Numerical techniques for NLO QCD and top quark physics at the LHC

Lazopoulos, Melnikov, FP hep-ph/0703273, arXiv:0709.4044

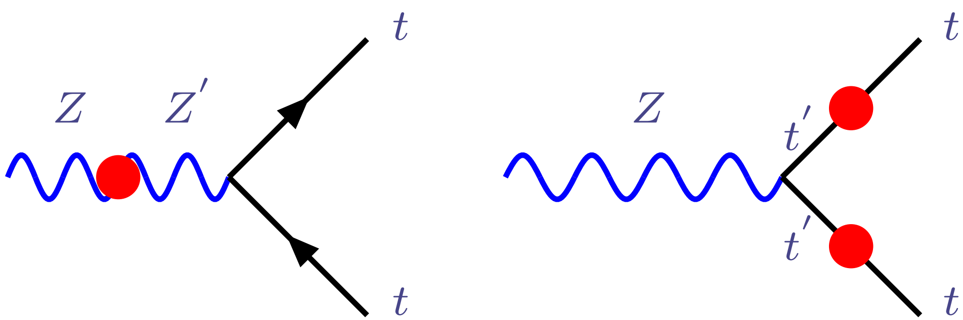
Lazopoulos, McElmurry, Melnikov, FP, in progress

Top quark electroweak couplings

- No direct measurements of $t\bar{t}V$ couplings exist ($V = \gamma, Z$)

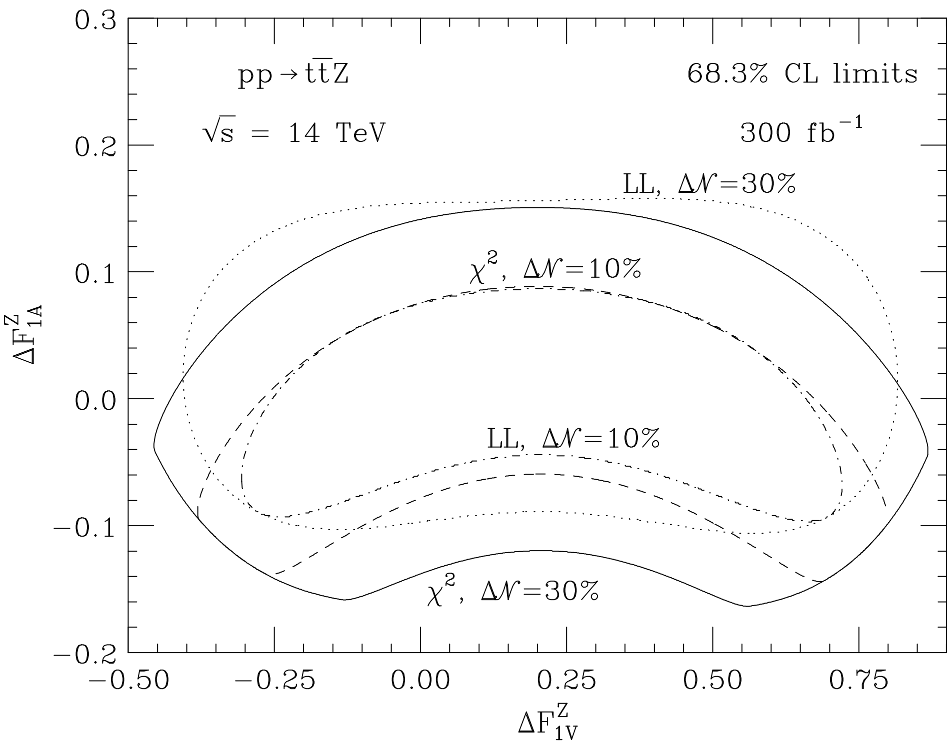
$$\Gamma_\mu^{t\bar{t}V} = -ie\left\{\gamma_\mu F_{1V}^V + \gamma_\mu\gamma_5 F_{1A}^V + \frac{\sigma_{\mu\nu}}{2m_t}(q+\bar{q})\left[iF_{2V}^V + \gamma_5 F_{2A}^V\right]\right\}$$

  - $F_{1V,A}^Z$ vector, axial couplings indirectly constrainted to few percent at LEP
  - No constraints on $F_{1V,A}^\gamma$ vector, axial couplings
  - Weak constraints on combinations of dipole couplings from LEP, $b \to s\gamma$
- Sensitive to mixing with extra gauge bosons and fermions



  - Present in strong dynamics, extra dimensions, Little Higgs, . . .
- Can't access at Tevatron; measure at LHC via $pp \to t\bar{t}V$

Measuring top quark couplings at the LHC



- LHC prospects studied by Baur et al., hep-ph/0412021,0512262
- Dominant uncertainty is normalization of signal cross section, ΔN
- Decrease from $\Delta N = \pm 30\%$ to $\pm 10\%$ gives factor of 2 improvement $\Rightarrow$ NLO QCD

NLO difficulties

- Sticking point: loops for $n = 5, 6, \ldots$ external legs
  - Standard analytic treatment (Passarino-Veltman reduction): $\sum I_n/D$
    - For $pp \to t\bar{t}H$, $D \sim \sin^2\theta_{t\bar{t}} \sin^2\phi_{t\bar{t}}$ (Dawson et al.)
      $\Rightarrow$ vanishes in non-negligible phase-space region; spurious, but tough to establish cancellation analytically
    - Identify problem areas, extrapolate numerics from safe region
  - Numerically integrate the Feynman parameter representation instead?
    - Thresholds where internal loop particles go on-shell
    - Feynman parameterization vanishes as $1/(-i\delta)^{n-2}$ $\Rightarrow$ unsuitable for numerics
    - Feynman parameterization has infrared singularities $\Rightarrow$ unsuitable for numerics
  - Simple algebriac complexity, production of numerical code with percent-level precision, . . .

Recent activity and results

- Much recent activity on new methods:
  - Expand reduction coefficients around fictitious singularities (Denner, Dittmaier)
  - Numerical solution of reduction equations (R. K. Ellis, Giele, Glover, Zanderighi)
  - Twistor-inspired (C. Berger, Bern, Dixon, Kosower; Britto, Cachazo, Feng; . . .)
  - Reduction at integrand level (Ossola, Papadopoulos, Pittau)
  - . . .

  $\Rightarrow$ both traditional analytic and new semi-numerical methods

Numerical approach

- Can we construct an approach to directly numerically integrate multi-leg loops?
- Must confront three main issues:
  - Find and extract soft/collinear singularities
  - Pick a good regulator for internal thresholds (Soper, Nagy)
  - Avoid problematic denominators by computing tensor integrals numerically
- Goal: quickly, effi ciently produce many NLO results

$\Rightarrow$ resulting code maybe not as quick as other methods

  - Envision usage as differential reweighting of LO codes

Loop integral singularities

- IR loop singularities governed by Landau equations
  - In Feynman parameter representation, must have $k_i^2 - m_i^2 = 0$ or $x_i = 0$ for every propagator
  - After k integration, all singularities occur as some $x_i \to 0$
  - Loop integral in Feynman parameter space:

$$\int_0^1 dx_i \delta(1 - \sum x_i) \Delta^{-n-\epsilon}$$

  - If IR singularity only when a single $x = 0$ ($\Delta = x\Delta'$), extract via

$$x^{-1+\epsilon} = \frac{1}{\epsilon}\delta(x) + \left[\frac{1}{x}\right]_+ + \dots$$

with

$$\int_0^1 dx f(x) \left[\frac{1}{x}\right]_+ = \int_0^1 dx \frac{f(x) - f(0)}{x}$$

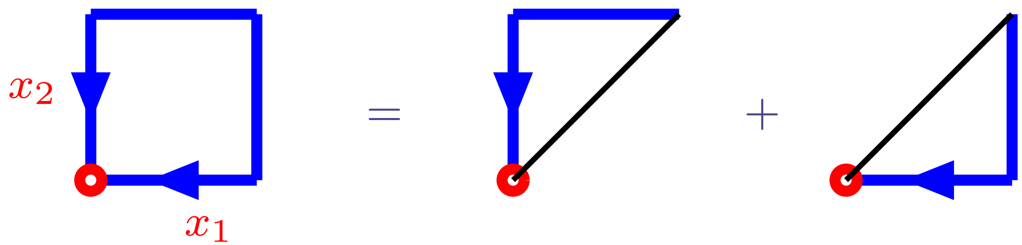
  - Simple, programmable procedure, numerical treatment possible

Sector decomposition

- How about when multiple x_i vanish?
  - Consider the simple example

$$I = \int_0^1 dx_1 dx_2 \, (x_1 + x_2)^{-2-\epsilon}$$

  - Divide the integration region by ordering the two variables:



  - Singularities factor in each region after the integration region is remapped into $[0, 1]$; consider the $x_2 < x_1$ region, and set $z = x_2/x_1$:

$$I(x_2 < x_1) = \int_0^1 dx_1 dz \, x_1^{-1-\epsilon} (1+z)^{-2-\epsilon}$$

  - Extract singular terms as before $\Rightarrow$ again a simple, programmable procedure

Generating cubic denominators

- Normal Feynman denominators of the form $(x_i S_{ij} x_j)^{-n-e} \Rightarrow$ quadratic
- Can generate cubic or higher structures via sector decomposition
- Example 1: $\Delta = x_1 T + x_2 x_3 S$ (generate linear term by choosing largest x_i
  - Sector 1: $x_2 = x_1 y_2$, $\Delta = x_1(T + y_2 x_3 S) \Rightarrow$ remains quadratic
  - Sector 2: $x_1 = x_2 y_1$, $\Delta = x_2(y_1 T + x_3 S) \Rightarrow$ also remains quadratic
- Example 2: $\Delta = x_1 T + x_2 x_3 S + x_4 U + x_2 x_4 V$
  - Decompose in the set of *three* variables x_1, x_4, x_2; only singular for $n > 3$
  - x_1 largest: $x_2 = x_1 y_2$, $x_4 = x_1 y_4$, $\Delta = x_1(T + y_2 x_3 S + y_4 U + x_1 y_2 y_4 V)$

$\Rightarrow$ cubic! Will come up later in the contour deformation

Regulating thresholds

- Feynman denominator can vanish in interior of x-space
  - Simple example:
  $$\int_0^1 dx_1 dx_2 \ \left(m^2 - x_1 x_2 s - i0\right)^{-1-\epsilon}$$
  - Occurs when unitarity cut leads to physical scattering process
  - Generic Feynman denominator has form
  $$\Delta = Z + Y_i x_i + \frac{1}{2} X_{ij} x_i x_j + \frac{1}{3} W_{ijk} x_i x_j x_k + \dots$$
  - Assume $W = 0$; deform contour by setting $x_i = y_i - i\tau_i$, get
  $$-i\tau_i [Y_i + \sum_j X_{ij} y_j]$$
  - To make sign-definite, choose
  $$\tau_i = \lambda y_i (1 - y_i)[Y_i + \sum_j X_{ij} y_j]$$
  - Imaginary part of Δ becomes
  $$-i\lambda y_i (1 - y_i)[Y_i + \sum_j X_{ij} y_j]^2$$

$\Rightarrow$ sign-definite, non-vanishing, easy to automate finding of $\Rightarrow$ a suitable regulator

Contour deformation with cubic terms

- With cubic terms, imaginary part becomes

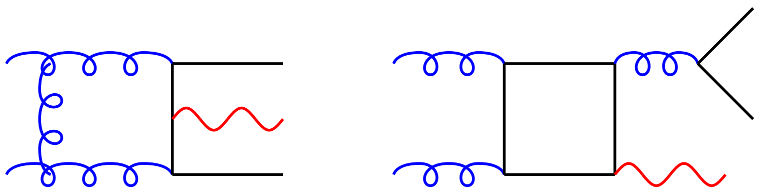
$$-i\tau_i[Y_i + X_{ij}y_j] + \frac{i}{3}W_{ijk}\tau_i\tau_j\tau_k$$

  - How to choose τ_i to make sign-definite, given arbitrary kinematic matrices?
  - $\tau_i \sim \lambda y_i(1 - y_i) < \lambda/4 \Rightarrow$ for small λ, W term unimportant
  - Begin with small λ, check that nothing breaks if it is increased

$\Rightarrow$ should be okay if contour is away from real axis at threshold location

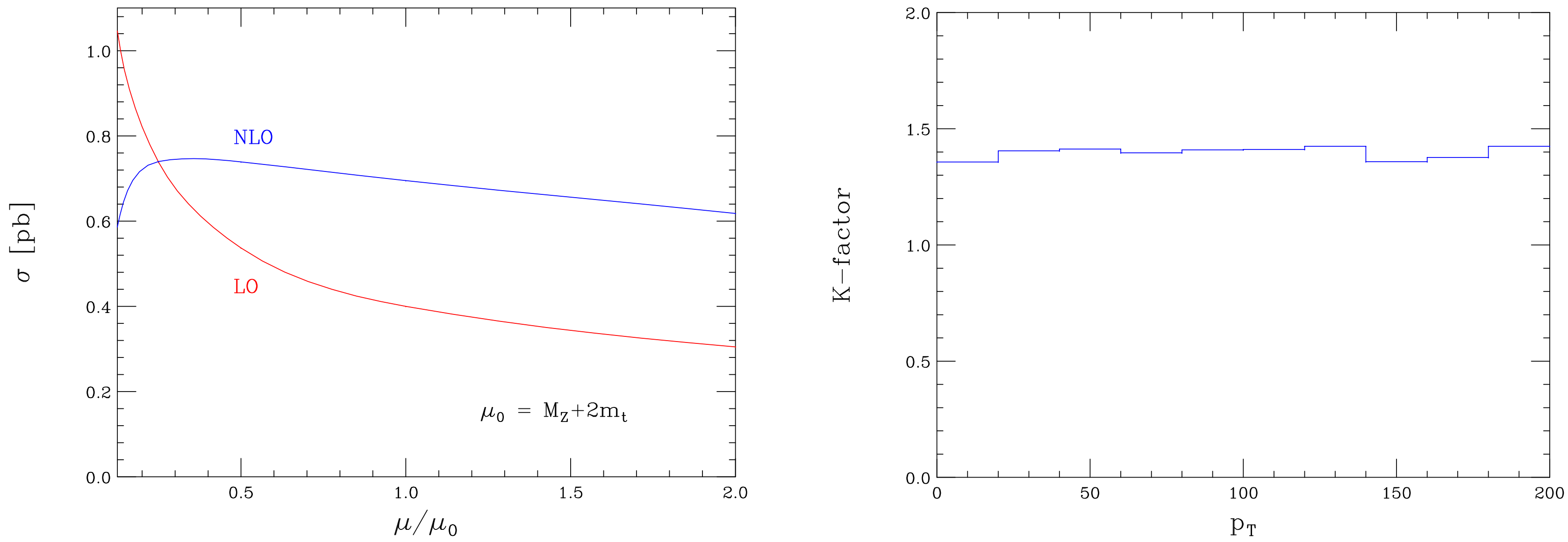
NLO QCD corrections to $gg \to t\bar{t}Z$

- Calculational details:
  - 162 one-loop diagrams, 50 real emission diagrams



  - Two classes of virtual diagrams: single-trace and two-trace
  - Two-trace class is finite, but Larin prescription used for γ_5 in intermediate steps
  - Two-cutoff phase-space slicing used for extraction of singularities from real-emission
  - At LO, two channels: gg (60%) and $q\bar{q}$ (40%); qg contributes at NLO
  - Only gg completed so far, others in progress

Results



- Drastic reduction of scale uncertainty
  - $\Delta N = \pm 5\%$ for variation in range $[\frac{\mu_0}{4}, \mu_0]$ after NLO
  - Improvement in anomalous coupling sensitivity beyond that hoped for in Baur et al.
- Corrections independent of kinematics
- Expect these to hold after $q\bar{q}$, qg inclusion