rays are Helium nuclei

- Rutherford and Royds

1912 Nucleus is shown to have very small radius ( a few  $\times 10^{-15}$  meters) at the center of the atom (a few  $\times 10^{-11}$  meters).

- Rutherford (and Geiger and Marsden)

# Artificial Transmutation

1919 - Rutherford succeeded in producing the first nuclear reaction in the laboratory.

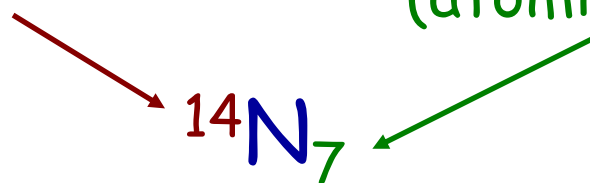
By colliding Nitrogen with  $\alpha$  rays, Rutherford succeeded in creating Hydrogen and Oxygen.

We can write this reaction as:



Nuclear Mass  
Number  $A = Z + N$

Nuclear charge  
(atomic number  $Z$ )



$A \approx$  atomic weight (not an integer)  
we will discuss the integer  $N$  in a bit

Both integers,  $A$  and  $Z$ , are "conserved".

A shorthand for this reaction:



# The Structure of the Nucleus

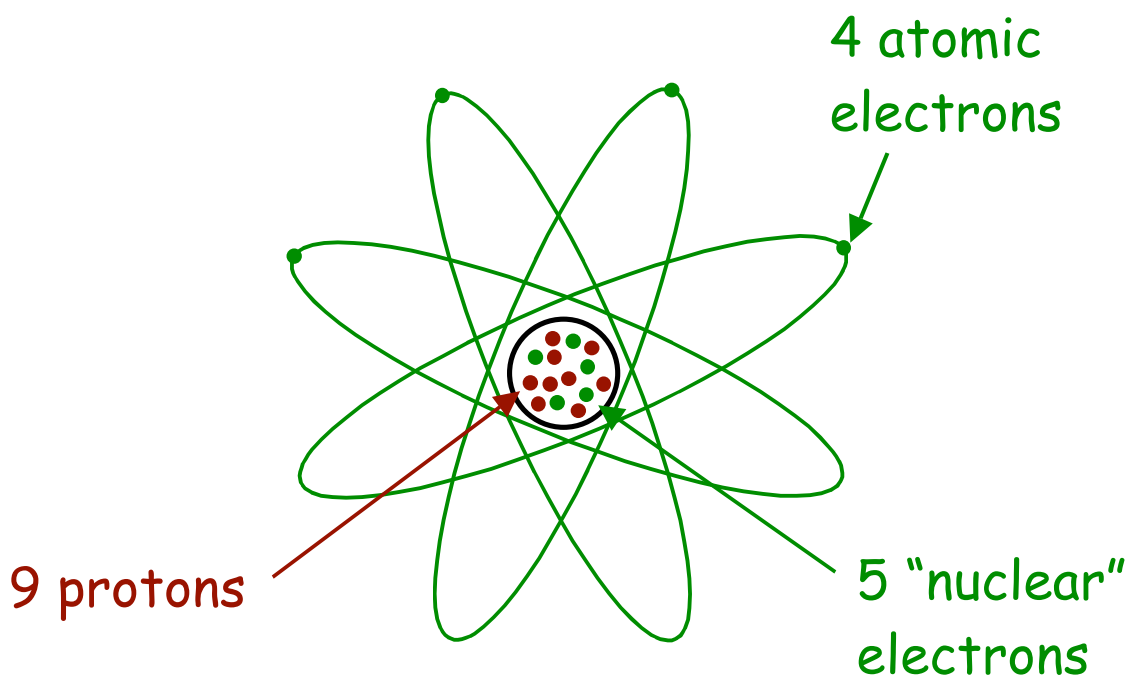
An early theory:

nucleus composed of **protons** and **electrons**:

**Protons** to get mass right plus enough **electrons** to get charge right.

Example: Beryllium has atomic weight 9 and atomic number 4.

- **9 protons** and **5 "nuclear" electrons**.  
(in addition to the 4 electrons orbiting around the nucleus).



Problems with this model:

- Uncertainty principle  $\Rightarrow$  nucleus would have too much energy
- Predicts incorrect spin for nuclei

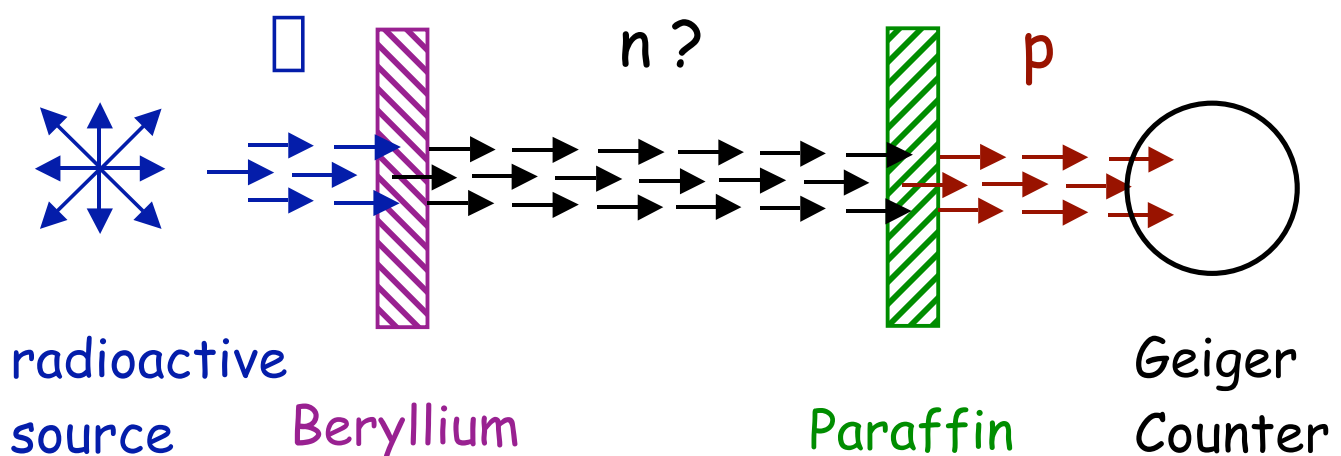
Rutherford suggested a new elementary particle in the nucleus, with roughly the same mass as the proton, and zero charge.

I.e., the "neutron".

# The Discovery of the Neutron

1930 - Bothe and Becker discover penetrating new type of radiation while bombarding **Beryllium** with  $\alpha$  particles.

Curie and Joliot showed that when this new radiation struck a **paraffin** target (which contains Hydrogen nuclei) it knocked out high energy **protons**.



James Chadwick - Using measurements of energy and momentum, showed that radiation must be a new (uncharged) particle of about the same mass as a proton.

Chadwick named it the neutron (symbol n).

The reaction for creating it was



or



The generic name for a neutron or proton is a nucleon.

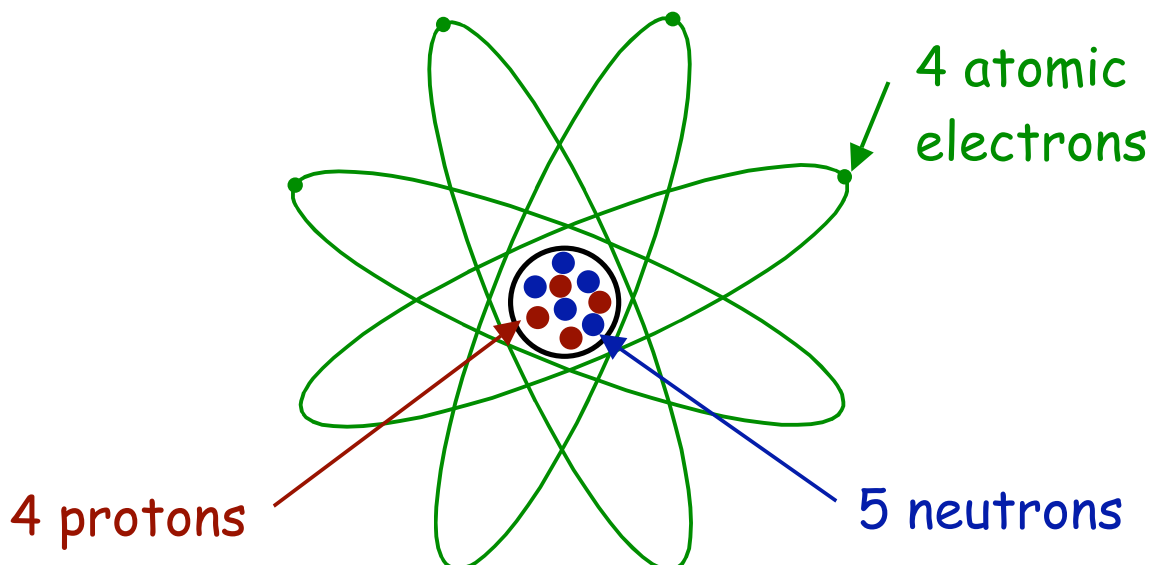
## The Structure of the Nucleus

The modern picture of the nucleus:  
**protons** and **neutrons**, but **no electrons**.

The charge is determined by the **number of protons** (same as **number of electrons**).  
The mass is the sum of the masses of the **protons** and **neutrons**.

Example, Beryllium (atomic weight 9,  
atomic number 4)

- **4 protons** and **5 neutrons** in the nucleus,  
**4 electrons** orbiting around it.



# Properties of Elementary Particles (circa 1932)

<u>particle</u>	<u>symbol</u>	<u>charge (C)</u>	<u>mass (kg)</u>
electron	${}^0e_{-1}$	$-1.6 \times 10^{-19}$	$9.1 \times 10^{-31}$
proton	${}^1p_1$	$+1.6 \times 10^{-19}$	$1.7 \times 10^{-27}$
neutron	${}^1n_0$	0	$1.7 \times 10^{-27}$

We write

$$A = \text{atomic weight} = N_{\text{protons}} + N_{\text{neutrons}}$$

$$Z = \text{atomic number} = N_{\text{protons}} = N_{\text{electrons}}$$

$$N = \text{neutron number} = N_{\text{neutrons}} = A - Z$$

Some examples:

	<u>A</u>	<u>Z</u>	<u>N</u>
${}^1\text{H}_1$	1	1	0
${}^4\text{He}_2$	4	2	2
${}^{12}\text{C}_6$	12	6	6
${}^{56}\text{Fe}_{26}$	56	26	30
${}^{238}\text{U}_{92}$	238	92	146

# Isotopes

The "name" of the element (Hydrogen, Iron, Lead, etc.) is determined by the **number of electrons = number of protons = Z**. It determines the chemical properties of the element.

Some elements exist in forms with different numbers of neutrons. These are called isotopes of the element.

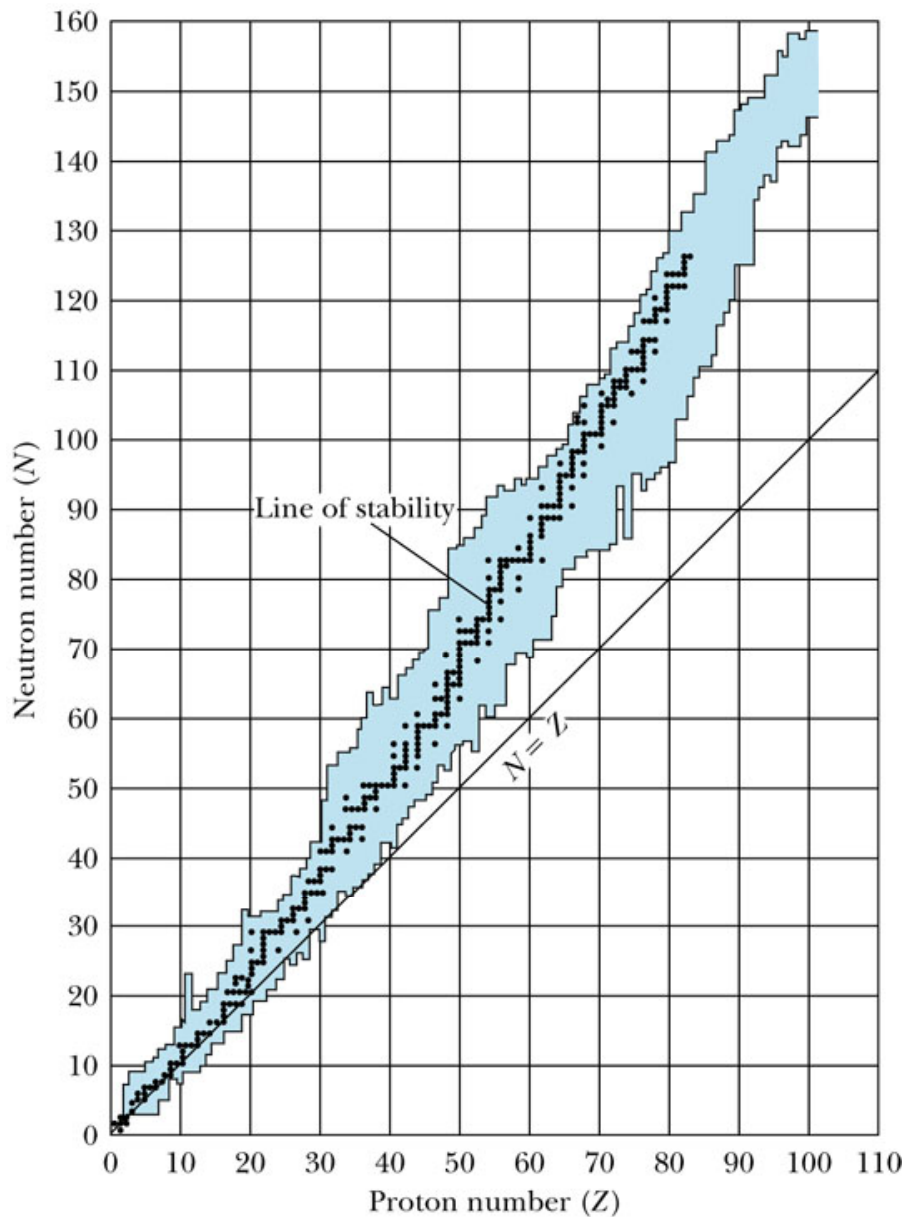
Example: 3 **isotopes** of Hydrogen:

		<u>A</u>	<u>Z</u>	<u>N</u>
Standard	${}^1\text{H}_1$	1	1	0
Deuterium	${}^2\text{H}_1$	2	1	1
Tritium	${}^3\text{H}_1$	3	1	2

# Stable and Unstable Isotopes

Not all isotopes  are stable ■

"Atomic Weight" averages over observable isotopes, so not an integer



## Sizes of Nuclei

Nuclei can usually be approximated by spheres of radius  $R$ , where

$$R = r_0 A^{1/3}$$

and

$$r_0 = 1.2 \times 10^{-15} \text{ m}$$

(Note: Volume  $\propto R^3 \propto A$ )

The unit  $10^{-15} \text{ m} = 1 \text{ femtometer (fm)}$ ,  
often called **1 fermi**.

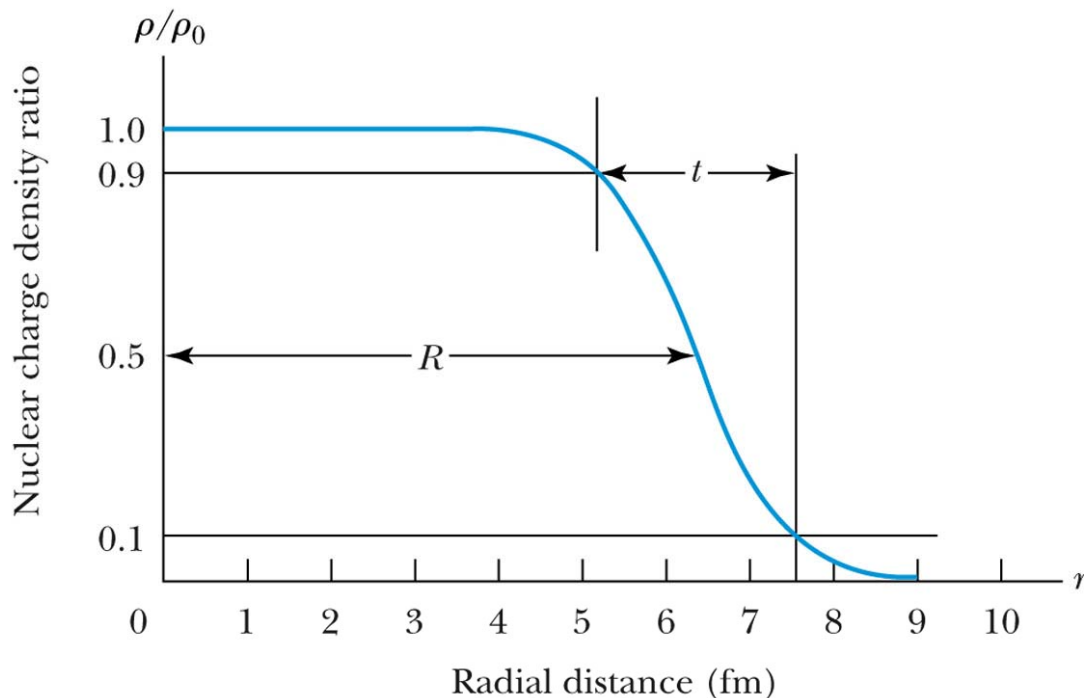
## Shapes of Nuclei

One can describe the shape of a nucleus by its "charge distribution".

1950's - measured by Robert Hofstadter at Stanford using 500 MeV electrons ( $\lambda_{\text{de Broglie}} \sim 2.5 \text{ fm}$ ).

The distribution can be parametrized by

$$\rho(r) = \frac{\rho_0}{1 + e^{(r-R)/a}}$$



$t \sim 2.2, a \sim .5 \text{ fm}$  for this nucleus

# The Nuclear Force

The nuclear (or "strong") force is what binds the **protons and neutrons** together into **nuclei**.

Properties:

- It is **attractive**.
- Within the nucleus, it is about 100x **stronger** than the Electromagnetic force (and about  $10^{38}$ x stronger than gravity).
- It is very **short range**.  
Outside of the nucleus, the EM force dominates, and the nucleus behaves just like a positive charge.

- It is charge-independent; that is, it acts the same on protons and neutrons.
- It is spin-dependent.

# Binding Energy

The nuclear force binds nucleons (**protons** and **neutrons**) together. Work must be done to separate them.

Conversely, energy is released when nucleons join together to form a stable nucleus.

The energy difference between a nucleus and its separate constituent nucleons is called the Binding Energy.

$$\text{Binding Energy} = (Z m_p + N m_n - {}^A M_Z) c^2$$

↑  
Mass of  
Bound nucleus

## Example: Helium

$$M(^1\text{H}) = 1.007825 \text{ u} \quad (* = m_p + m_e)$$

$$m_n = 1.008665 \text{ u}$$

$$M(^4\text{He}_2) = 4.002603 \text{ u} \quad (* = m_\alpha + 2m_e)$$

$$\text{Atomic Mass Unit: } u = 931.5 \text{ MeV}/c^2$$

$$\begin{aligned} \text{B. E.} &= (2 M(^1\text{H}) + 2 m_n - M(^4\text{He}_2))c^2 \\ &= 0.030377 \text{ u } c^2 \\ &= 28.30 \text{ MeV} \end{aligned}$$

\*These are actually the atomic masses of Hydrogen and Helium, which include the electron masses. Note that the electron masses cancel out of the formula.

# Radioactive Decay

In any nuclear reaction, the following quantities are conserved:

1. Nucleon Number,  $A$
2. Charge
3. Energy
4. Momentum

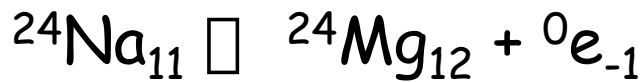
## $\alpha$ Decay



The masses of the two nuclei on the RHS add up to about  $1.1 \times 10^{-29}$  kg less than the mass on the LHS. The difference is made up in Kinetic Energy (using  $E=mc^2$ ).

Most of the energy (about 6 MeV) is taken by the  $\alpha$  particle.

## β Decay

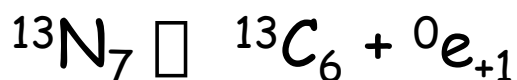


The electron takes away 1 unit of electric charge, increasing  $Z$  of the nucleus by 1. Atomic mass  $A$  is unchanged.

(It's as if a **neutron** changes into a **proton** and an **electron**.)

## Positive β Decay

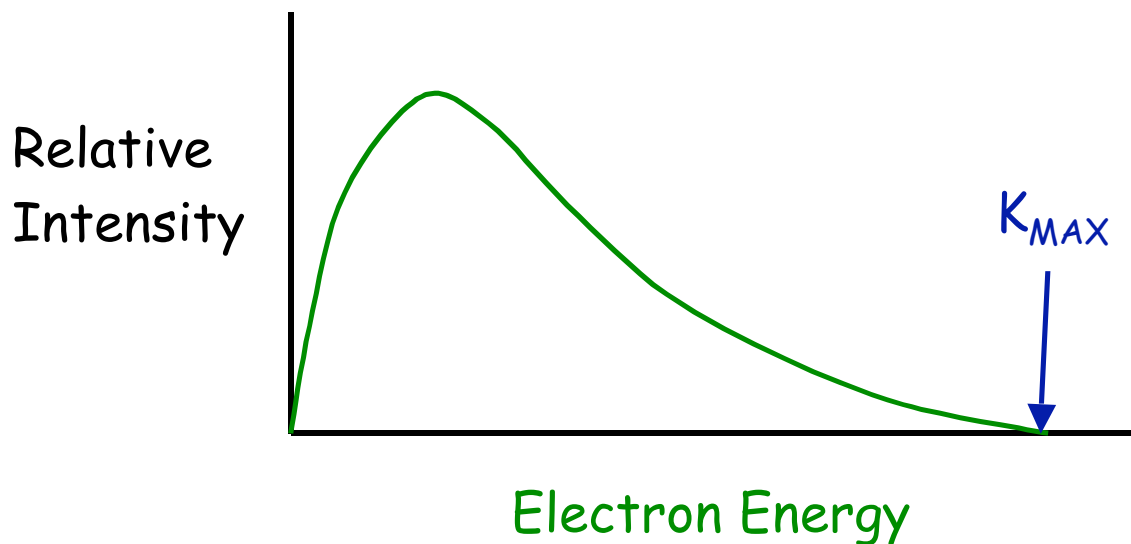
The anti-particle to the electron (the positron, predicted by the Dirac Equation) can also take part in β decay.



(It's as if a **proton** changes into a **neutron** and a **positron**.)

For  $\beta$  decay (**positive** or **negative**), one would expect the electron to come out with a single energy, just as for  $\alpha$  decay.

However, experiment showed a continuous spectrum of energies:



In 1930 Wolfgang Pauli solved this by suggesting that the electron energy was shared with a new particle, the neutrino.

The **neutrino** is chargeless and (almost) massless.

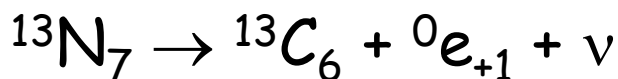
1956 - **neutrino** finally detected.

Difficult to detect because it does not interact through EM or strong force, only through the weak force.

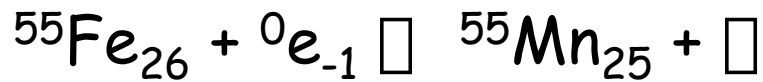
1998 - evidence of a nonzero mass confirmed.

Its **mass is of order  $10^{-3} \text{ eV}/c^2$**  or less (compared to an **electron mass of  $m_e = 5.11 \times 10^{-3} \text{ eV}/c^2$** ).

We can now write the  $\beta$  decays as:



## Electron Capture



For higher-Z nuclides, it is possible for electron capture to occur.

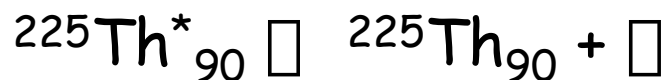
The effect is the same as for **positive  $\beta$  decay**: a **proton** is converted to a **neutron**.

When electron capture occurs, the hole left by the captured inner electron will be filled by an outer electron which drops down, while emitting an X-ray of characteristic wavelength.

## γ Decay

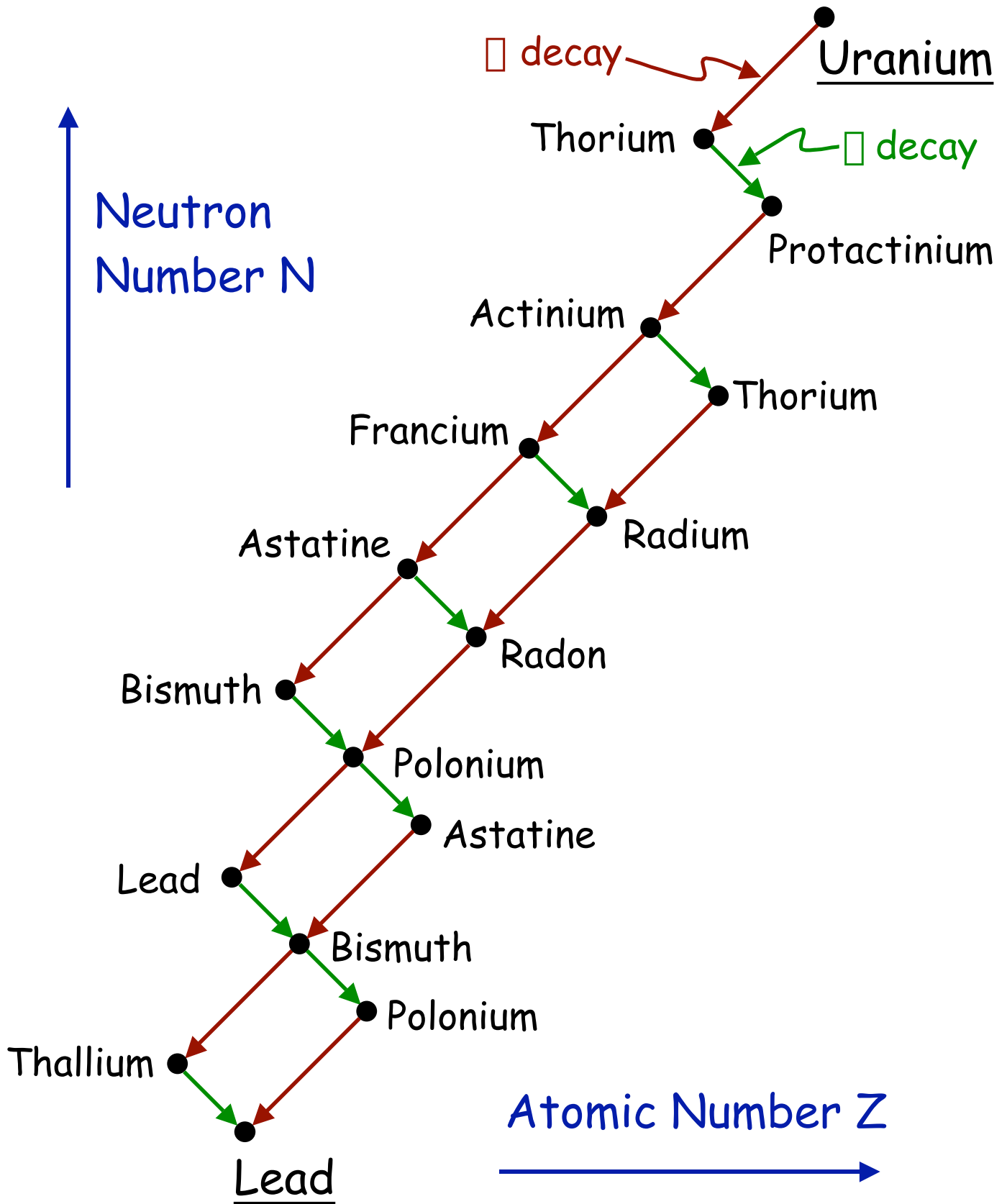
Just like an atom, a nucleus can also have excited states. Often, an α or β decay of a radioactive nucleus will leave the daughter nucleus in an excited state.

The excited nucleus will then decay to the ground state with the emission of a high energy photon (γ ray).



Obviously, this leaves  $A$  and  $Z$  unchanged.

# A radioactive decay "chain" (Segre chart)



## Decay times

Radioactive decay is a probabilistic event.

For large numbers of nuclei, the number that decay in a short time will be proportional to the total number  $N$  and the time  $\Delta t$ :

$$\Delta N = -\lambda N \Delta t$$

The solution to this differential equation is

$$N(t) = N(0) e^{-\lambda t}$$

# Activity

The Activity of a radioactive substance is defined as the number of decays per unit time:

$$\text{Activity: } R = - dN/dt = \lambda N(t)$$

The SI unit is the Becquerel:

$$1 \text{ Bq} = 1 \text{ decay/second}$$

An older, but still used, unit is the Curie:

$$1 \text{ Ci} = 3.7 \times 10^{10} \text{ decays/second}$$

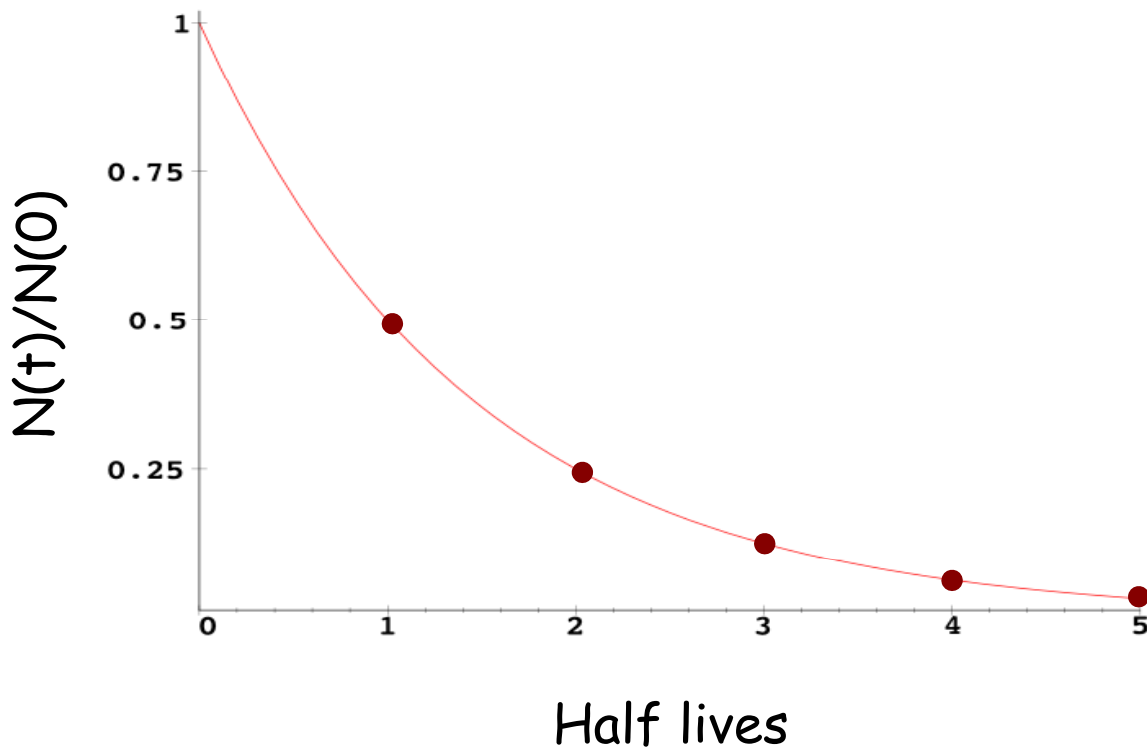
1g of  $^{226}\text{Ra}$

The activity falls also falls off exponentially:

$$R(t) = R(0) e^{-\lambda t}$$

$\lambda$  is the decay constant.

# Half-life



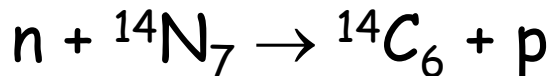
The half-life ( $t_{\frac{1}{2}}$ ) is the time for the number of radioactive nuclei to drop by a factor of 2. It is easy to show

$$t_{\frac{1}{2}} = \ln(2)/\lambda = 0.693/\lambda$$

$$N(t) = \left(\frac{1}{2}\right)^{t/t_{\frac{1}{2}}}$$

# Carbon Dating

Radioactive  $^{14}\text{C}$  is continually produced in the atmosphere by bombardment of  $^{14}\text{N}$  by neutrons produced by cosmic rays:



There is a natural equilibrium ratio of  $^{14}\text{C}$  to  $^{12}\text{C}$  of  $R_0 = 1.2 \times 10^{-12}$ . This ratio occurs in the carbon in the  $\text{CO}_2$  taken up by living organisms.

But when a living organism dies, the  $^{14}\text{C}$  decays and the ratio of  $^{14}\text{C}/^{12}\text{C}$  decreases, allowing us to date the time of death of the organism by

$$R(t) = R_0 e^{-\lambda t}$$