Evolution Equations of Twist-3 Correlation Functions relevant to SSAs

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Based on work with Zhong-Bo Kang, 0811.3101[hep-ph]

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The Question

- ❑ How to probe the hadron structure beyond the PDFs?
  beyond the probability distributions?

$$\sigma(Q,s_T) = H_0 \otimes \underbrace{f_2 \otimes f_2}_{\text{Too large to compete!}} + (1/Q)\, H_1 \otimes f_2 \otimes \underbrace{f_3}_{\text{Three-parton correlation}} + \mathcal{O}(1/Q^2)$$

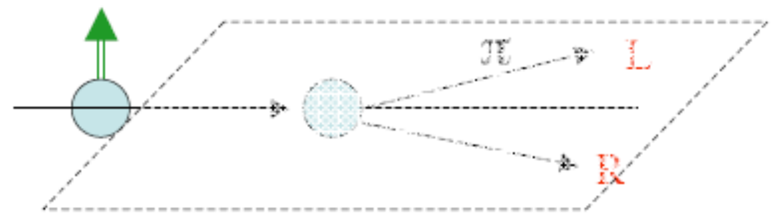
- ❑ Idea:

Take a difference of the cross sections, and
Hope that the leading power term cancels

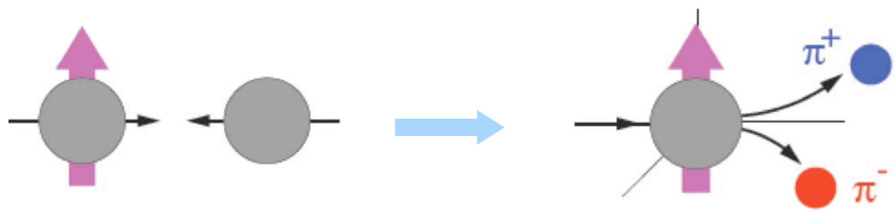
$$\Delta\sigma(Q,s_T) \equiv [\sigma(Q,s_T) - \sigma(Q,-s_T)]/2$$

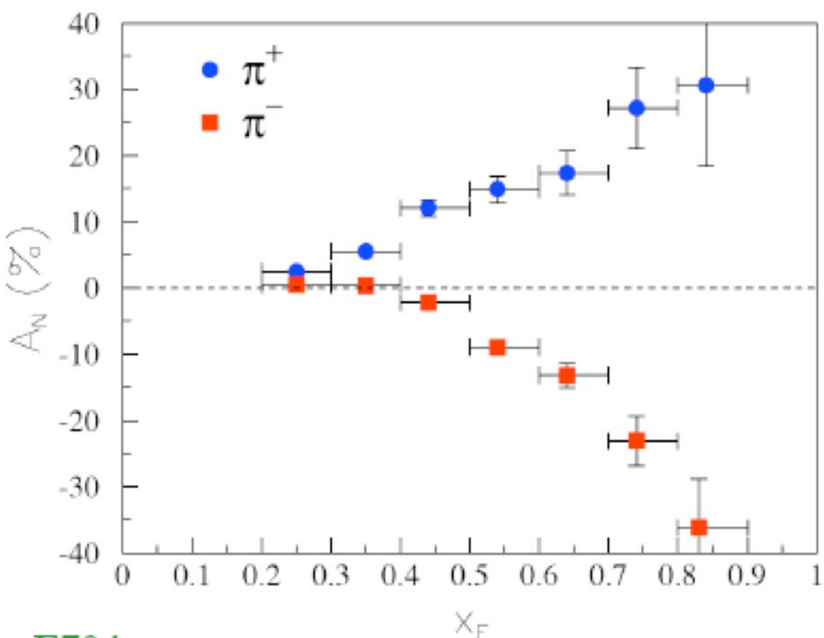
Asymmetry in hadronic collisions

- **Hadronic** $p\uparrow + p \to \pi(l)X$:



$$A_N = \frac{1}{P_{\rm beam}} \frac{N^{\pi}_{\rm left} - N^{\pi}_{\rm right}}{N^{\pi}_{\rm left} + N^{\pi}_{\rm right}}$$





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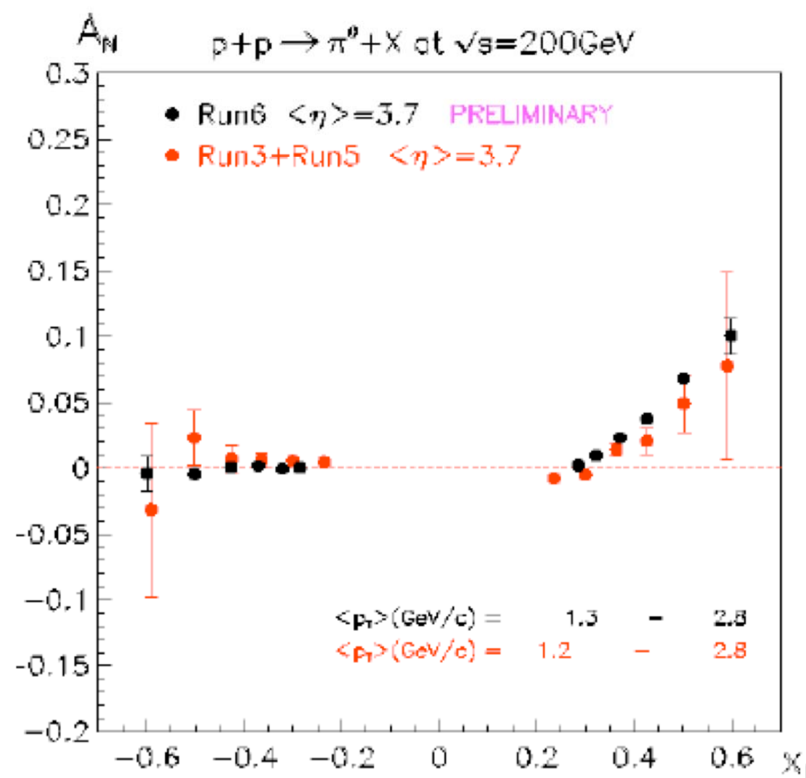


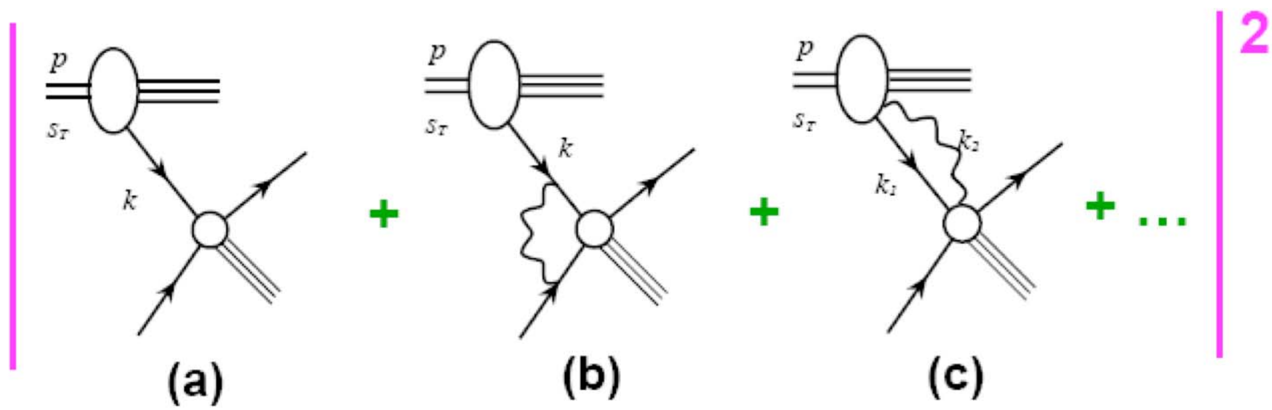
STAR (BRAHMS, too)

SSA in collinear factorization

Efremov, Teryaev, 1982, Qiu, Sterman, 1991

- When all observed scales >> Λ_{QCD}, collinear factorization should work:

$\sigma(s_T)$ ~



- Leading spin dependent part of the cross section
  - ➡ Interference between amplitudes (a) and (b) or (c)
- The hadronic phase – the "i"
  - ➡ Re[(a)] interferes with Im[(b)] or Im[(c)]
- $\mathrm{Re}[(a)] \times \mathrm{Im}[(b)] \propto m_Q \;\delta q(s_\perp)$

Twist-3 three-parton correlations

❑ Set I:

Qiu, Sterman, 1991, 1998
Ji, 1992, Kang, Qiu, 2008

$$\widetilde{T}_{q,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\, \langle P, s_T|\overline{\psi}_q(0)\,\frac{\gamma^+}{2}\,\left[\epsilon^{s_T\sigma n\bar{n}} F_\sigma{}^+(y_2^-)\right]\psi_q(y_1^-)|P, s_T\rangle$$

$$\widetilde{T}^{(f,d)}_{G,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\, \frac{1}{P^+}\langle P, s_T|F^{+\rho}(0)\left[\epsilon^{s_T\sigma n\bar{n}} F_\sigma{}^+(y_2^-)\right] F^{+\lambda}(y_1^-)|P, s_T\rangle(-g_{\rho\lambda})$$

❑ Set II:

$$\widetilde{T}_{\Delta q,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\, \langle P, s_T|\overline{\psi}_q(0)\,\frac{\gamma^+\gamma^5}{2}\,\left[i\, s_T^\sigma\, F_\sigma{}^+(y_2^-)\right]\psi_q(y_1^-)|P, s_T\rangle$$

$$\widetilde{T}^{(f,d)}_{\Delta G,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\, \frac{1}{P^+}\langle P, s_T|F^{+\rho}(0)\left[i\, s_T^\sigma\, F_\sigma{}^+(y_2^-)\right] F^{+\lambda}(y_1^-)|P, s_T\rangle\,(i\epsilon_{\perp\rho\lambda})$$

Two possible color contractions: if_{abc}, d_{abc}

Two possible tri-gluon correlation functions

Connection to Twist-2 PDFs

- **Set I:**

  Spin-averaged twist-2 PDFs + an operator Insertion

$$\int \frac{dy_2^-}{2\pi}\, e^{ix_2 P^+ y_2^-}\, \left[\epsilon^{s_T \sigma n \bar{n}}\, F_\sigma{}^+(y_2^-)\right] = i \int \frac{dy_2^-}{2\pi}\, e^{ix_2 P^+ y_2^-}\, \left[i\, \epsilon_\perp^{\rho\sigma}\, s_{T\rho}\, F_\sigma{}^+(y_2^-)\right]$$

- **Set II:**

  Spin-dependent twist-2 HDFs + an operator Insertion

$$i \int \frac{dy_2^-}{2\pi}\, e^{ix_2 P^+ y_2^-}\, \left[s_T^\sigma\, F_\sigma{}^+(y_2^-)\right]$$

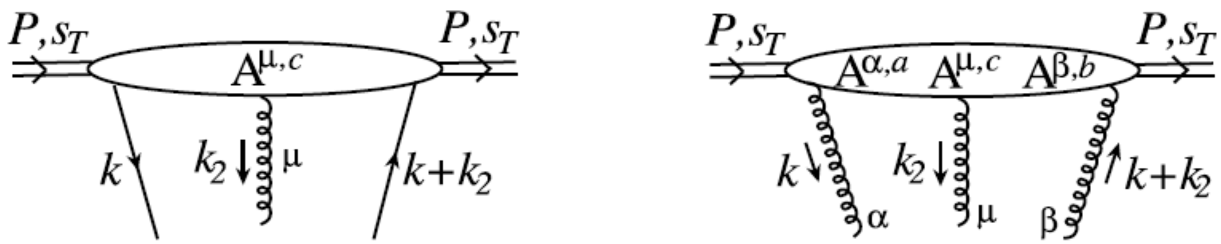
- **Extra 'i'**

  Phase needed for the nonvanishing SSAs

  Do not contribute to parity conserving double-spin asymmetry!

Feynman Diagram Representation

❑ Diagrams:



❑ Cut vertices in the light-cone gauge:

$$\mathcal{V}^{\rm LC}_{q,F} = \frac{\gamma^+}{2P^+}\,\delta\left(x-\frac{k^+}{P^+}\right) x_2\,\delta\left(x_2-\frac{k_2^+}{P^+}\right)\left(i\,\epsilon^{s_T\sigma n\bar{n}}\right)\left[-g_{\sigma\mu}\right]\mathcal{C}_q\,,$$

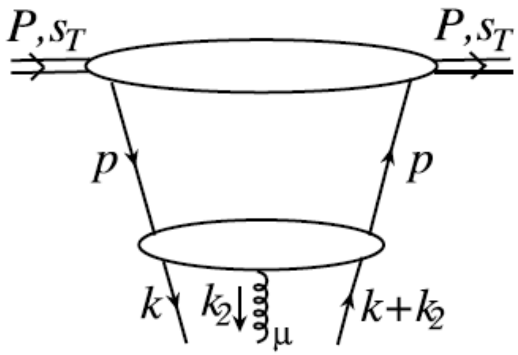
$$\mathcal{V}^{\rm LC}_{G,F} = x(x+x_2)\left(-g_{\alpha\beta}\right)\delta\left(x-\frac{k^+}{P^+}\right) x_2\,\delta\left(x_2-\frac{k_2^+}{P^+}\right)\left(i\,\epsilon^{s_T\sigma n\bar{n}}\right)\left[-g_{\sigma\mu}\right]\mathcal{C}_g^{(f,d)}$$

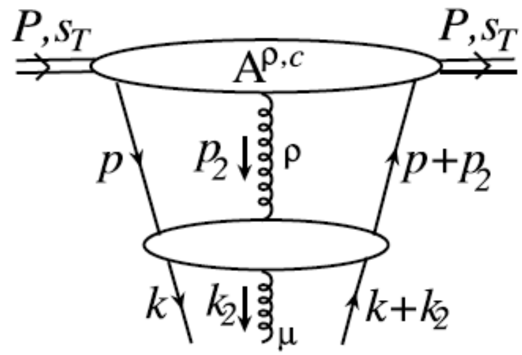
$$\mathcal{V}^{\rm LC}_{\Delta q,F} = \frac{\gamma^+\gamma^5}{2P^+}\,\delta\left(x-\frac{k^+}{P^+}\right) x_2\,\delta\left(x_2-\frac{k_2^+}{P^+}\right)\left(-s_T^\sigma\right)\left[-g_{\sigma\mu}\right]\mathcal{C}_q$$

$$\mathcal{V}^{\rm LC}_{\Delta G,F} = x(x+x_2)\left(i\epsilon_\perp^{\beta\alpha}\right)\delta\left(x-\frac{k^+}{P^+}\right) x_2\,\delta\left(x_2-\frac{k_2^+}{P^+}\right)\left(-s_T^\sigma\right)\left[-g_{\sigma\mu}\right]\mathcal{C}_g^{(f,d)}$$

Evolution Equations

- Perturbative change of the correlation functions:





- Evolution equation for the flavor non-singlet channel:

$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \widetilde{\mathcal{T}}_{q,F}(x, x+x_2, \mu_F, s_T)$$

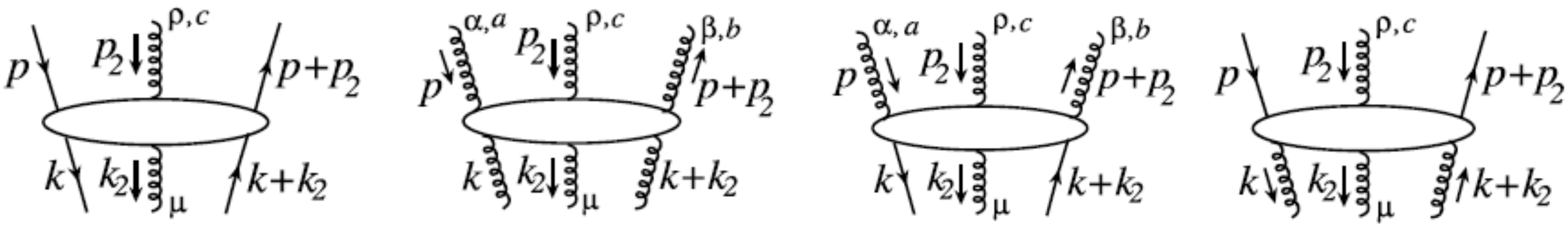
$$= \int d\xi \, d\xi_2 \, \Big[\widetilde{\mathcal{T}}_{q,F}(\xi, \xi+\xi_2, \mu_F, s_T) \, K_{qq}(\xi, \xi+\xi_2, x, x+x_2, \alpha_s)$$

$$+ \widetilde{\mathcal{T}}_{\Delta q,F}(\xi, \xi+\xi_2, \mu_F, s_T) \, K_{q\Delta q}(\xi, \xi+\xi_2, x, x+x_2, \alpha_s) \Big]$$

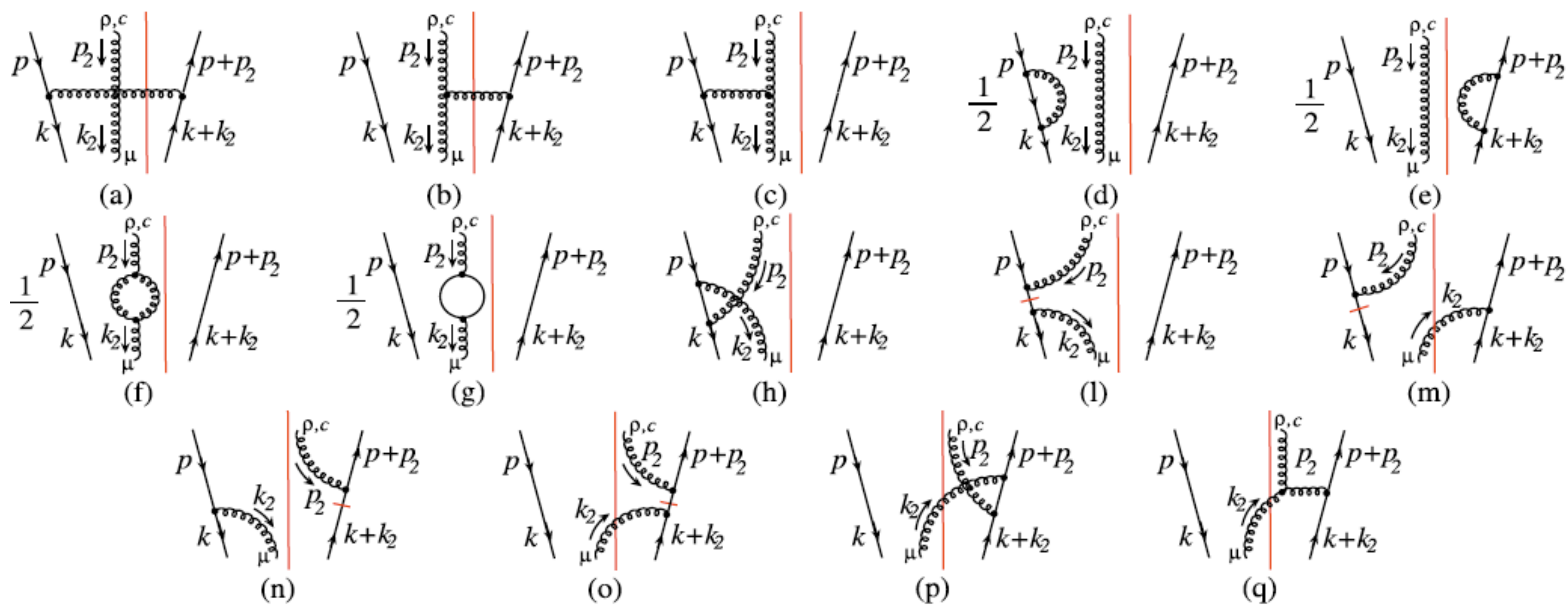
Plus all flavor singlet channels

Evolution Kennels

❑ Feynman diagrams:



❑ Leading order:



LO Evolution Equations - I

- Diagonal contribution - Quarks:

$$\frac{\partial T_{q,F}(x,x,\mu_F)}{\partial \ln \mu_F^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left\{P_{qq}(z)\,T_{q,F}(\xi,\xi,\mu_F)\right.$$
$$+\frac{C_A}{2}\left[\frac{1+z^2}{1-z}\left[T_{q,F}(\xi,x,\mu_F)-T_{q,F}(\xi,\xi,\mu_F)\right]+z\,T_{q,F}(\xi,x,\mu_F)\right]$$
$$+\frac{C_A}{2}\left[T_{\Delta q,F}(x,\xi,\mu_F)\right]$$
$$\left.+P_{qg}(z)\left(\frac{1}{2}\right)\left[T_{G,F}^{(d)}(\xi,\xi,\mu_F)+T_{G,F}^{(f)}(\xi,\xi,\mu_F)\right]\right\}$$

$$\frac{\partial T_{\bar{q},F}(x,x,\mu_F)}{\partial \ln \mu_F^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left\{P_{qq}(z)\,T_{\bar{q},F}(\xi,\xi,\mu_F)\right.$$
$$+\frac{C_A}{2}\left[\frac{1+z^2}{1-z}\left[T_{\bar{q},F}(\xi,x,\mu_F)-T_{\bar{q},F}(\xi,\xi,\mu_F)\right]+z\,T_{\bar{q},F}(\xi,x,\mu_F)\right]$$
$$+\frac{C_A}{2}\left[T_{\Delta \bar{q},F}(x,\xi,\mu_F)\right]$$
$$\left.+P_{qg}(z)\left(\frac{1}{2}\right)\left[T_{G,F}^{(d)}(\xi,\xi,\mu_F)-T_{G,F}^{(f)}(\xi,\xi,\mu_F)\right]\right\};$$

Feature of evolution kernels, Difference in quark and antiquark!

LO Evolution Equations - II

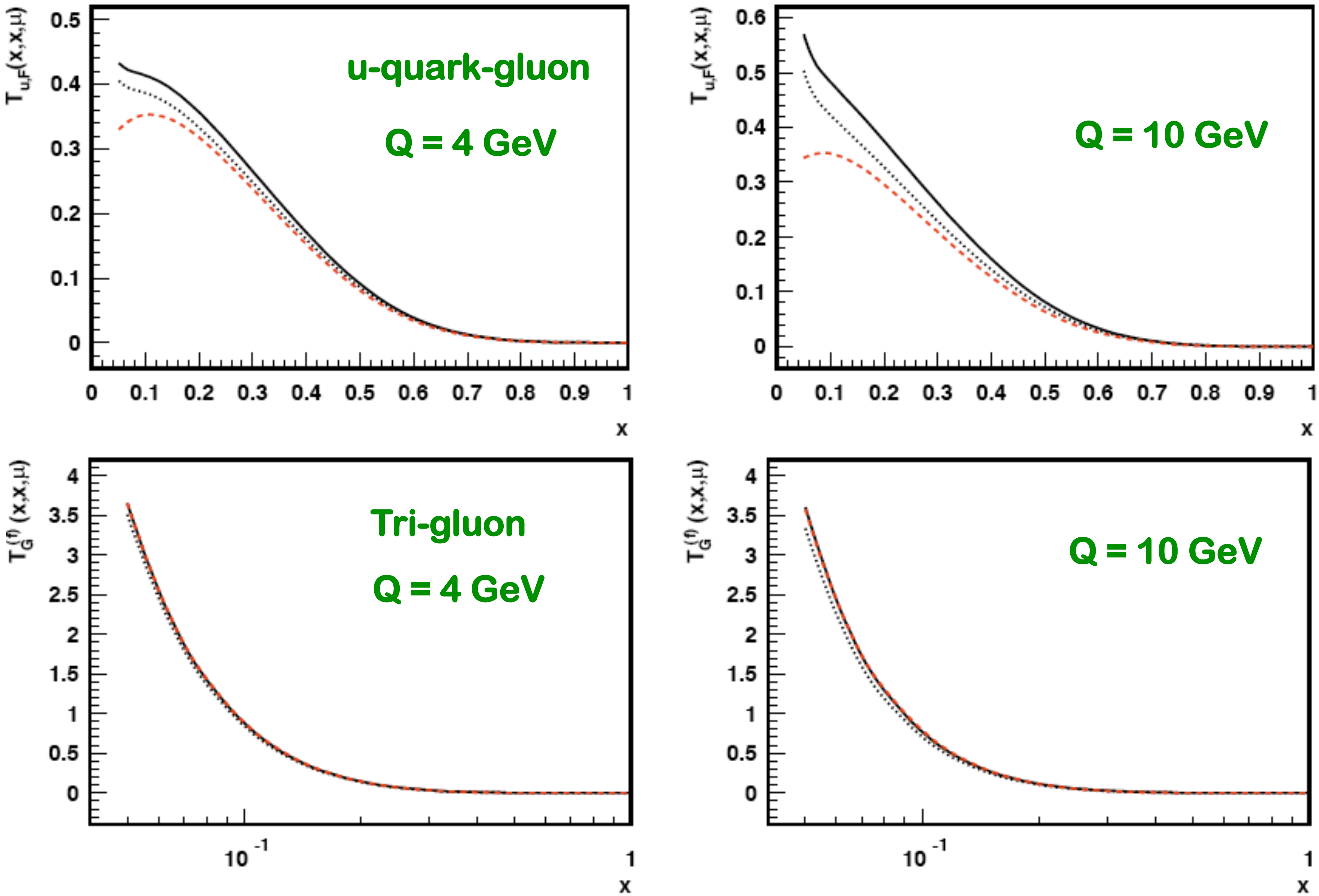
❑ Diagonal contribution - gluons:

$$
\begin{aligned}
\frac{\partial T^{(f)}_{G,F}(x,x,\mu_F)}{\partial \ln \mu_F^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\Bigg\{ & P_{gg}(z)\, T^{(f)}_G(\xi,\xi,\mu_F) \\
& + \frac{C_A}{2}\left[2\left(\frac{z}{1-z}+\frac{1-z}{z}+z(1-z)\right)\left[T^{(f)}_{G,F}(\xi,x,\mu_F)-T^{(f)}_{G,F}(\xi,\xi,\mu_F)\right]\right. \\
& \qquad \left. +2\left(1-\frac{1-z}{2z}-z(1-z)\right)T^{(f)}_{G,F}(\xi,x,\mu_F)\right] \\
& + \frac{C_A}{2}\left[(1+z)\, T^{(f)}_{\Delta G,F}(x,\xi,\mu_F)\right] \\
& + P_{gq}(z)\left(\frac{N_c^2}{N_c^2-1}\right)\sum_q \left[T_{q,F}(\xi,\xi,\mu_F)-T_{\bar{q},F}(\xi,\xi,\mu_F)\right]\Bigg\}\,;
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial T^{(d)}_{G,F}(x,x,\mu_F)}{\partial \ln \mu_F^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\Bigg\{ & P_{gg}(z)\, T^{(d)}_{G,F}(\xi,\xi,\mu_F) \\
& + \frac{C_A}{2}\left[2\left(\frac{z}{1-z}+\frac{1-z}{z}+z(1-z)\right)\left[T^{(d)}_{G,F}(\xi,x,\mu_F)-T^{(d)}_{G,F}(\xi,\xi,\mu_F)\right]\right. \\
& \left. +2\left(1-\frac{1-z}{2z}-z(1-z)\right)T^{(d)}_{G,F}(\xi,x,\mu_F)\right] \\
& + \frac{C_A}{2}\left[(1+z)\, T^{(d)}_{\Delta G,F}(x,\xi,\mu_F)\right] \\
& + P_{gq}(z)\left(\frac{N_c^2-4}{N_c^2-1}\right)\sum_q \left[T_{q,F}(\xi,\xi,\mu_F)+T_{\bar{q},F}(\xi,\xi,\mu_F)\right]\Bigg\}\,.
\end{aligned}
$$

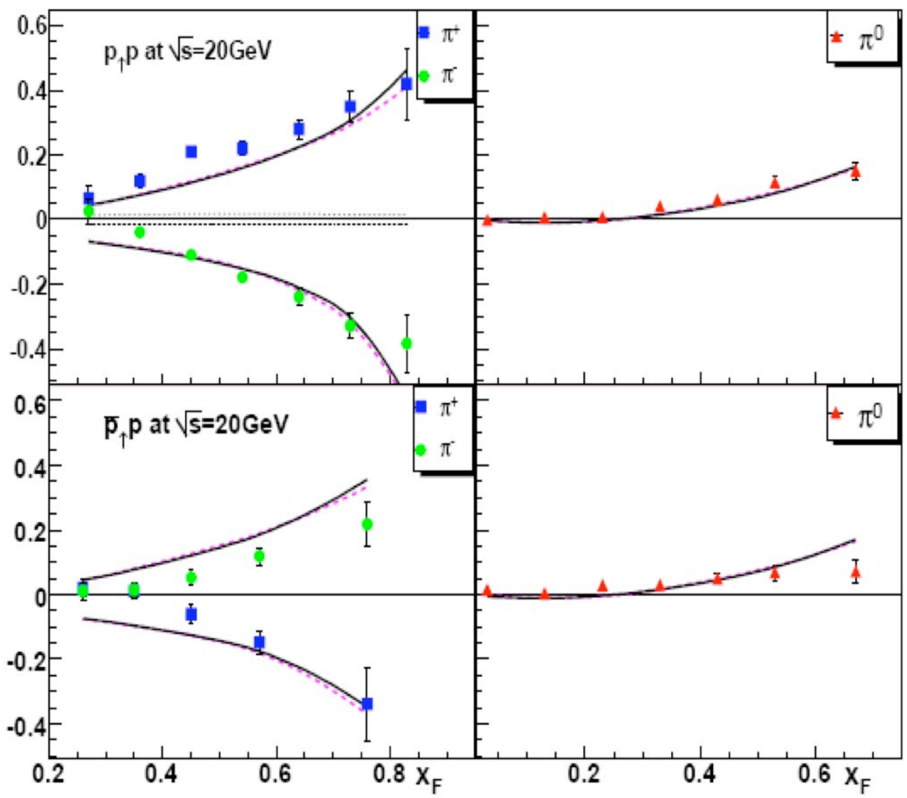
Difference from the TMD PDFs!

Q^2-Dependence of Correlation Functions

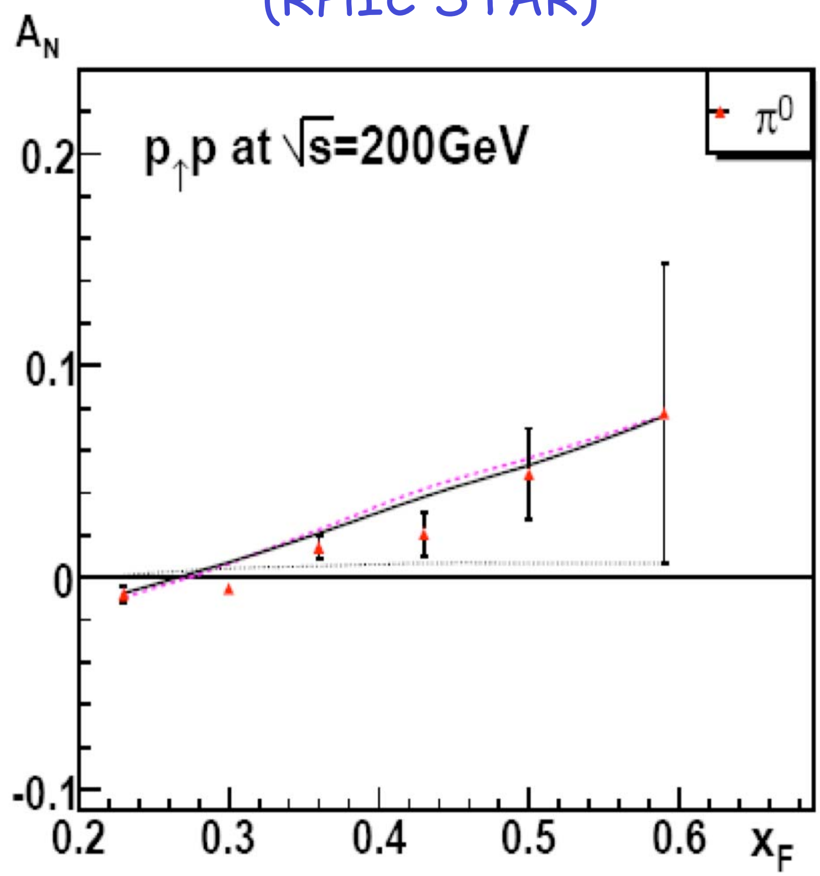


Example: Asymmetries from the $T_{q,F}(x,x)$

(FermiLab E704)



(RHIC STAR)



Kouvaris, Qiu, Vogelsang, Yuan, 2006

$T_F(x_1, x_2)$ **only in forward region**

Nonvanish twist-3 function ⟶ **Nonvanish transverse motion**

Physics of the $T_{q,F}(x,x)$?

- Twist-3 correlation $T_F(x,x)$:

$$T_F(x,x) = \int \frac{dy_1^-}{4\pi} \mathrm{e}^{ixP^+y_1^-} \times \langle P,\vec{s}_T|\bar{\psi}_a(0)\gamma^+ \left[\int dy_2^- \, \epsilon^{s_T\sigma n\bar{n}} \, F_\sigma^{\ +}(y_2^-)\right] \psi_a(y_1^-)|P,\vec{s}_T\rangle$$

- Twist-2 quark distribution:

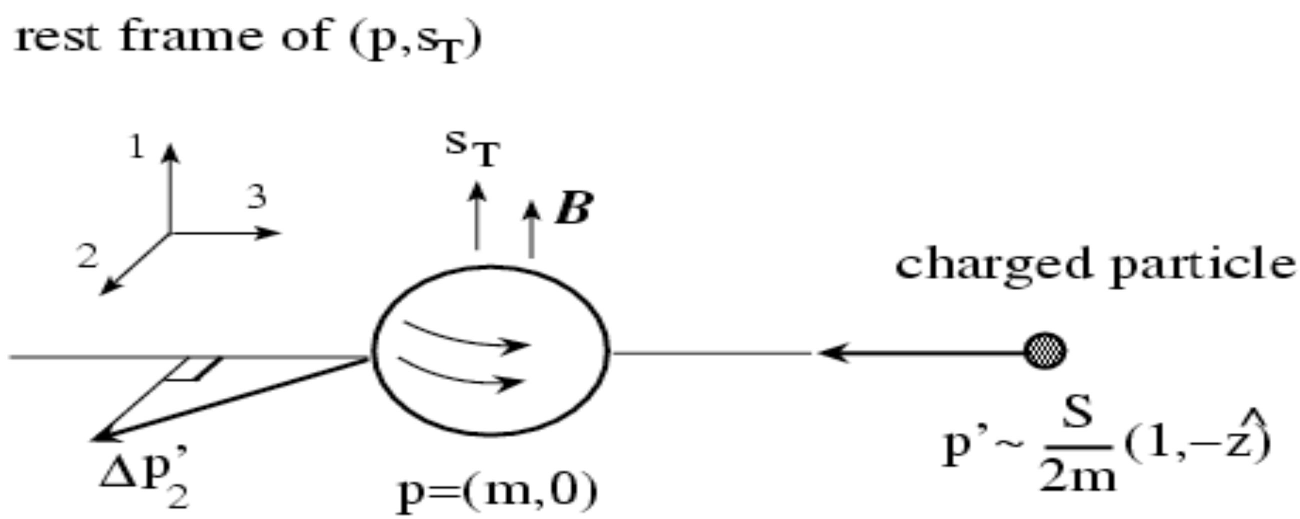
$$q(x) = \int \frac{dy_1^-}{4\pi} \mathrm{e}^{ixP^+y_1^-} \langle P,\vec{s}_T|\bar{\psi}_a(0)\gamma^+ \psi_a(y_1^-)|P,\vec{s}_T\rangle$$

- **Second set – helicity PDFs with the insertion:**

$$i\int \frac{dy_2^-}{2\pi} \, e^{ix_2P^+y_2^-} \, [s_T^\sigma \, F_\sigma^{\ +}(y_2^-)]$$

What the $T_{q,F}(x,x)$ tries to tells us?

- **Consider a classical (Abelian) situation:**



– change of transverse momentum

$$\frac{d}{dt}p'_2 = e(\vec{v}' \times \vec{B})_2 = -ev_3 B_1 = ev_3 F_{23}$$

– in the c.m. frame

$$(m, \vec{0}) \rightarrow \bar{n} = (1, 0, 0_T), \quad (1, -\hat{z}) \rightarrow n = (0, 1, 0_T)$$

$$\Longrightarrow \quad \frac{d}{dt}p'_2 = e\, \epsilon^{s_T \sigma n \bar{n}}\, F_\sigma{}^+$$

– total change: $\boxed{\Delta p'_2 = e \int dy^- \epsilon^{s_T \sigma n \bar{n}}\, F_\sigma{}^+(y^-)}$

Summary

- ❑ After more than 20 years of effort, the universality of PDFs has been well-tested

- ❑ Single transverse-spin asymmetry offers another testing ground for QCD dynamics and hadron structure beyond the probability distributions

- ❑ RHIC has polarized proton beams and root(s)=500 GeV! Additional SIDIS measurements

Backup slices

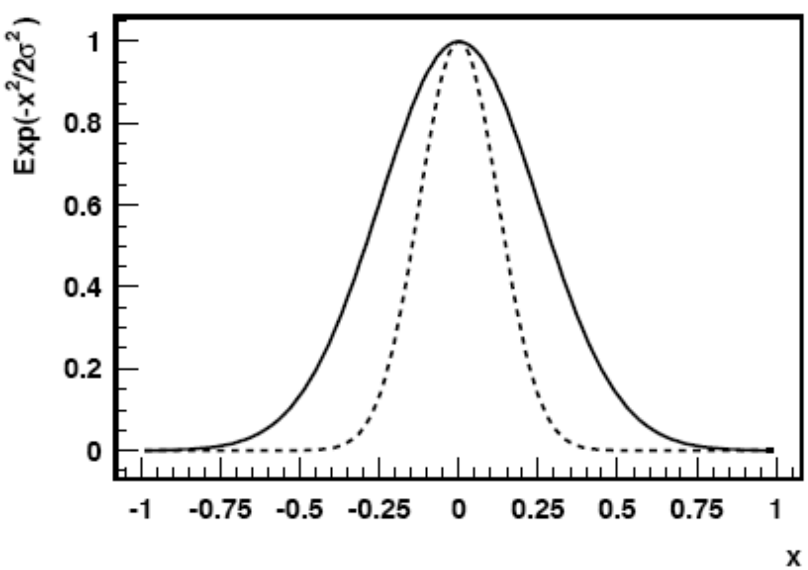
Model for off-diagonal Correlations

❑ Symmetry:

$$T_{q,F}(x_1,x_2,\mu_F) \;=\; \frac{1}{2}\left[T_{q,F}(x_1,x_1,\mu_F)+T_{q,F}(x_2,x_2,\mu_F)\right]\, e^{-\frac{(x_1-x_2)^2}{2\sigma^2}}$$

$$T_{G,F}^{(f,d)}(x_1,x_2,\mu_F) \;=\; \frac{1}{2}\left[T_{G,F}^{(f,d)}(x_1,x_1,\mu_F)+T_{G,F}^{(f,d)}(x_2,x_2,\mu_F)\right]\, e^{-\frac{(x_1-x_2)^2}{2\sigma^2}}$$

$$T_{G,F}^{(f,d)}(x_1,x_2,\mu_F) = \frac{1}{2}\left[T_{G,F}^{(f,d)}(x_1,x_1,\mu_F)+\frac{x_2}{x_1}\,T_{G,F}^{(f,d)}(x_2,x_2,\mu_F)\right]\, e^{-\frac{(x_1-x_2)^2}{2\sigma^2}}$$



$\sigma = 1/4$ (solid) and $\sigma = 1/8$ (dashed)