Direct Numerical Integration of One-loop Feynman Diagrams for N-photon Amplitudes

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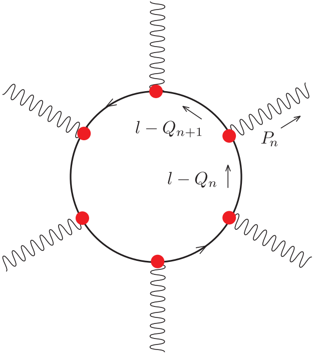
A Brief Review

• The importance of calculating virtual loop Feynman diagrams with multiple external legs

• For simplicity, we consider the amplitude for $\gamma + \gamma \rightarrow (N-2)\,\gamma$ through a (massless) electron loop.



• Singularities and numerical Monte Carlo integration over the loop momentum l and $\{p_1, ..., p_n\}$

$$\mathcal{M} = \int \frac{d^4 l}{(2\pi)^4}\ (-\mathrm{i}e)^N N(l) \prod_{i=1}^{N} \frac{\mathrm{i}}{(l - Q_i)^2 + \mathrm{i}0} \quad . \tag{1}$$

Deform the loop momentum to ℓ and let

$$\ell^\mu(l) = l^\mu + \mathrm{i}\kappa^\mu(l) \quad . \tag{2}$$

With this notation, the integral becomes

$$\mathcal{M} = \int \frac{d^4 l}{(2\pi)^4}\ (-\mathrm{i}e)^N \det(\partial\ell/\partial l)\ N(\ell(l)) \prod_{i=1}^{N} \frac{\mathrm{i}}{(l - Q_i)^2 + \mathrm{i}0} \quad . \tag{3}$$

- The direct numerical integration and its advantage

- The $+\mathrm{i}0$ prescription

We parameterize the deformation using $0 < \lambda(l) < \lambda_{\mathrm{f}}(l)$,

$$\kappa(l) = \lambda(l)\,\kappa_0(l) \quad . \tag{4}$$

and thus the denominator of propagator i becomes

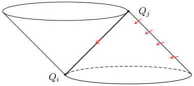
$$(l - Q_i + \mathrm{i}\kappa(l))^2 = (l - Q_i)^2 - \lambda(l)^2\,\kappa_0(l)^2 + 2\mathrm{i}\,\lambda(l)\,(l - Q_i)\cdot\kappa_0(l) \quad . \tag{5}$$

The prescription requires $(l - Q_i)\cdot\kappa_0(l) \geq 0$ on any of the surfaces $(l - Q_i)^2 = 0$.

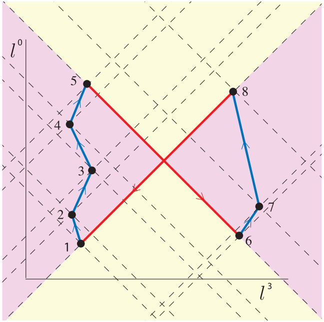
The geometric interpretation is for l on the cone $(l - Q_i)^2 = 0$, $\kappa_0(l)$ satisfies the condition $(l - Q_i)\cdot\kappa_0(l) > 0$ by pointing toward the interior of the cone.

The Deformation

- An example: two light-like seperate light cones



- Geometric arrangements of the light cones



- The general formula

$$\kappa_0 = -\sum_{j=1}^{N} c_j(l - Q_j) + \tilde{c}_+ (P + \bar{P}) - \tilde{c}_- (P + \bar{P}) \quad . \qquad (6)$$

The coefficients c_j and $\tilde{c}_\pm$ are non-negative functions of l.

- How far to deform?

$$\lambda(l) = \min[\lambda_{\mathrm{c}}, \lambda_0(l), \min_i \{\lambda_i(l)\}] \quad . \qquad (7)$$

To determine λ_i, we consider the ith denominator,

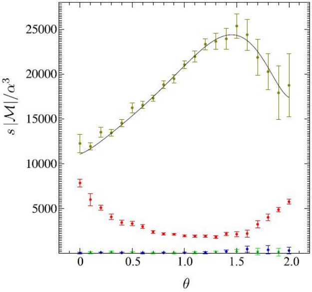
$$D_i = (l - Q_i + \mathrm{i}\lambda\kappa_0)^2 = (l - Q_i)^2 + 2\mathrm{i}\lambda\kappa_0 \cdot (l - Q_i) - \lambda^2\kappa_0^2 \quad , \qquad (8)$$

and the function D_i vanishes at values of λ given by

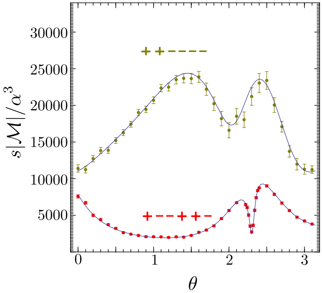
$$\lambda = \frac{1}{\kappa_0^2} \left\{ \mathrm{i}\ \kappa_0 \cdot (l - Q_i) \pm \sqrt{\kappa_0^2 (l - Q_i)^2 - [\kappa_0 \cdot (l - Q_i)]^2} \right\} \quad . \qquad (9)$$

Results

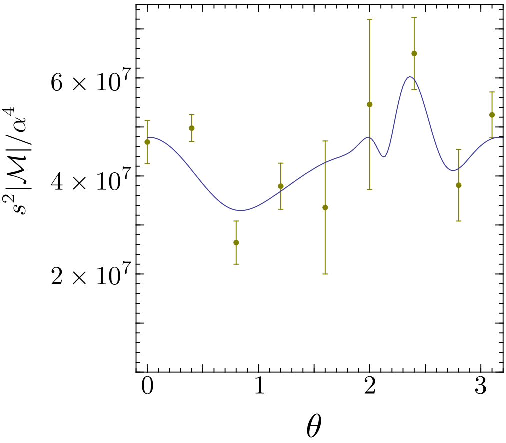
Using Feynman Parameters



Using direct integration



- Eight photons



Conclusions

• The method of direct numerical integration has better convergence over the method that makes use of the Feynman parameters for 6 external photon legs.

• The method of direct numerical integration can have results for 8 external photon legs, while the method of Feynman parameters breaks down.