

Collinear factorization for DIS at large x_B and low Q^2

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Motivation and outline

Why large x_B and low Q^2 ?

- Large uncertainties in quark and gluon PDF at $x > 0.5$
- Precise PDF at large x are needed, e.g.,
 - at LHC
 - 1) New physics as excess in large p_T spectra \Leftrightarrow large x PDF
 - 2) DGLAP evolution feeds large x , low Q^2 into lower x , large Q^2
 - d/u ratio at $x=1$ \Leftrightarrow non-perturbative structure of the nucleon
- JLAB has precision DIS data at large x_B , BUT low Q^2
 - need of theoretical control over
 - 1) higher twist $\propto \Lambda^2/Q^2$
 - 2) target mass corrections (TMC) $\propto x_B^2 m_N^2/Q^2$
 - 3) jet mass corrections (JMC) $\propto m_f^2/Q^2$

} **this talk**

OPE and Target Mass Corrections

[Georgi, Politzer 1976; see review of Schienbein et al. 2007]

$$\int d^4z e^{-iq \cdot z} \langle N | T [j^{\dagger\mu}(z) j^\nu(0)] | N \rangle = \sum_k f^{\mu_1 \dots \mu_{2k}} A_{2k} \langle N | \underbrace{\mathcal{O}_{\mu_1 \dots \mu_{2k}}(0)}_{\text{symmetric, traceless}} | N \rangle$$

$$A_{2k} = \int_0^1 dy y^{2k} F(y) \quad F(y) \sim \Sigma e_q^2 q(y) \text{ (at LO)} = \text{“quark function”}$$

➡ Mellin transform, resum, transform back:

$$F_2^{\text{GP}}(x, Q^2) = \frac{x^2}{r^3} F(\xi) + 6 \frac{M^2 x^3}{Q^2 r^4} \int_\xi^1 d\xi' F(\xi') + 12 \frac{M^4 x^4}{Q^4 r^5} \int_\xi^1 d\xi' \int_{\xi'}^1 d\xi'' F(\xi'')$$

$$\xi = \frac{2x_B}{1 + \sqrt{1 + 4x_B^2 m^2 / Q^2}} = \text{Nachtmann variable}$$

➡ Threshold problem: $x_B \leq 1$ implies $0 \leq y \leq \xi_{\text{th}} \stackrel{\text{def}}{=} \xi(x_B=1)$

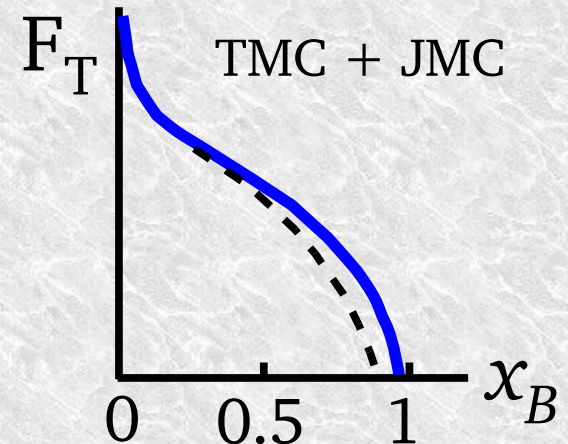
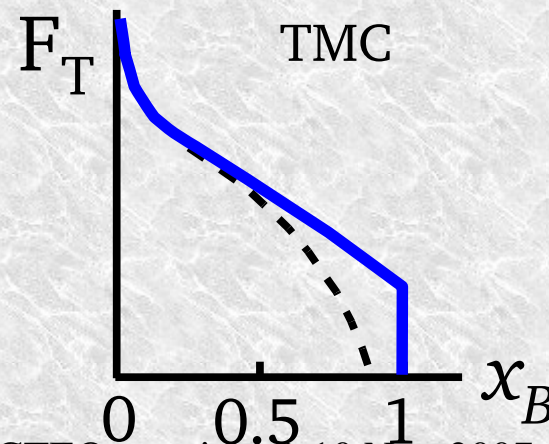
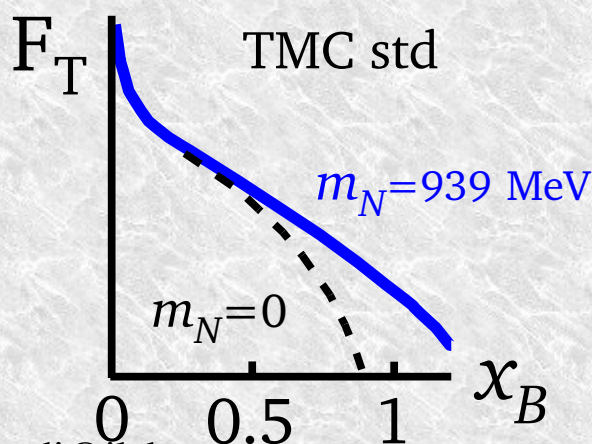
➡ Inverse Mellin transform does not give back $F(y)$!! [Johnson, Tung 1979]

➡ Unphysical region: $q(y)$ has support over $0 < y < 1$

➡ $F_2(x_B) > 0$ also for $x_B > 1$!!

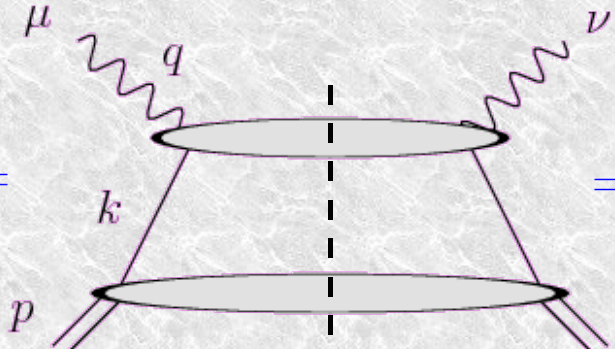
Collinear factorization - outline

- Target Mass Corrections – $O(x_B^2 m_N^2/Q^2)$
 - momentum space, no need of Mellin transf.
 - analiticity of handbag diagram
⇒ no “unphysical region” at $x_B > 1$ (!!)
 - any order in α_s at leading twist
- Jet Mass Corrections – $O(m_J^2/Q^2)$
 - leading order in α_s , leading twist
- Conclusion:
 - factorized formula with TMC + JMC
 - remarks, open issues



Target mass corrections

Kinematics with $m_N \neq 0$



$$W^{\mu\nu}(p, q) = \frac{1}{8\pi} \int d^4z e^{-iq \cdot z} \langle p | j^{\dagger\mu}(z) j^\nu(0) | p \rangle$$

◆ Collinear frames:

$$p^\mu = p^+ \bar{n}^\mu + \frac{m^2}{2p^+} n^\mu$$

$$q^\mu = -\xi x p^+ \bar{n}^\mu + \frac{Q^2}{2\xi x p^+} n^\mu$$

$$k^\mu = x p^+ \bar{n}^\mu + \frac{k^2 + k_T^2}{2x p^+} n^\mu + k_T^\mu$$

where:

$$x = \frac{k^+}{p^+} \quad \xi = \frac{2x_B}{1 + \sqrt{1 + 4x_B^2 m_N^2 / Q^2}}$$

Lorentz invariants:

$$x_B = \frac{-q^2}{2p \cdot q} \quad Q^2 = -q^2$$

$$x_q = \frac{-q^2}{2k \cdot q} \quad m_N^2 = p^2$$

Light cone vectors:

$$\bar{n} = (1/\sqrt{2}, \vec{0}_\perp, 1/\sqrt{2})$$

$$n = (1/\sqrt{2}, \vec{0}_\perp, -1/\sqrt{2})$$

$$a^\pm = (a_0 \pm a_3)/\sqrt{2}$$

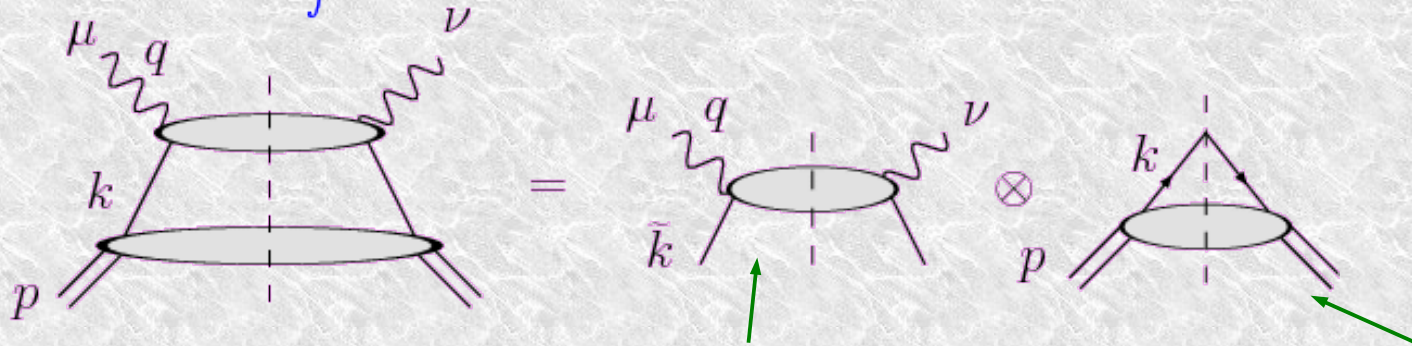
◆ Bjorken limit: $\xi \rightarrow x_B \quad x_q \rightarrow x_B / x$

Factorization theorem with $m_N \neq 0$

[see also Qiu's talk at CTEQ meeting 2005]

- Expand around $\tilde{k}^\mu = xp^+ \bar{n}^\mu$ $\tilde{k}^2 = 0$ $\tilde{x}_q = \frac{-q^2}{2\tilde{k} \cdot q} = \frac{\xi}{x}$

$$W_N^{\mu\nu}(p, q) = \sum_f \int \frac{dx}{x} \mathcal{H}_f^{\mu\nu}(\tilde{k}, q) \varphi_{f/N}(x, Q^2) + O(\Lambda^2/Q^2)$$



perturbative: doesn't know about the target's mass

dynamical TMC only from nucleon w.f.

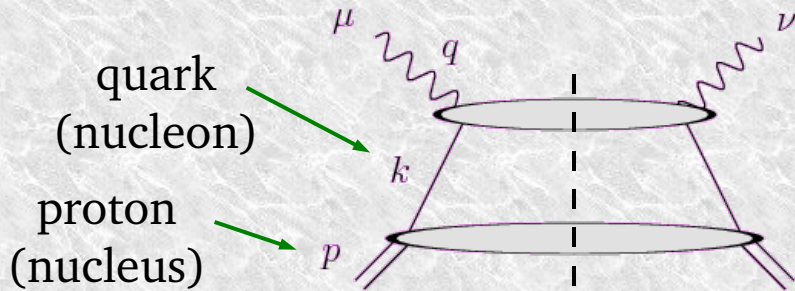
- Helicity structure functions F_T , F_L projected out of $W^{\mu\nu}$: e.g.,

$$F_T(x_B, Q^2) = \sum_f \int \frac{dx}{x} h_{fT}(\underbrace{\tilde{x}_q}_{= \xi/x}, Q^2) \varphi_{f/N}(x, Q^2) + O(\Lambda^2/Q^2)$$

no kinematic prefactors [Aivazis, Olness, Tung 1994]

Kinematic constraints

- General handbag diagram:



$$\alpha = \frac{k \cdot q}{p \cdot q} = \frac{x_B}{x_q}$$

- Analitycity, ∞ momentum frame $\Rightarrow x_B \leq \alpha \leq 1$

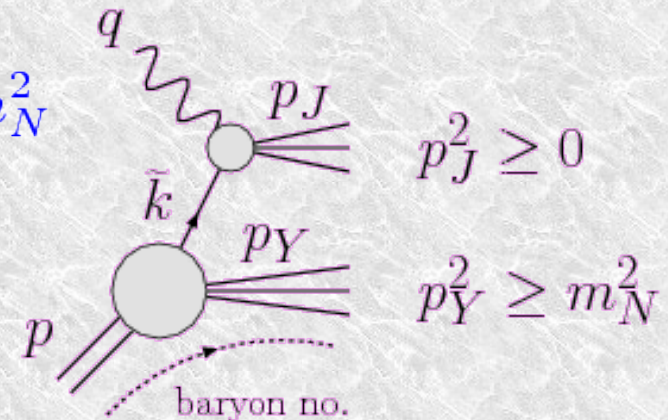
\Rightarrow for us, $k = \tilde{k}$, $\tilde{\alpha} = \xi / x \Rightarrow \xi \leq x \leq \xi / x_B$

- Simple proof for on-shell $\tilde{k}^2=0$, $\tilde{\alpha} = \tilde{k} \cdot q / p \cdot q$ (works also for heavy quarks)

$\Rightarrow 0 \leq (\tilde{k} + q)^2 = Q^2 \left(\frac{1}{\tilde{x}_q} - 1 \right) \Rightarrow \tilde{x}_q \leq 1 \Rightarrow \tilde{\alpha} \geq x_B$

$\Rightarrow s = (p + q)^2 = (p_J + p_Y)^2 \geq p_J^2 + p_Y^2 \geq p_J^2 + m_N^2$

$$\left. \begin{aligned} p_J^2 &= \left(\frac{\tilde{\alpha}}{\tilde{x}_B} - 1 \right) Q^2 \\ s - m_N^2 &= \left(\frac{1}{\tilde{x}_B} - 1 \right) Q^2 \end{aligned} \right\} \Rightarrow \tilde{\alpha} \leq 1$$



No unphysical region!

$$F_T(x_B, Q^2) = \sum_f \int_{\xi}^{x_B} \frac{dx}{x} h_{fT}\left(\frac{\xi}{x}, Q^2\right) \varphi_{f/N}(x, Q^2)$$

$$F_T(x_B, Q^2) = 0 \quad \text{at } x_B \geq 1$$

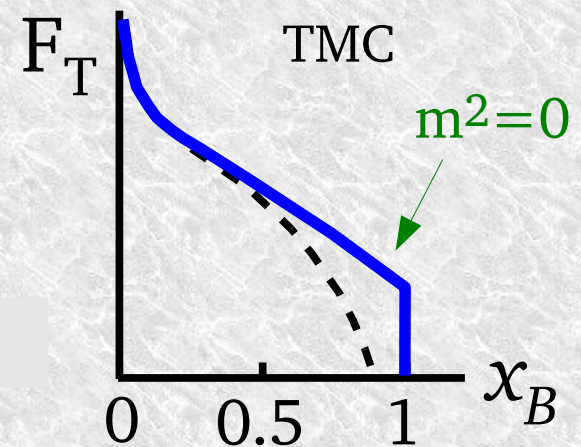
- Bjorken limit recovers “massless” structure functions ($m_N=0$)

$$F_T(x_B, Q^2) \longrightarrow \sum_f \int_{x_B}^1 \frac{dx}{x} h_{fT}\left(\frac{x_B}{x}, Q^2\right) \varphi_{f/N}(x, Q^2) \equiv F_T^{(0)}(x_B, Q^2)$$

- But... at leading order,

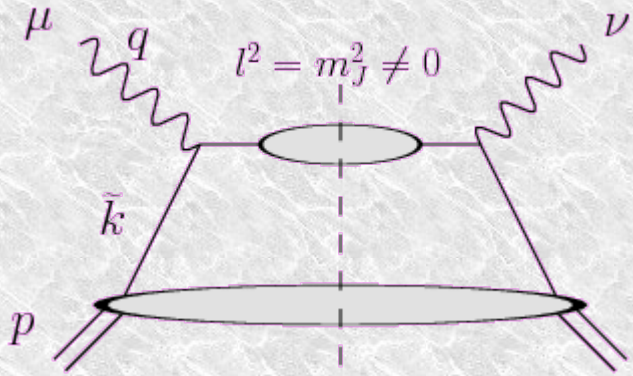
$$h_{fT}\left(\frac{\xi}{x}, Q^2\right) = x \frac{1}{2} e_f^2 \delta\left(\frac{\xi}{x} - 1\right) = \begin{array}{c} \text{m}^2=0 \\ \swarrow \quad \searrow \\ \begin{array}{c} q \\ \text{---} \\ \bar{k} \end{array} \end{array}$$

$$F_T(x_B, Q^2) = \frac{1}{2} \sum_f e_f^2 \phi(\xi) = F_T^{(0)}(\xi, Q^2) \quad \text{at } x_B < 1$$



Jet smearing, heuristically

- ◆ Ansatz: jet with a non zero mass, smoothly distributed in m_J^2



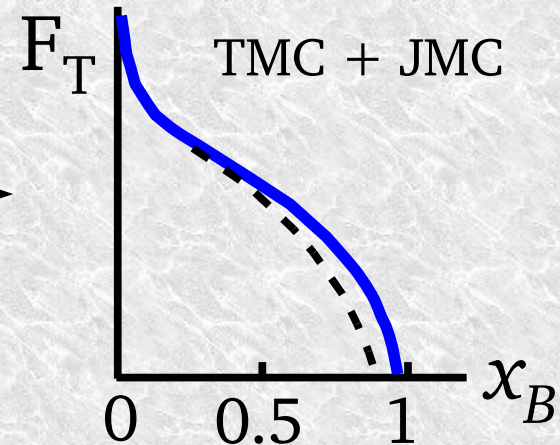
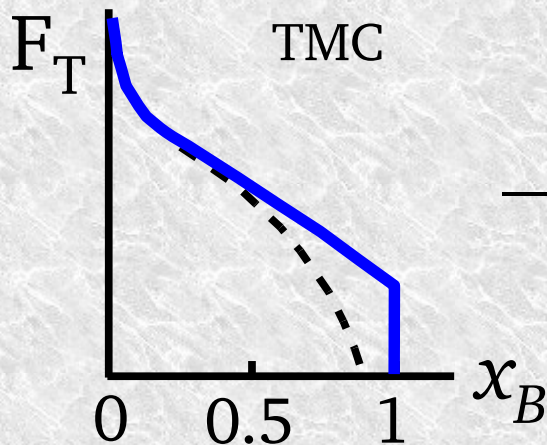
$$(k + q)^2 = m_J^2 \quad \longrightarrow \quad \delta \left[x - \xi \left(1 + \frac{m_J^2}{Q^2} \right) \right]$$

jet mass distribution

$$F_T(x_B, Q^2) = \int_0^\infty dm_J^2 J(m_J^2) \int_\xi^{\frac{\xi}{x_B}} dx \frac{1}{2} e_q^2 \delta \left[x - \xi \left(1 + \frac{m_J^2}{Q^2} \right) \right] \varphi_f(x, Q^2)$$

note the
int. limits

$$= \int_0^{\frac{1-x_B}{x_B} Q^2} dm_J^2 J(m_J^2) F_T^{(0)} \left(\xi \left(1 + \frac{m_J^2}{Q^2} \right), Q^2 \right)$$

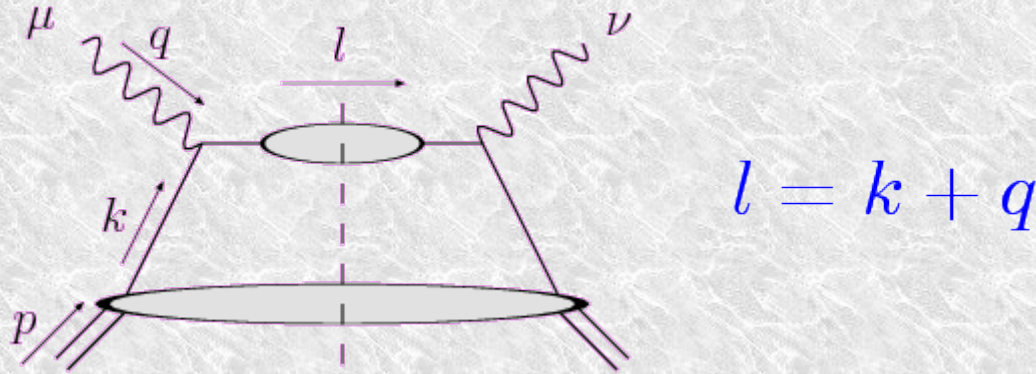


Jet mass corrections

Collinear factorization with a jet function

[see also Collins, Rogers, Stasto, 2007]

- Handbag diagram with a quark jet



$$W^{\mu\nu}(p, q) = \frac{e_q^2}{8\pi} \int \frac{d^4k}{(2\pi)^4} \text{Tr} [\hat{T}(k) \gamma^\nu \hat{J}(l) \gamma^\mu]$$

- A hat denotes a Dirac matrix:

$$\hat{T}(k) = \begin{array}{c} i \quad j \\ \diagup \quad \diagdown \\ k \quad | \\ \text{---} \text{---} \text{---} \end{array} = \int d^4z e^{iz \cdot k} \langle p | \bar{\psi}_j(z) \psi_i(0) | p \rangle$$

$$\hat{J}(l) = \begin{array}{c} | \\ \text{---} \text{---} \text{---} \\ l \end{array} = \int d^4z e^{iz \cdot l} \langle 0 | \bar{\psi}_j(z) \psi_i(0) | 0 \rangle$$

(color factors are included in \hat{T})

Factorization procedure

[Ellis, Furmanski, Petronzio, 1983]

- Expand on a basis of Dirac matrices

$$\hat{T}(k) = \tau_1(k)\hat{\mathbb{1}} + \tau_2(k)\not{k} + \tau_3(k)\gamma_5 + \tau_4(k)\not{k}\gamma_5$$

\nearrow =0 for massless quarks $\nwarrow \swarrow$ cancel for unpolarized targets

$$\hat{J}(l) = j_1(l)\hat{\mathbb{1}} + j_2(l)\not{l} + j_3(l)\gamma_5 + j_4(l)\not{l}\gamma_5$$

\nearrow enter traces with odd no. of γ 's $\nwarrow \swarrow$ =0 in pure QCD + EM (parity invariance)

- Dominance of k^+ , l^- in Breit frame suggests to define

$$\tau_2(k) = \frac{1}{4k^+} \text{Tr}[\not{n}\hat{T}(k)] = \frac{1}{4k^+} \int d^4z e^{iz \cdot k} \langle p | \bar{\psi}_j(z) \gamma^+ \psi(0) | p \rangle$$

$$j_2(k) = \frac{1}{4l^-} \text{Tr}[\not{\bar{n}}\hat{J}(l)] = \frac{1}{4l^-} \int d^4z e^{iz \cdot l} \langle 0 | \bar{\psi}_j(z) \gamma^- \psi(0) | 0 \rangle$$

Jet spectral representation

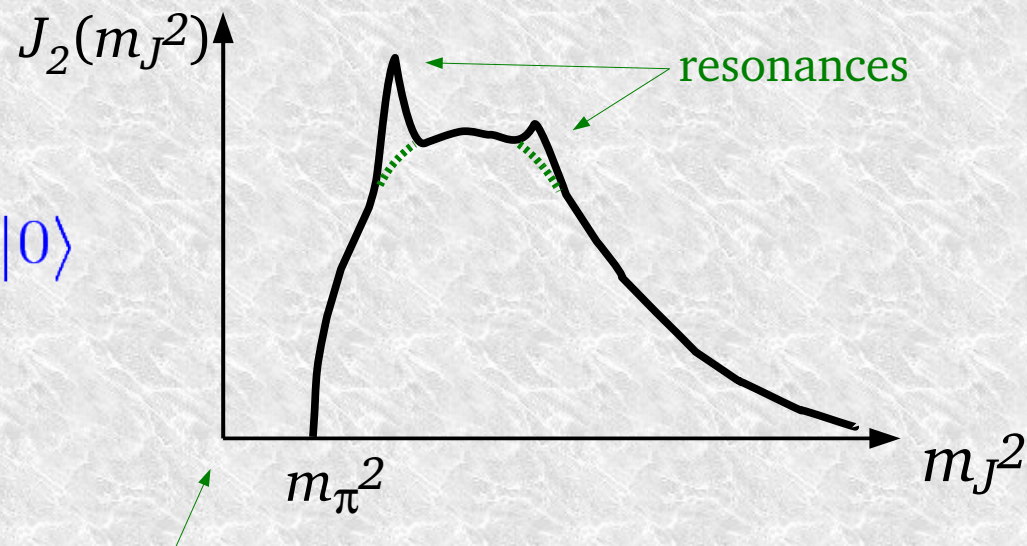
$$\begin{aligned}
 \left[\text{jet} \right] &= \sum_n (2\pi)^4 \delta^{(4)}(l - \sum_1^n p_i^h) \left| \text{hadron} \right|^2 \\
 &= \int_0^\infty dm_j^2 [J_1(m_j^2) \hat{\mathbb{I}} + J_2(m_j^2) \not{l}] 2\pi \delta(l^2 - m_j^2) \theta(l^0)
 \end{aligned}$$

$$j_2(l) = \int_0^\infty dm_j^2 J_2(m_j^2) 2\pi \delta(l^2 - m_j^2) \theta(l^0) \quad \text{with} \quad \int_0^\infty dm_j^2 J_2(m_j^2) = 1$$

- ◆ $J_2(m_j^2)$ measurable in lattice QCD!

$$J_2(m_j^2) \propto F.T. \langle 0 | \bar{\psi}_j(z) \gamma^- \psi_i(0) | 0 \rangle$$

- ◆ I am assuming color neutralization through a (neglected) soft exchange with the target jet



QCD vacuum is confining:
no zero mass hadrons

Collinear expansion - 1

$$W^{\mu\nu}(p, q) = \int \frac{d^4k}{(2\pi)^4} \frac{e_q^2}{8\pi} \underbrace{\text{Tr}[k\gamma^\nu \not{l}\gamma^\mu]}_{= \frac{1}{2\pi} H_*^{\mu\nu}(k, l)} j_2(l) \tau_2(k) \mathbb{K}(k, p, q)$$

↑
kinematic constraints

$$k^\mu = xp^+ \bar{n}^\mu + \frac{k^2 + k_T^2}{2xp^+} n^\mu + k_T^\mu$$

$$l^\mu = (x - \xi)p^+ \bar{n}^\mu + \left(\frac{k^2 + k_T^2}{2xp^+} + \frac{Q^2}{2\xi p^+} \right) n^\mu + k_T^\mu$$

1) Expand $H_*(k, l)$ around $\tilde{k} \equiv xp^+ \bar{n}^\mu$ [$\tilde{l} \equiv \tilde{k} + q$]

$$H_*^{\mu\nu}(k, l) = H_*^{\mu\nu}(\tilde{k}, \tilde{l}) + \frac{\partial H_*^{\mu\nu}}{\partial k^\alpha} (k^\alpha - \tilde{k}^\alpha) + \dots$$

↑
leading twist

↑
contributes to Higher Twists [Qiu '90]

NOTE:

➡ up to now no approximations

➡ especially, I did not approximate the final state kinematic

Collinear expansion - 2

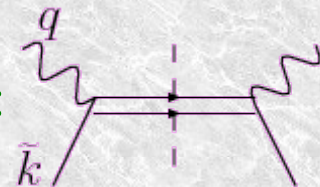
2) Use spectral representation

3) Assume $k^-, k_T \ll (x/\xi)Q^2 \Rightarrow j_2(l) \approx \int_0^\infty dm_J^2 J_2(m_J^2) 2\pi\delta(\tilde{l}^2 - m_J^2) \theta(l^0)$

$$W^{\mu\nu}(p, q) = \int_0^\infty dm_J^2 J_2(m_J^2) \int \frac{d^4k}{(2\pi)^4} H_*^{\mu\nu}(\tilde{k}, \tilde{l}) \delta(\tilde{l}^2 - m_J^2) 2\tau_2(k) \mathbb{K}(k, p, q)$$

unapproximated!

“fat quark” line:



NOTE:

- ➡ Involves a shift in the final state momentum l – **evil !! see [CRS]** but $J_2(m_J^2)$ is unapproximated (improvement over $m_J^2=0$ case)
- ➡ OK if $\int d^4l$ dominated by l such that $j_2(l)$ has small slope.

In terms of the spectral representation we need,

$$\frac{1 - x_B}{x_B} Q^2 \gtrsim m_J^2|_{\text{peak}}$$

Collinear expansion - 3

4) Ignore kinematic limits on k^- , k_T : $\mathbb{K}(k, p, q) \approx \mathbb{K}(\tilde{k}, p, q)$

$$W^{\mu\nu}(p, q) = \int_0^\infty dm_j^2 J_2(m_j^2) \int \frac{dx}{x} H_*^{\mu\nu}(\tilde{k}, \tilde{l}) \delta(\tilde{l}^2 - m_j^2) \varphi_q(x) \mathbb{K}(\tilde{k}, p, q)$$

where $\varphi_q(x) = \int \frac{dz^-}{2\pi} e^{iz^-k^+} \langle p | \bar{\psi}(z^-n) \frac{\gamma \cdot \bar{n}}{2} \psi(0) | p \rangle$

➔ needed to define collinear PDF

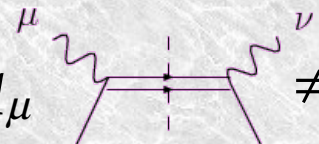
➔ does not respect 4-momentum conservation – **evil !!** – e.g.,

$$s = (p_J + p_Y)^2 \geq 4k_T^2 \quad \implies \quad 4k_T^2 \leq \frac{1-\xi}{\xi} Q^2 \left(1 + \xi \frac{m_N^2}{Q^2}\right)$$

5) Set $m_j^2=0$ inside $H_*(\tilde{k}, \tilde{l})$ [CRS]

$$H_*^{\mu\nu}(\tilde{k}, \tilde{l}) \approx H_*^{\mu\nu}(\tilde{k}, \hat{l}) \quad \text{with} \quad \hat{l}^\mu = \frac{Q^2}{2\xi p^+} n^\mu$$

Needed to:

➔ respect gauge invariance (otherwise q_μ  $\neq 0$)

➔ use Ward ids in proof of factorization

➔ **not so evil:** does not touch the final state kinematic

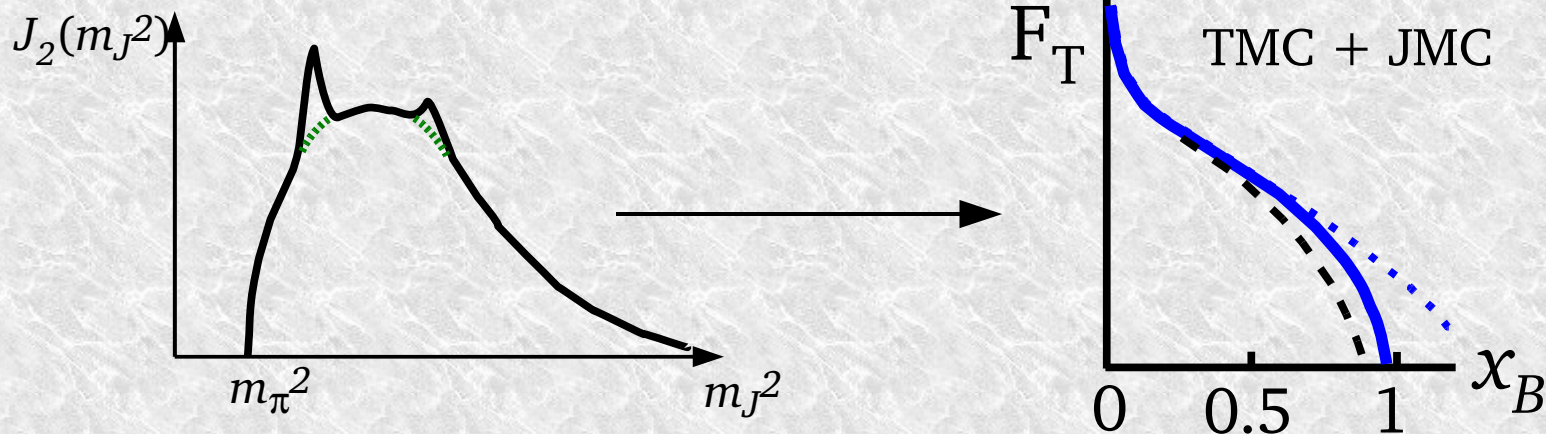
Conclusion

★ Collinearly factorized DIS at LO with Target and Jet Mass Corrections

➔ respects $x_B \leq 1$, goes smoothly to 0:

$$W^{\mu\nu}(p, q) = \int_0^\infty dm_j^2 J_2(m_j^2) \int_\xi^{\frac{\xi}{x_B}} \frac{dx}{x} \underbrace{\frac{1}{8\pi} \frac{e_q^2 \text{Tr}(\tilde{k} \gamma^\nu \hat{\psi} \gamma^\mu) 2\pi \delta(\tilde{l}^2 - m_j^2)}{2}}_{\mathcal{H}^{\mu\nu}} \varphi_q(x) = \frac{\xi}{Q^2} \delta\left(x - \xi\left(1 + \frac{m_j^2}{Q^2}\right)\right)$$

$$F_T(x_B, Q^2) = \int_0^{\frac{1-x_B}{x_B} Q^2} dm_j^2 J(m_j^2) F_T^{(0)}\left(\xi\left(1 + \frac{m_j^2}{Q^2}\right), Q^2\right)$$



Remarks

- ★ Kinematic approximations are explicit
 - ➔ estimates of validity in (x_B, Q^2) plane

- ★ $J_2(m_f^2)$ has explicit operator definition
 - ➔ Accessible in lattice QCD
 - ➔ Clear physical picture, allows for phenomenology

- ★ Fully consistent with collinear factorization
 - ➔ generalizable to other processes
 - ➔ fully consistent with CTEQ global analysis

- ★ Collinear factorization is pushed to its limit (or close to it)
 - ➔ further improvement requires:
 - ✓ jet-target soft interactions, finite rapidity separation
 - ✓ unapproximated final-state kinematics
 - ➔ Fully unintegrated parton distributions of [CRS]

Open Issues

★ Next-to-leading orders (see also [CRS])

- ➔ real gluon emission: already in $J_2(l)$?
- ➔ how to compute the hard tensor?

★ Higher twist terms

- ➔ Just $x_B \rightarrow \xi (1 + m_f^2/Q^2)$?
- ➔ Dynamical TM correlations among different twists because of equations of motion? [EFP]
- ➔ Expansion around $l = \hat{l} + \dots$? (in analogy to $k = \tilde{k} + \dots$)

★ Fits to experimental data

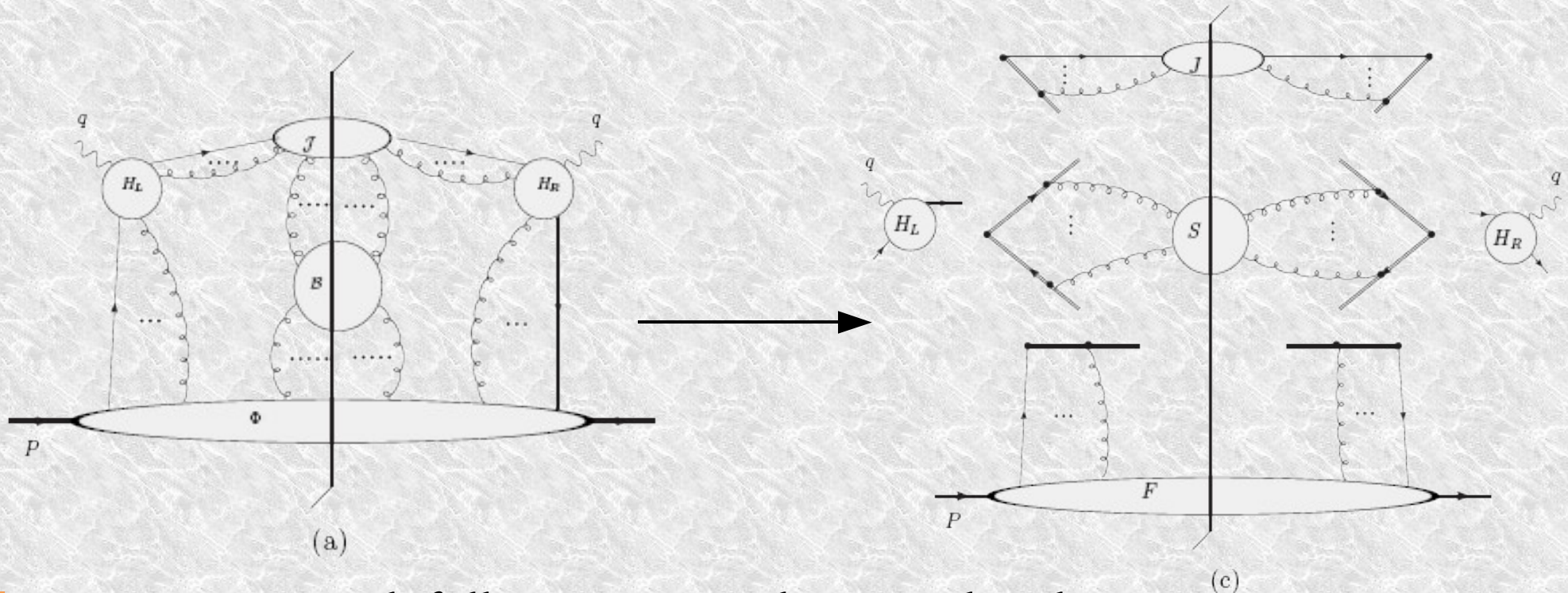
- ➔ $J_2(m_f^2) \Rightarrow$ parametrization / lattice QCD based phenomenology
- ➔ HT $\Rightarrow C[\xi (1 + m_f^2/Q^2)] / Q^2$?
- ➔ dynamical TMC in HT terms ?

The end

Appendices

“Proof” of collinear factorization - 1

- Generalized handbag diagram with a quark jet [Collins, Rogers, Stasto, 2007]



- Factorization with fully unintegrated parton distributions (for an abelian theory of massive gluons – QCD to come soon) [CRS]

$$P_{\mu\nu} W^{\mu\nu} = \int \frac{d^4 k_T}{(2\pi)^4} \frac{d^4 k_J}{(2\pi)^4} \frac{d^4 k_S}{(2\pi)^4} (2\pi)^4 \delta^{(4)}(q + P - k_T - k_J - k_S) \times$$

$$\times |H(Q, \mu)|^2 S_2(k_S, y_s, \mu) F(k_T, y_p, y_s, \mu) J(k_J, y_s, \mu).$$

soft PCF
target PCF
jet PCF

“Proof” of collinear factorization - 2

Start from

$$P_{\mu\nu} W^{\mu\nu} = \int \frac{d^4 k_T}{(2\pi)^4} \frac{d^4 k_J}{(2\pi)^4} \frac{d^4 k_S}{(2\pi)^4} (2\pi)^4 \delta^{(4)}(q + P - k_T - k_J - k_S) \times \\ \times |H(Q, \mu)|^2 S_2(k_S, y_s, \mu) F(k_T, y_p, y_s, \mu) J(k_J, y_s, \mu).$$

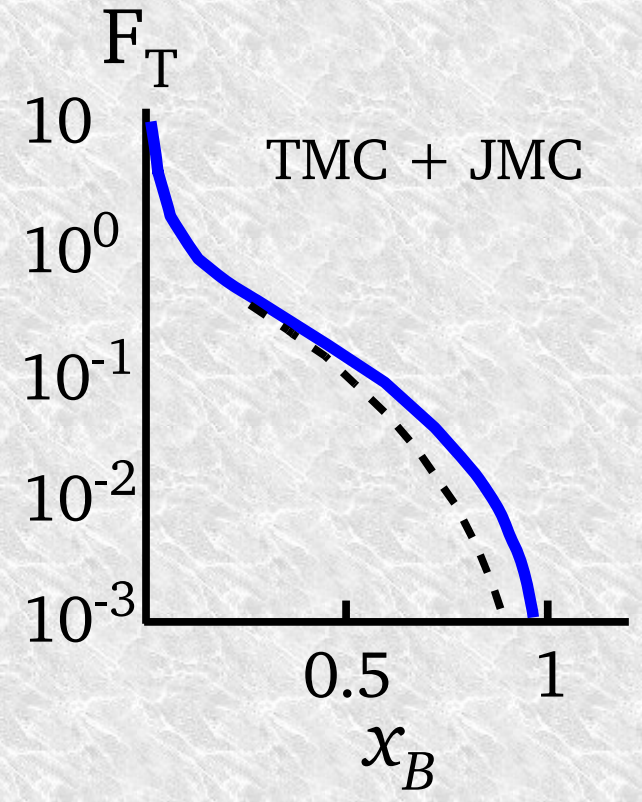
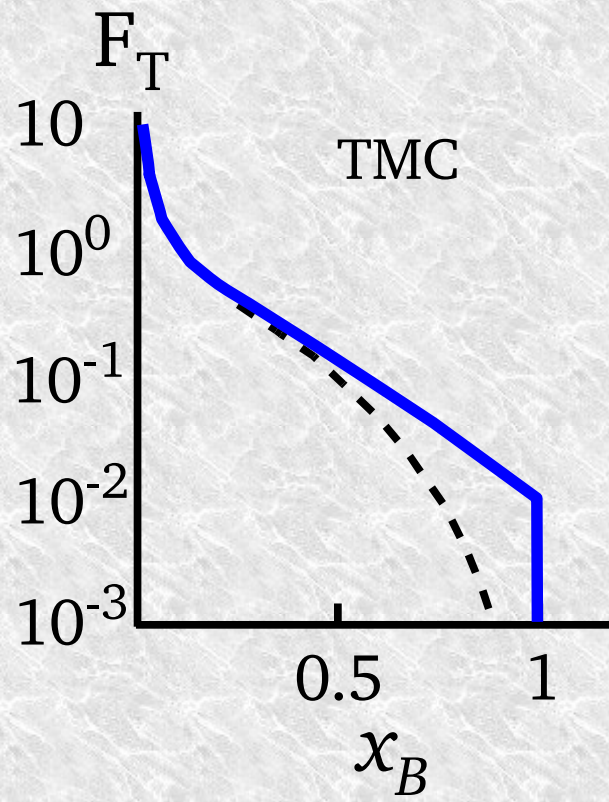
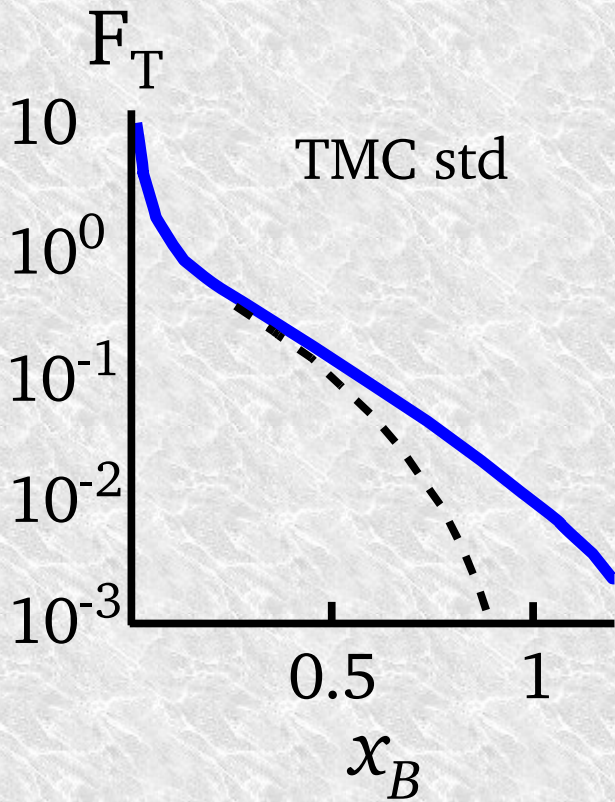
$$\tilde{F}(w, y_p, y_s, \mu) = \langle p | \bar{\psi}(w) V_w^\dagger(n_s) I_{n_s; w, 0} \frac{\gamma^+}{2} V_0(n_s) \psi(0) | p \rangle.$$

$$J(k_J, y_s, m) = \langle 0 | \bar{\psi}(w) V_w^\dagger(-n_s) I_{-n_s; w, 0} \gamma^- V_0(-n_s) \psi(0) | 0 \rangle$$

$$V_w(n) = P \exp\left(-ig \int_0^\infty d\lambda n \cdot A(w + \lambda n)\right)$$

- ➔ neglect soft jet-target interactions, use $P - k_T = k$, $k_J = l$
- ➔ the hard function H is the same as our $h_{T,L,\dots}$
- ➔ integrate out k_J , use spectral representation for $J(k_J)$
- ➔ expand H , repeat approximations 3, 4
- ➔ use $n_s \cdot A = 0$ gauge

➤ Transverse structure function at LO in α_s with CTEQ5L parton distributions



(the only cartoon)