NLO QCD corrections to VBF and VVV production

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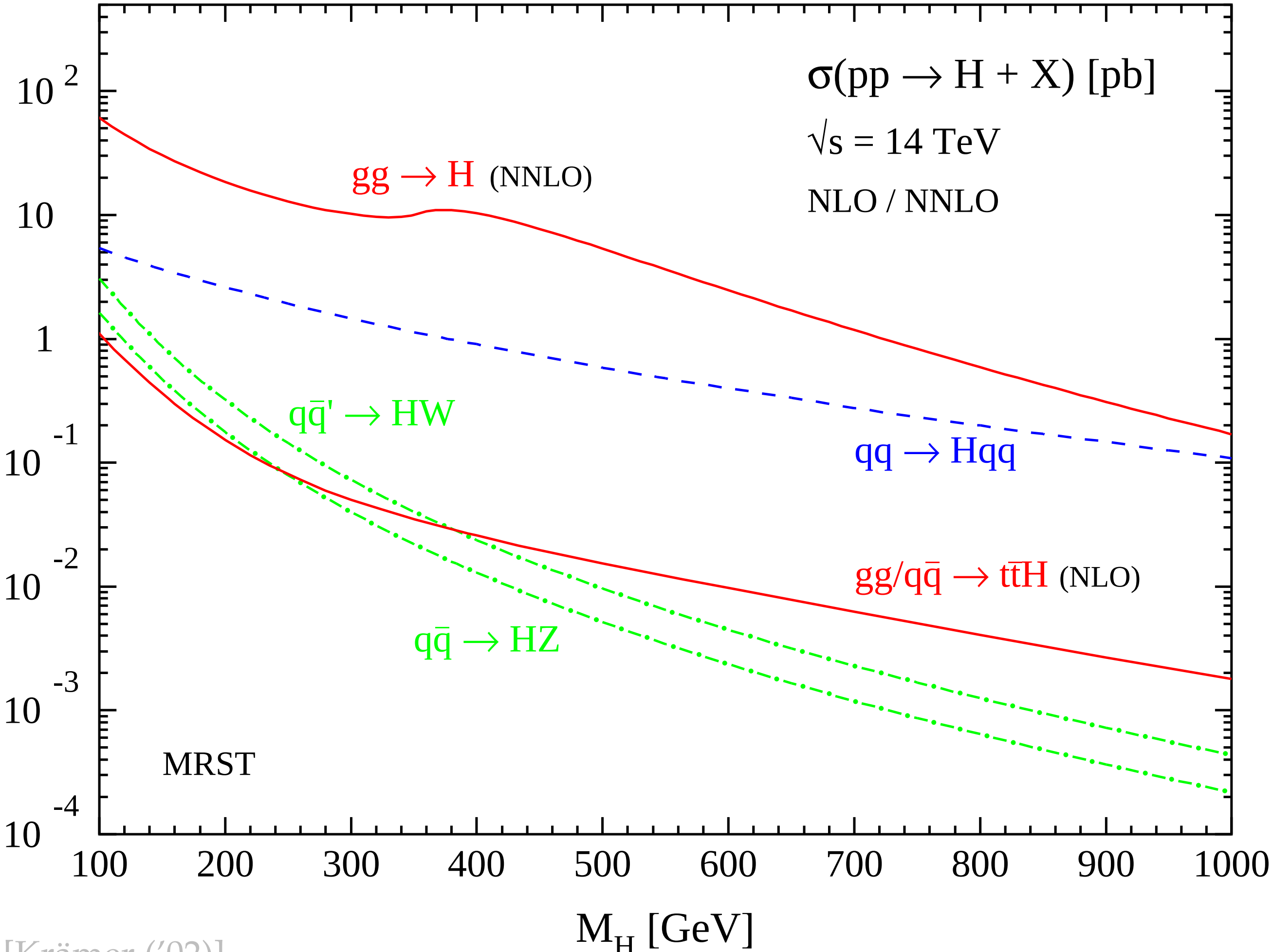
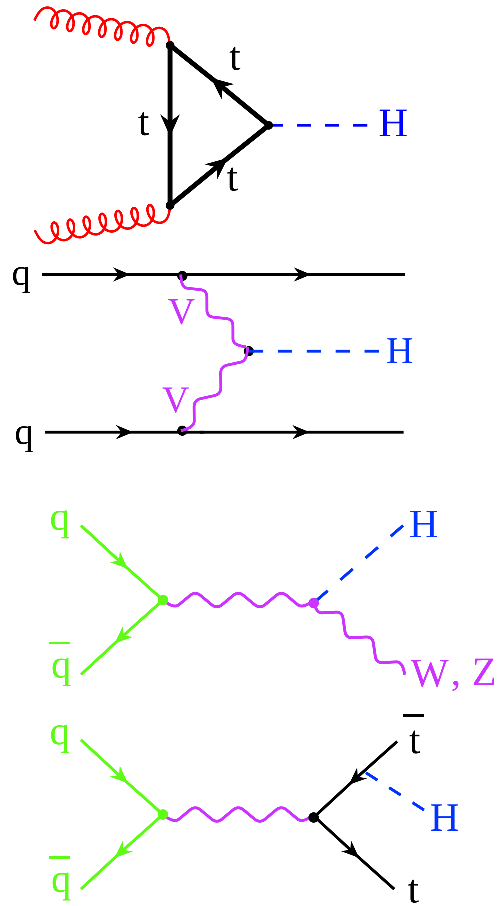
- VBF Processes
- NLO for WWW, WWZ and WZZ production
- Phenomenology and results for LHC
- Conclusions

Total Higgs production cross sections at the LHC

Vector boson fusion is an important ingredient in Higgs search at the LHC





[Krämer ('02)]

QCD corrections to VBF processes

Precise predictions require QCD corrections

$qq \to qqH$

Han, Valencia, Willenbrock (1992); Figy, Oleari, DZ: hep-ph/0306109; Campbell, Ellis, Berger (2004)

- Higgs coupling measurements

$qq \to qqZ$ and $qq \to qqW$

Oleari, DZ: hep-ph/0310156

- $Z \to \tau\tau$ as background for $H \to \tau\tau$
- measure central jet veto acceptance at LHC

$qq \to qqWW$, $qq \to qqZZ$, $qq \to qqWZ$

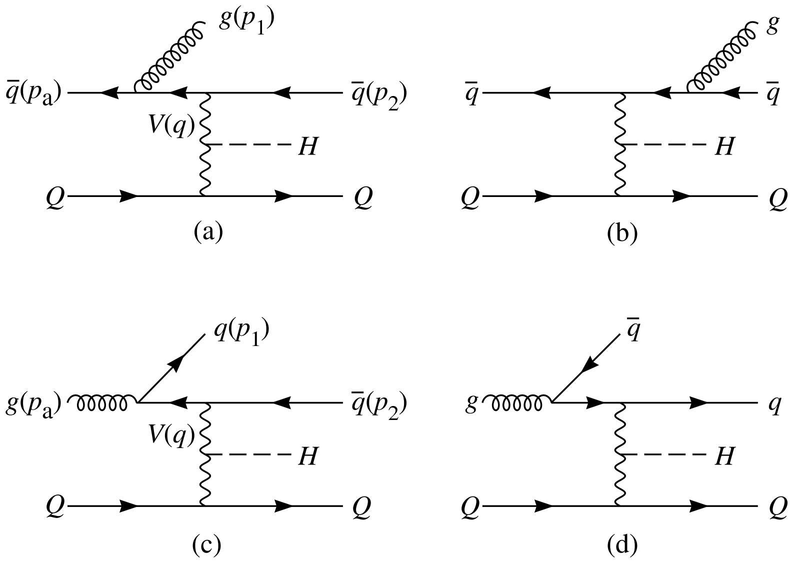
Jäger, Oleari, Bozzi, DZ: hep-ph/0603177, hep-ph/0604200, hep-ph/0701105

- $qqWW$ is background to $H \to WW$ in VBF
- underlying process is weak boson scattering:
  $WW \to WW$, $WW \to ZZ$, $WZ \to WZ$ etc.
  $\Longrightarrow$ measure weak boson scattering

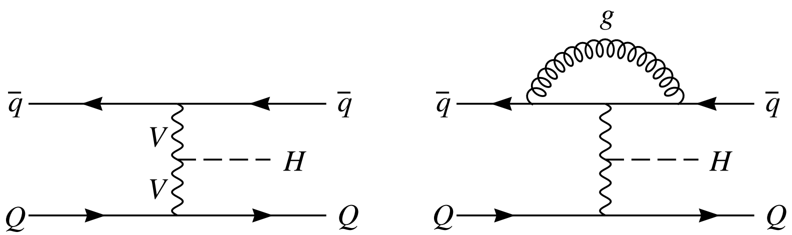
Generic features of QCD corrections to VBF

t-channel color singlet exchange $\Longrightarrow$ QCD corrections to different quark lines are independent

real emission contributions: upper line



Born and vertex corrections to upper line



No t-channel gluon exchange at NLO

Features are generic for all VBF processes

Real emission

Calculation is done using Catani-Seymour subtraction method

Consider $q(p_a)Q \to g(p_1)q(p_2)QH$. Subtracted real emission term

$$|\mathcal{M}_{\text{emit}}|^2 - 8\pi\alpha_s \frac{C_F}{Q^2} \frac{x^2+z^2}{(1-x)(1-z)} |\mathcal{M}_{\text{Born}}|^2 \qquad \text{with } 1-x = \frac{p_1 \cdot p_2}{(p_1+p_2)\cdot p_a}, \quad 1-z = \frac{p_1 \cdot p_a}{(p_1+p_2)\cdot p_a}$$

is integrable $\Longrightarrow$ do by Monte Carlo

Integral of subtracted term over $d^3\mathbf{p}_1$ can be done analytically and gives

$$\frac{\alpha_s}{2\pi} C_F \left(\frac{4\pi\mu_R^2}{Q^2}\right)^{\epsilon} \Gamma(1+\epsilon) |\mathcal{M}_{\text{Born}}|^2 \left[\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + 9 - \frac{4}{3}\pi^2\right] \delta(1-x)$$

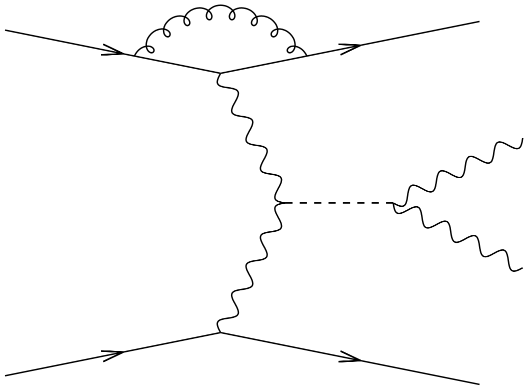
after factorization of splitting function terms (yielding additional “finite collinear terms”)

The divergence must be canceled by virtual corrections for all VBF processes

only variation: meaning of Born amplitude $\mathcal{M}_{\text{Born}}$

Higgs production

Most trivial case: Higgs production
Virtual correction is vertex correction only



virtual amplitude proportional to Born

$$\mathcal{M}_V = \mathcal{M}_{\text{Born}} \frac{\alpha_s(\mu_R)}{4\pi} C_F \left(\frac{4\pi\mu_R^2}{Q^2}\right)^{\epsilon} \Gamma(1+\epsilon) \left[-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} + \frac{\pi^2}{3} - 7\right] + \mathcal{O}(\epsilon)$$

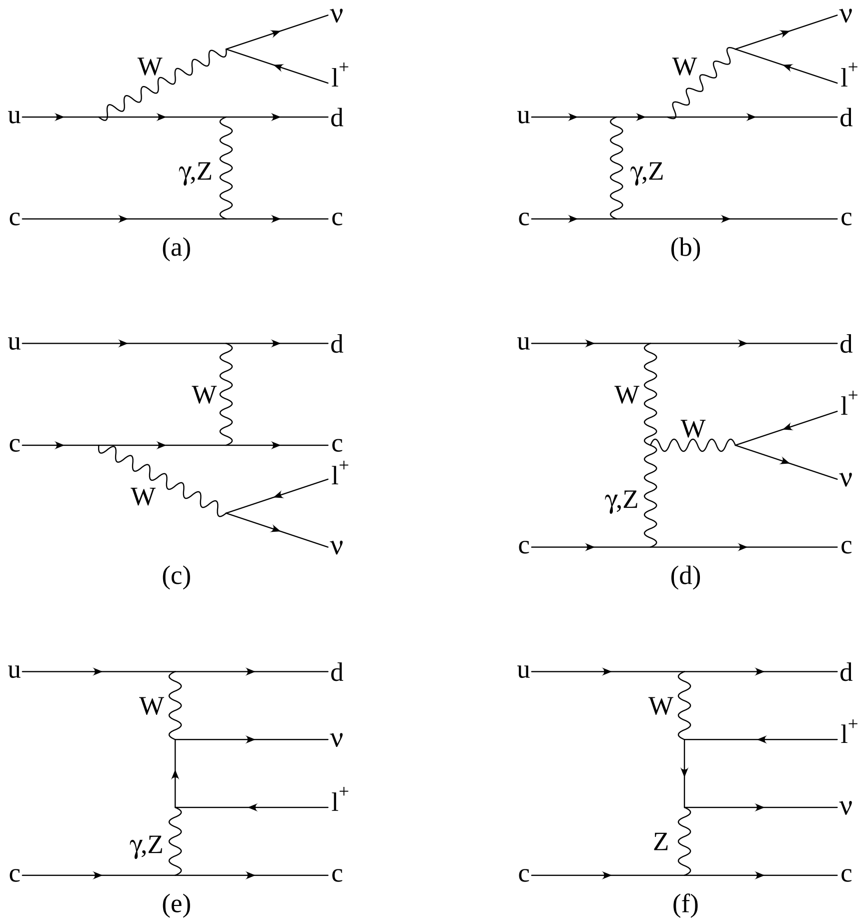
- Divergent piece canceled via Catani Seymour algorithm

Remaining virtual corrections are accounted for by trivial factor multiplying Born cross section

$$|\mathcal{M}_{\text{Born}}|^2 \left(1 + 2\alpha_s \frac{C_F}{2\pi} c_{\text{virt}}\right)$$

- Factor 2 for corrections to upper and lower quark line
- Same factor to Born cross section absorbs most of the virtual corrections for other VBF processes

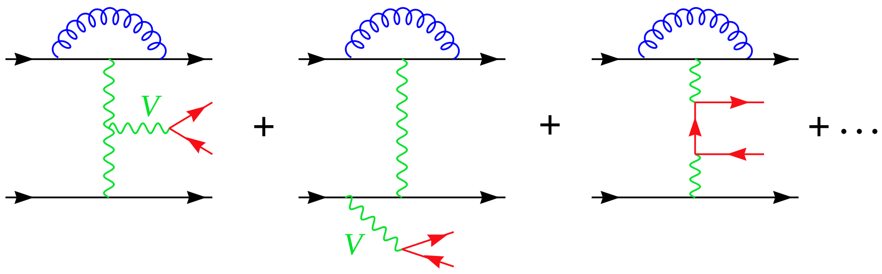
W and Z production



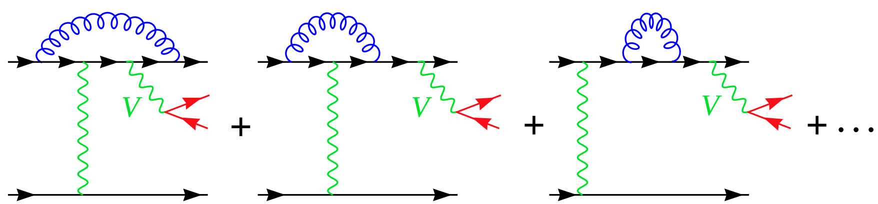
- $10 \cdots 24$ Feynman graphs
- ⇒ use amplitude techniques, i.e. numerical evaluation of helicity amplitudes
- However: numerical evaluation works in d=4 dimensions only

Virtual contributions

Vertex corrections: same as for Higgs case



New: Box type graphs (plus gauge related diagrams)



For each individual pure vertex graph $\mathcal{M}^{(i)}$ the vertex correction is proportional to the corresponding Born graph

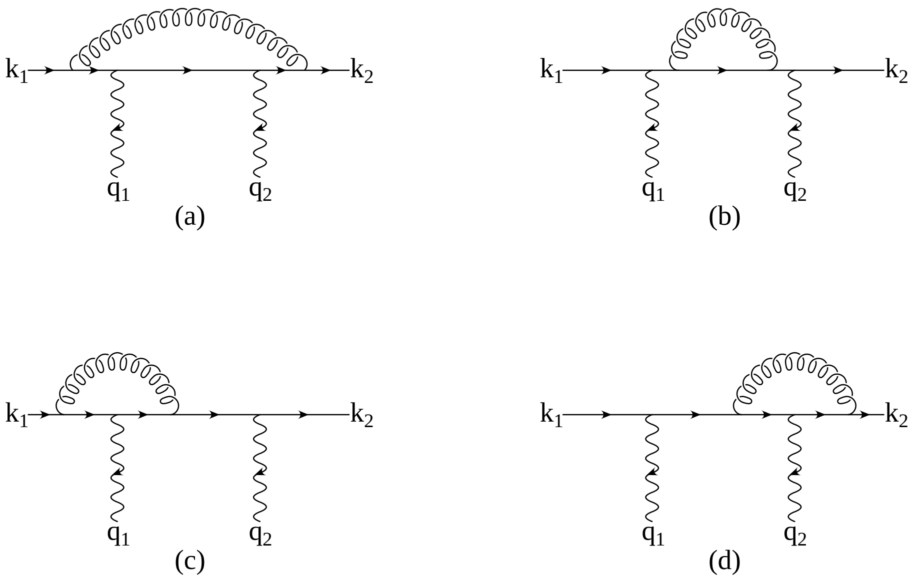
$$\mathcal{M}_V^{(i)} = \mathcal{M}_B^{(i)} \frac{\alpha_s(\mu_R)}{4\pi} C_F \left(\frac{4\pi\mu_R^2}{Q^2}\right)^\epsilon \Gamma(1+\epsilon) \left[-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} + \frac{\pi^2}{3} - 7\right]$$

Vector boson propagators plus attached quark currents are effective polarization vectors

build a program to calculate the finite part of the sum of the graphs

Boxline corrections

Virtual corrections for quark line with 2 EW gauge bosons



The external vector bosons correspond to $V \to l_1 \bar{l}_2$ decay currents or quark currents

Divergent terms in 4 Feynman graphs combine to multiple of corresponding Born graph

$$\mathcal{M}^{(i)}_{\text{boxline}} = \mathcal{M}^{(i)}_B F(Q) \left[-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} + \frac{\pi^2}{3} - 7 \right]$$
$$+ \frac{\alpha_s(\mu_R)}{4\pi} C_F \widetilde{\mathcal{M}}_\tau(q_1, q_2)(-e^2) g_\tau^{V_1 f_1} g_\tau^{V_2 f_2}$$
$$+ \mathcal{O}(\epsilon)$$

with $F(Q) = \frac{\alpha_s(\mu_R)}{4\pi} C_F \left(\frac{4\pi\mu_R^2}{Q^2}\right)^\epsilon \Gamma(1+\epsilon)$

$\widetilde{\mathcal{M}}_\tau(q_1, q_2) = \widetilde{\mathcal{M}}_{\mu\nu} \epsilon_1^\mu \epsilon_2^\nu$ is universal virtual $qqVV$ amplitude: use like HELAS calls in MadGraph

Handling of IR and collinear divergences

Use tensor decomposition a la Passarino-Veltman

Split $B_0 \quad \cdots \quad D_{ij}$ functions into divergent and finite parts

With $s = (q_1 + q_2)^2$, $t = (k_2 + q_2)^2 = (k_1 - q_1)^2$ we get, for example,

$$\begin{aligned}
B_0(q^2) &= \frac{\Gamma(1+\epsilon)}{(-s)^\epsilon} \left[\frac{1}{\epsilon} + 2 - \ln \frac{q^2 + i0^+}{s} + \mathcal{O}(\epsilon) \right] \\
&= \frac{\Gamma(1+\epsilon)}{(-s)^\epsilon} \left[\frac{1}{\epsilon} + \widetilde{B}_0(q^2) + \mathcal{O}(\epsilon) \right] \\
D_0(k_2, q_2, q_1) &= \frac{\Gamma(1+\epsilon)}{(-s)^\epsilon} \left[\frac{1}{st} \left(\frac{1}{\epsilon^2} + \frac{1}{\epsilon} \ln \frac{q_1^2 q_2^2}{t^2} \right) + \widetilde{D}_0(k_2, q_2, q_1) + \mathcal{O}(\epsilon) \right] \\
D^{\mu\nu}(k_2, q_2, q_1) &= \frac{\Gamma(1+\epsilon)}{(-s)^\epsilon} \left(\frac{1}{\epsilon} \left(k_1^\mu k_1^\nu d_2(q_1^2, t) + k_2^\mu k_2^\nu d_2(q_2^2, t) \right) + \widetilde{D}^{\mu\nu}(k_2, q_2, q_1) + \mathcal{O}(\epsilon) \right)
\end{aligned}$$

with $d_2(q^2, t) = 1/(s(q^2 - t)^2) \left[t \ln(q^2/t) - (q^2 - t) \right]$

Finite $\widetilde{D}_{ij}$ have standard PV recursion relations $\Longrightarrow$ determine them numerically

Virtual corrections

Born sub-amplitude is multiplied by same factor as found for pure vertex corrections
$\Rightarrow$ when summing all Feynman graphs the divergent terms multiply the complete $\mathcal{M}_B$

Complete virtual corrections

$$\mathcal{M}_V = \mathcal{M}_B \; F(Q) \left[-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} + \frac{\pi^2}{3} - 7 \right] + \widetilde{\mathcal{M}}_V$$

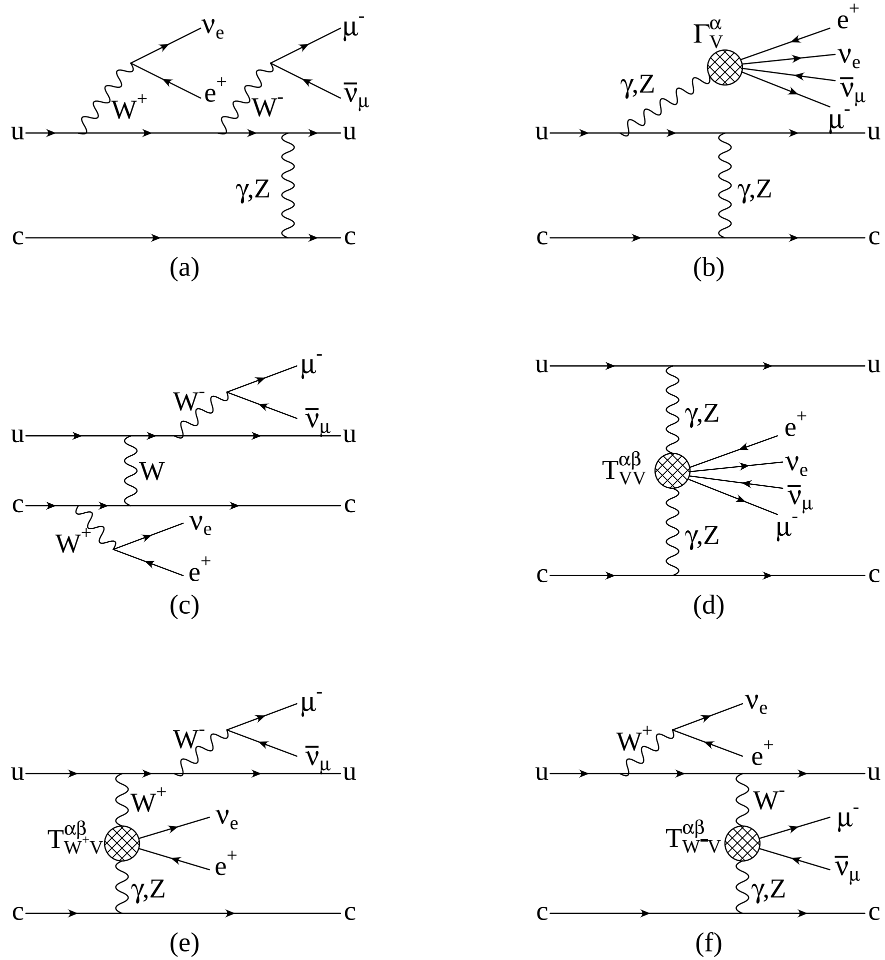
where $\widetilde{\mathcal{M}}_V$ is finite, and is calculated with amplitude techniques.
The interference contribution in the cross-section calculation is then given by

$$2\,\mathrm{Re}\,[\mathcal{M}_V \mathcal{M}_B^*] = |\mathcal{M}_B|^2 F(Q) \left[-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} + \frac{\pi^2}{3} - 7 \right] + 2\,\mathrm{Re}\left[\widetilde{\mathcal{M}}_V \mathcal{M}_B^* \right]$$

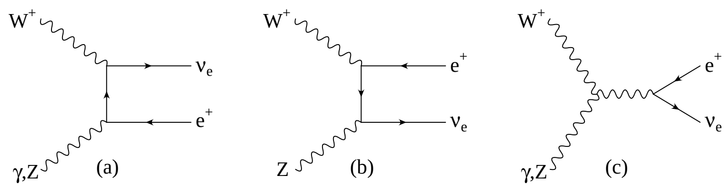
The divergent term, proportional to $|\mathcal{M}_B|^2$, cancels against the subtraction terms just like in the Higgs case.

3 weak bosons on a quark line: $qq \to qqWW$, $qqZZ$, $qqWZ$ at NLO

- example: WW production via VBF with leptonic decays: $pp \to e^+ \nu_e \mu^- \bar{\nu}_\mu + 2j$
- Spin correlations of the final state leptons
- All resonant and non-resonant Feynman diagrams included
- NC $\Longrightarrow$ 181 Feynman diagrams at LO
- CC $\Longrightarrow$ 92 Feynman diagrams at LO



Use modular structure, e.g. leptonic tensor

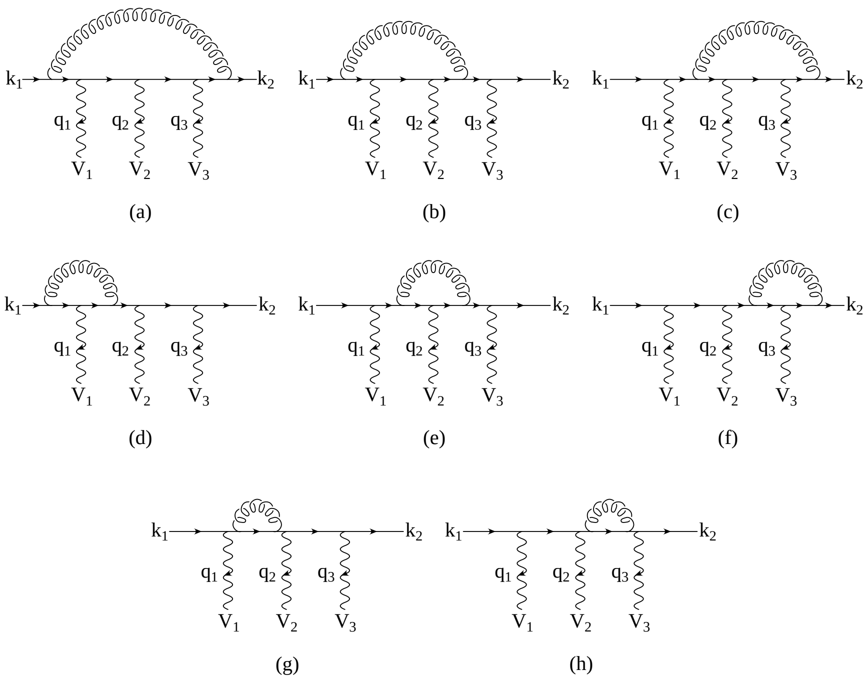


Calculate once, reuse in different processes

Speedup factor ≈ 70 compared to MadGraph for real emission corrections

New for virtual: pentline corrections

Virtual corrections involve up to pentagons



The external vector bosons correspond to $V \to l_1 \bar{l}_2$ decay currents or quark currents

The sum of all QCD corrections to a single quark line is simple

$$\mathcal{M}_V^{(i)} = \mathcal{M}_B^{(i)} \frac{\alpha_s(\mu_R)}{4\pi} C_F \left(\frac{4\pi\mu_R^2}{Q^2}\right)^{\epsilon} \Gamma(1+\epsilon) \left[-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} + c_{\text{virt}}\right] + \widetilde{\mathcal{M}}^{(i)}_{V_1V_2V_3,\tau}(q_1,q_2,q_3) + \mathcal{O}(\epsilon)$$

- Divergent pieces sum to Born amplitude: canceled via Catani Seymour algorithm
- Use amplitude techniques to calculate finite remainder of virtual amplitudes

Pentagon tensor reduction with Denner-Dittmaier is stable at 0.1% level

Gauge invariance tests

Numerical problems flagged by gauge invariance test: use Ward identities for pentline and boxline contributions

$$q_2^{\mu_2}\widetilde{\mathcal{E}}_{\mu_1\mu_2\mu_3}(k_1, q_1, q_2, q_3) = \widetilde{\mathcal{D}}_{\mu_1\mu_3}(k_1, q_1, q_2 + q_3) - \widetilde{\mathcal{D}}_{\mu_1\mu_3}(k_1, q_1 + q_2, q_3)$$

With Denner-Dittmaier recursion relations for E_{ij} functions the ratios of the two expressions agree with unity (to 10% or better) at more than 99.8% of all phase space points.

Ward identities reduce importance of computationally slow pentagon contributions when contracting with $W^\pm$ polarization vectors

$$J_\pm^\mu = x_\pm\, q_\pm^\mu + r_\pm^\mu$$

choose $x_\pm$ such as to minimize pentagon contribution from remainders $r_\pm$ in all terms like

$$J_+^{\mu_1} J_-^{\mu_2}\widetilde{\mathcal{E}}_{\mu_1\mu_2\mu_3}(k_1, q_+, q_-, q_0) = r_+^{\mu_1}\, r_-^{\mu_2}\,\widetilde{\mathcal{E}}_{\mu_1\mu_2\mu_3}(k_1, q_+, q_-, q_0) + \text{box contributions}$$

Resulting true pentagon piece contributes to the cross section at permille level $\Longrightarrow$ totally negligible for phenomenology

Phenomenology

Study LHC cross sections within typical VBF cuts

- Identify two or more jets with k_T-algorithm ($D = 0.8$)

$$p_{Tj} \geq 20 \text{ GeV}\,, \qquad |y_j| \leq 4.5$$

- Identify two highest p_T jets as tagging jets with wide rapidity separation and large dijet invariant mass

$$\Delta y_{jj} = |y_{j_1} - y_{j_2}| > 4, \qquad M_{jj} > 600 \text{ GeV}$$

- Charged decay leptons ($\ell = e,\ \mu$) of W and/or Z must satisfy

$$p_{T\ell} \geq 20 \text{ GeV}\,, \qquad |\eta_\ell| \leq 2.5\,, \qquad \triangle R_{j\ell} \geq 0.4\,,$$
$$m_{\ell\ell} \geq 15 \text{ GeV}\,, \qquad \triangle R_{\ell\ell} \geq 0.2$$

and leptons must lie between the tagging jets

$$y_{j,min} < \eta_\ell < y_{j,max}$$

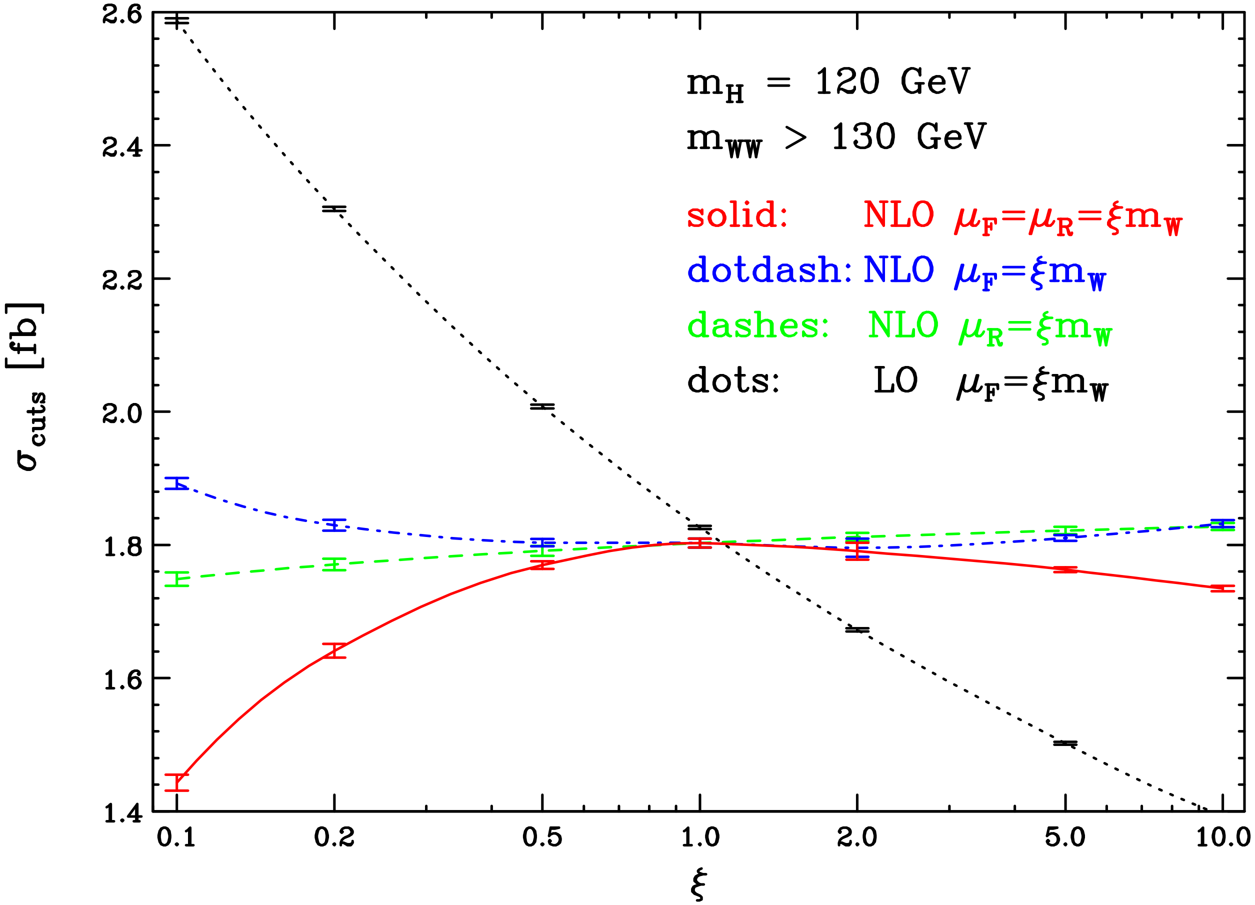
For scale dependence studies we have considered

$\mu = \xi\, m_V$ fixed scale $\qquad$ $\mu = \xi\, Q_i$ weak boson virtuality : $Q_i^2 = 2k_{q_1} \cdot k_{q_2}$

WW production: $pp \to jje^+\nu_e\mu^-\bar{\nu}_\mu X$ @ LHC

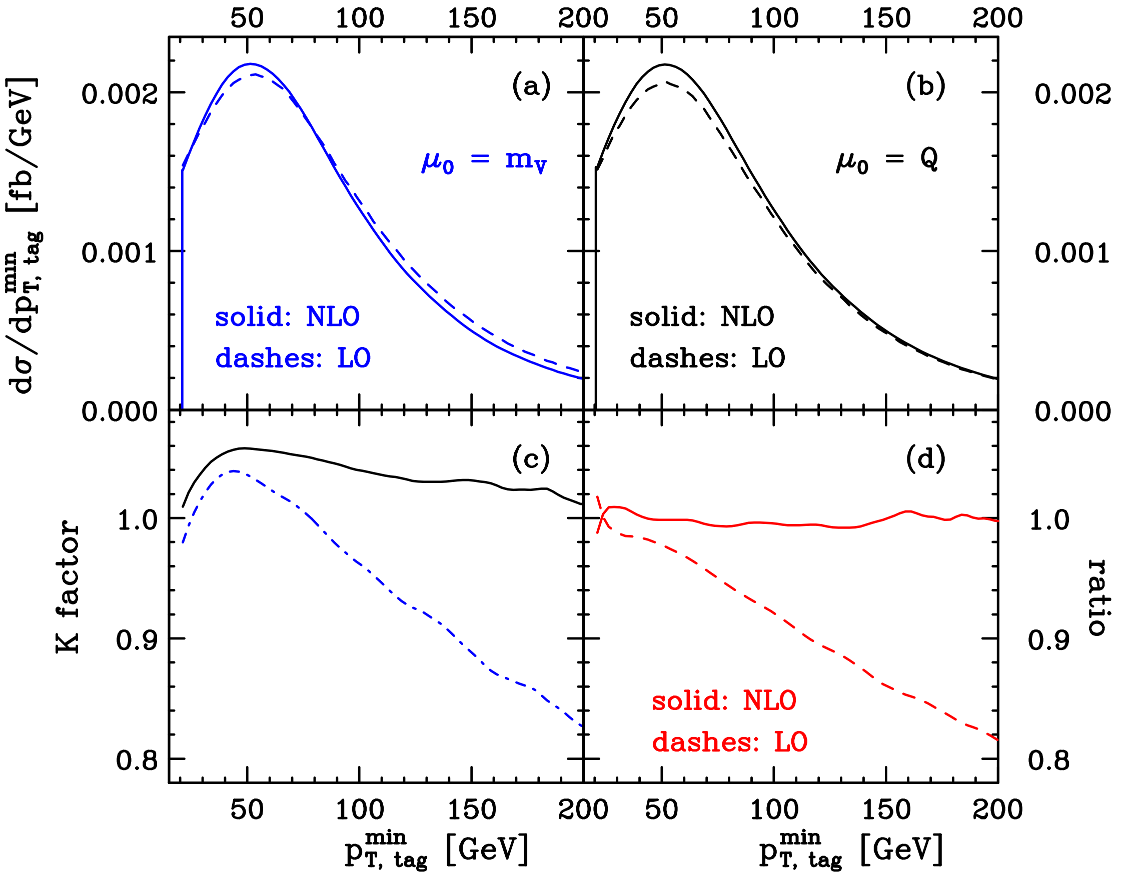
Stabilization of scale dependence at NLO

Jäger, Oleari, DZ hep-ph/0603177



WZ production in VBF, $WZ \to e^+ \nu_e \mu^+ \mu^-$

Transverse momentum distribution of the softer tagging jet



- Shape comparison LO vs. NLO depends on scale
- Scale choice $\mu = Q$ produces approximately constant K-factor
- Ratio of NLO curves for different scales is unity to better than 2%: scale choice matters very little at NLO

Use $\mu_F = Q$ at LO to best approximate the NLO results

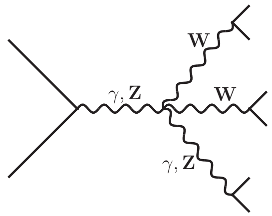
Crossing: VVV Production

- The pentline graphs directly correspond to production of three (virtual) electroweak bosons in $q\bar{q} \to VVV$
- Virtual QCD corrections fully contained in modules for boxline and pentline routines (and a trivial overall factor for the vertex amplitudes)
- Crossing is trivial for the basic helicity amplitudes of the fermion lines. Analytic continuation implemented for all scalar integrals: boxline and pentline routines work directly for crossed processes.
- New: subtraction of real emission singularities. Use Catani Seymour for Drell-Yan type processes. Implemented by Vera Hankele and tested against $q\bar{q} \to W^+W^-$ as implemented in MCFM.

Work in collaboration with V. Hankele, S. Prestel, C. Oleari and F. Campanario

Motivation

- Standard Model background for SUSY processes with multi-lepton + $\not{p}_T$ signature
- Possibility to obtain information about quartic electroweak couplings.



- QCD corrections to $pp \to$ VVV + X on experimentalist's wishlist:

  [The QCD, EW, and Higgs Working Group: hep-ph/0604120]

process ($V \in \{Z, W, \gamma\}$)	relevant for
1. $pp \to V\,V\,\mathrm{jet}$	$t\bar{t}H$, new physics
2. $pp \to t\bar{t}\,b\bar{b}$	$t\bar{t}H$
3. $pp \to t\bar{t} + 2\,\mathrm{jets}$	$t\bar{t}H$
4. $pp \to V\,V\,b\bar{b}$	VBF$\to H \to VV$, $t\bar{t}H$, new physics
5. $pp \to V\,V + 2\,\mathrm{jets}$	VBF$\to H \to VV$
6. $pp \to V + 3\,\mathrm{jets}$	various new physics signatures
7. $pp \to V\,V\,V$	SUSY trilepton

Status of the calculations

- Calculation of QCD corrections to ZZZ production without Higgs-contribution and leptonic decays.
  [Lazopoulos, Melnikov, Petriello; hep-ph/0703273]

- Calculation of QCD corrections to ZZZ, W^+W^-Z, $W^+W^-W^+$ and ZZW^+ production without Higgs-contribution and leptonic decays.
  [Binoth, Ossola, Papadopoulos, Pittau; arXiv:0804:0350]

- Calculation of QCD corrections to W^+W^-Z production with leptonic decays.
  [Hankele, DZ; arXiv:0712.3544]

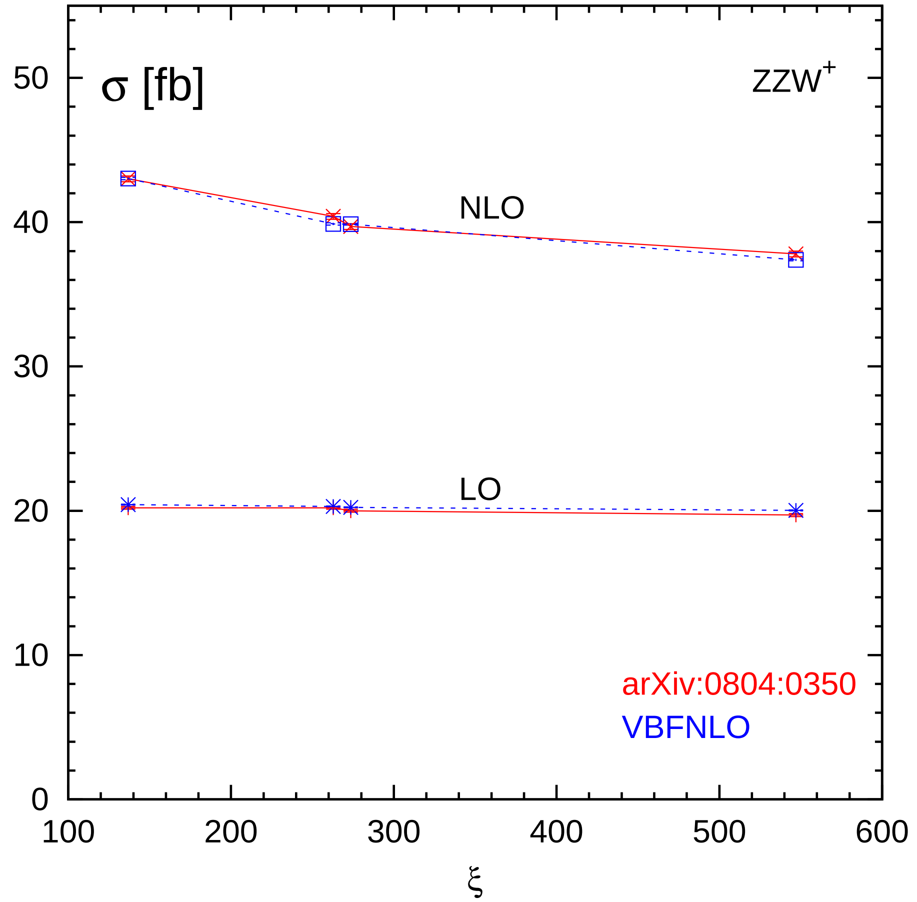
- Calculation of QCD corrections to $ZZW^\pm$ and $W^\pm W^\mp W^\pm$ production with leptonic decays.
  [Campanario, Hankele, Oleari, Prestel, DZ; arXiv:0809.0790]

Implemented into the Fortran program `VBFNLO`.

Release of new version including tri-boson processes: arxiv:0811.4559

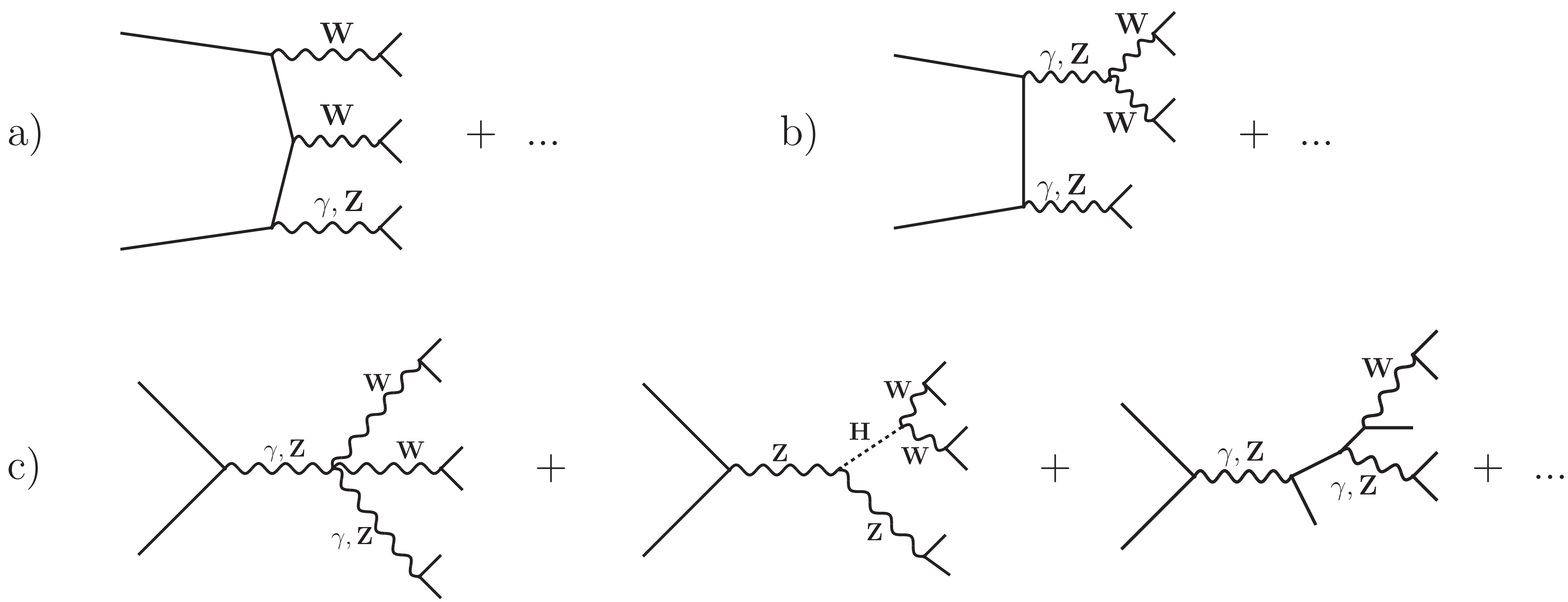
Comparison of various tri-boson codes

Numerous checks on the final results. For example comparison of ZZW^+ in narrow width approximation and without Higgs contribution with [Binoth, Ossola, Papadopoulos, Pittau; arXiv:0804:0350]



$\Rightarrow$ Agreement at the level of the accuracy of the Monte Carlo runs.

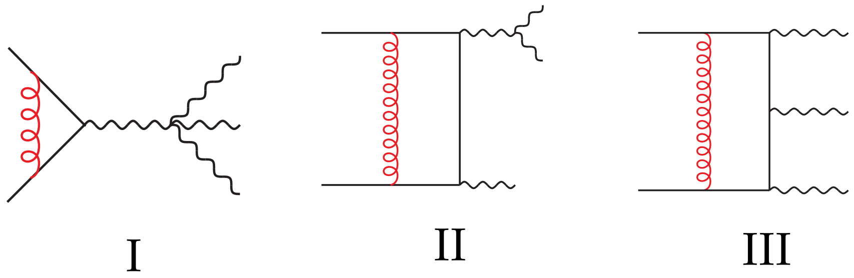
Contributions to WWZ production in our code

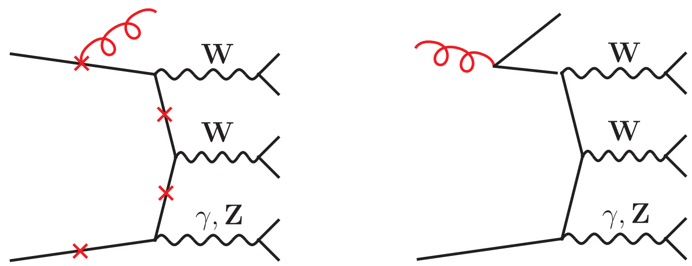


- All resonant and non-resonant matrix elements as well as spin correlations of final state leptons and Higgs contribution included.
- Interference terms due to identical particles in the final state have been neglected.
- All fermion mass effects neglected. ($H\tau\tau$-coupling = 0)

1-loop matrix elements and real emission matrix elements

Three different topologies:





- I Vertex correction proportional to Born matrix element.
- II Maximally 4-point integrals appear.
- III Up to five external legs (Pentagons):
  - Two independent calculations.
  - Numerically stable results with Denner Dittmaier method.

- Two different classes: final state gluon and initial state gluon.
- Each of them consists of several hundred Feynman-Graphs.
- No initial state gluon contribution at LO.

Input variables for LHC phenomenology

- **PDFs**: CTEQ6L1 at LO and CTEQ6M, $\alpha_S(m_Z) = 0.118$ at NLO.

- **Cuts and Masses**:

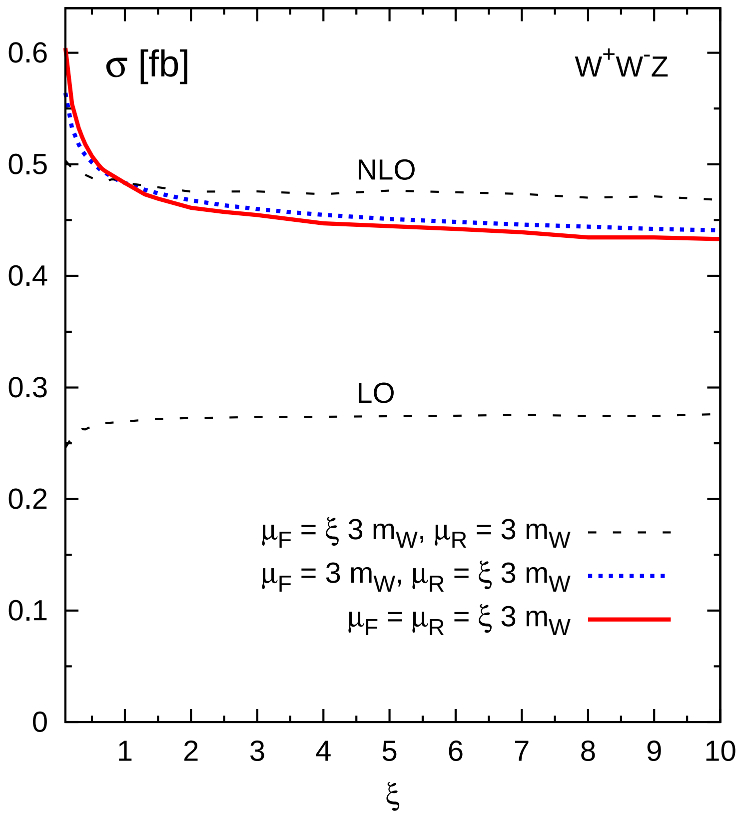
$$p_{T_\ell} > 10 \text{ GeV}, \qquad |\eta_\ell| < 2.5, \qquad m_{\ell^+\ell^-} > 15 \text{ GeV}, \qquad m_H = 120 \text{ GeV}.$$

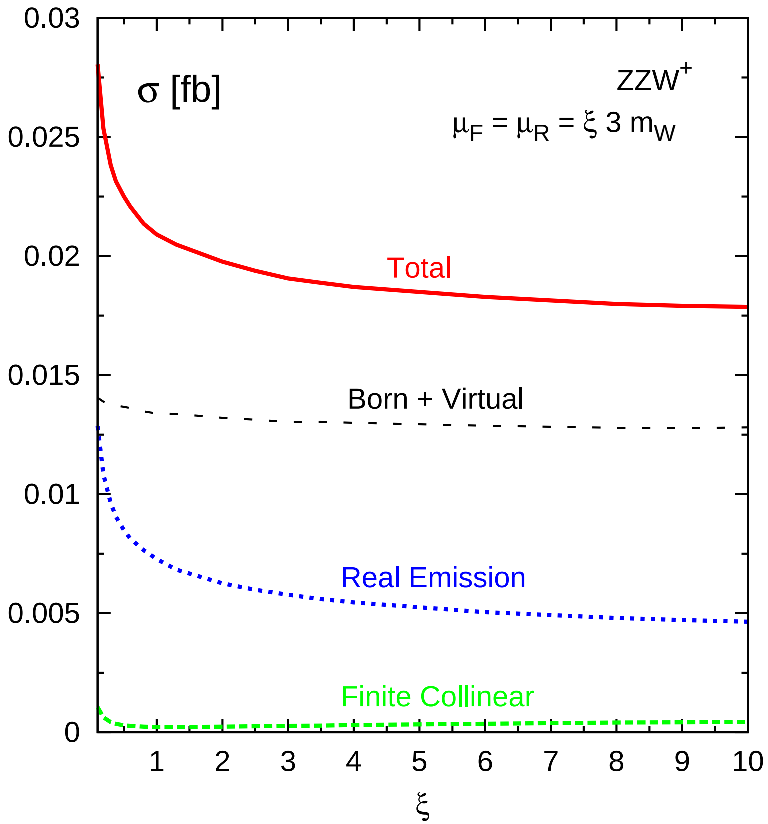
- **Renormalization- and Factorization Scale**: $\mu_F = \mu_R = 3\, m_W$.

Following results are for electrons and/or muons in the final state:

$\Longrightarrow$ Combinatorial factor of 8/4 for the $W^+W^-Z/ZZW^\pm$ production compared to three different lepton families in the final state.

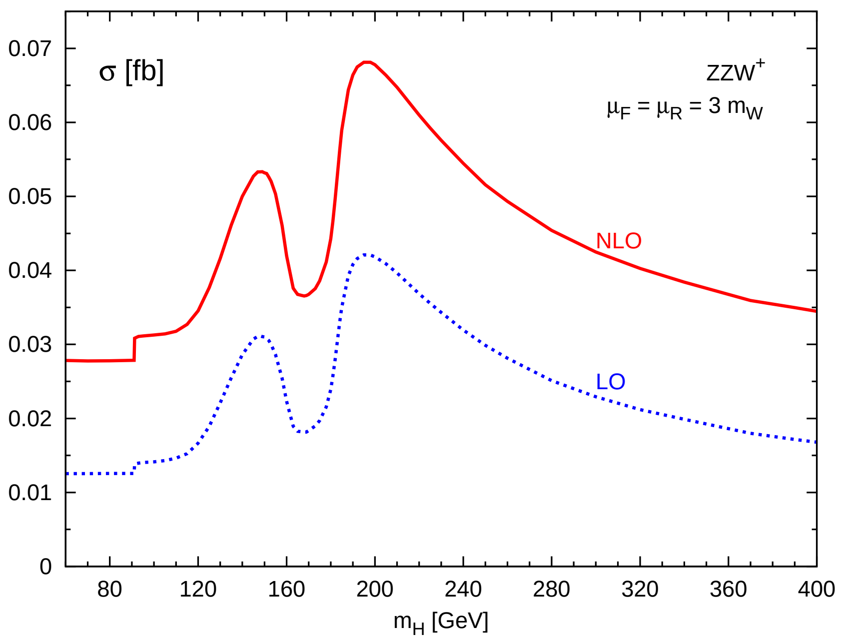
Scale Dependence

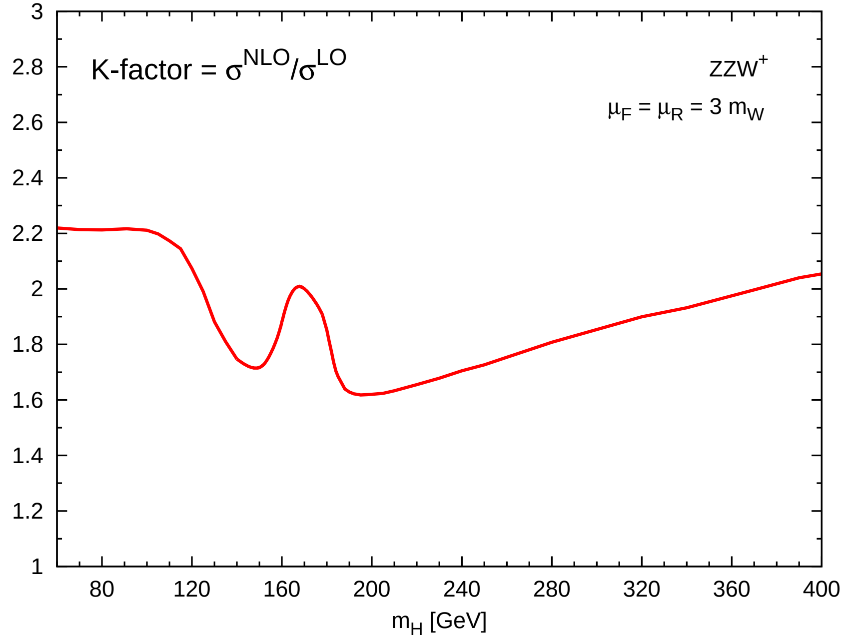


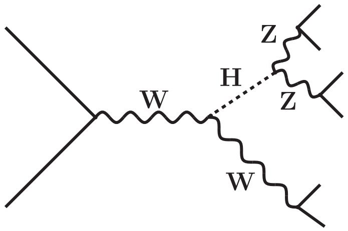


- At LO only small μ_F-dependence, no $\alpha_s(\mu_R)$.
- At NLO scale dependence is dominated by $\alpha_s(\mu_R)$.
- Real emission contribution drives overall scale dependence at NLO.

Higgs mass dependence

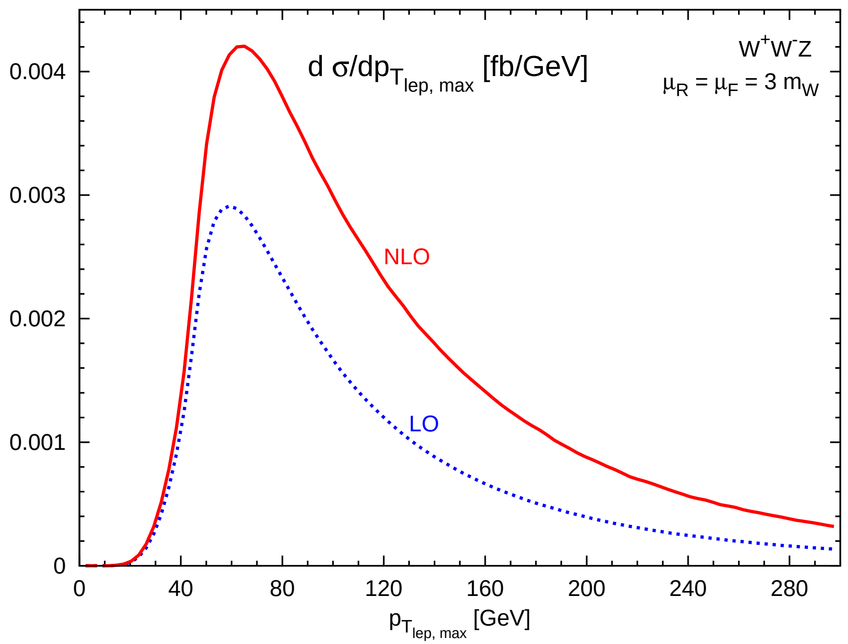


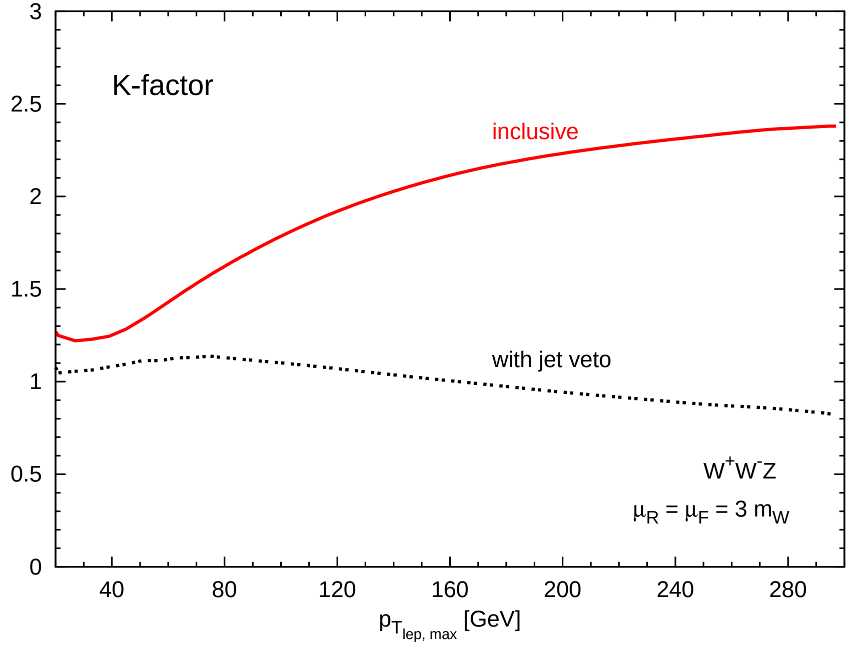




- Cross section reflects behavior of $BR(H \to ZZ)$
- K-factor is reduced by Higgs contribution.
  K-factor for $pp \to ZH$ production is about K = 1.3
  [Han and Willenbrock, Phys. Lett. B **273** (1991) 167.]

Differential cross section and K-factor for the highest-p_T-lepton





- K-factor increases with transverse momentum (p_T) by almost a factor of 2.
- Strong phase space dependence due to events with high p_T jets recoiling against the leptons.
- Veto on jets with $p_T > 50$ GeV leads to flat K-factor.

Conclusions

- NLO QCD corrections to $pp \rightarrow$ VVV + X are Standard Model background processes for new-physics searches and are sensitive to quartic electroweak couplings.
- All off-shell diagrams as well as the Higgs-contributions have been considered.
- The K-factor is sizeable and NLO corrections lead to substantial shape changes of lepton distributions.
- Sizable scale dependence of the NLO cross section, small scale dependence at LO.
- New release of `VBFNLO` includes NLO QCD corrections for W^+W^-Z, $ZZW^\pm$ and $W^\pm W^\mp W^\pm$ production at hadron colliders: arxiv:0811.4559

  Code is available at
  http://www-itp.particle.uni-karlsruhe.de/~vbfnloweb