Three-jet calculation at NLO level for hadron colliders

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INTRODUCTION, MOTIVATION

In the point of the QCD the multi-jet cross section can be used to

- test the prediction of perturbative QCD
- determine the $\alpha_s(Q^2)$ strong coupling and the parton distribution function of proton ($f_a(x, Q^2)$)

In the point of the new physics search when QCD particles are presented the QCD contributions could be very important both to the signal and background process (e.g.: $pp \to H + 0, 1\text{jet} \to \gamma\gamma + 0, 1\text{jet}$).

Because of the large strong coupling the LO order predictions strongly depend on the unphysical renormalization and factorization scales thus the higher order corrections are important to stabilize the theoretical prediction.

NLO CROSS SECTION

$$\sigma(p_A, p_B) = \sum_{a,b} \int_0^1 d\eta_a d\eta_b f_a(\eta_b, \mu_F^2) f_b(\eta_b, \mu_F^2)$$
$$\times \left[\hat{\sigma}_{a,b}^{LO}(\eta_a p_A, \eta_b p_B) + \hat{\sigma}_{a,b}^{NLO}(\eta_a p_A, \eta_b p_B)\right]$$

where

$$\hat{\sigma}_{a,b}^{LO}(p_a, p_b) = \int_m d\hat{\sigma}_a^B(p_a, p_b) = \int d\Gamma^{(m)} |M_{a,b}|^2 F_J^{(m)}(p_a, p_b, p_1, .., p_m)$$

and the NLO correction

$$\hat{\sigma}_a^{NLO}(p_a, p_b) = \int_{m+1} \hat{\sigma}_{a,b}^R(p_a, p_b) + \int_m \hat{\sigma}_{a,b}^V(p_a, p_b) + \int_m \hat{\sigma}_{a,b}^C(p_a, p_b)$$

This integrals (R,V,C) are separately divergent but their sum is finite in $d = 4$ dimension.

NLO CROSS SECTION: SUBTRACTION TERM

We used the dipole method to regularize the divergent integrals [Catani- Seymour]

$$d\sigma^{NLO}_{a,b} = [d\sigma^{R}_{a,b} - d\sigma^{A}_{a,b}] + d\sigma^{A}_{a,b} + d\sigma^{V}_{a,b} + d\sigma^{C}_{a,b} \quad ,$$

where $d\sigma^{A}_{a,b}$ is a local approximation of $d\sigma^{R}_{a,b}$ and

- it has to exactly match the singular behaviour of $d\sigma^{R}_{a,b}$
- it has to be exactly integrable in $d = 4 - 2\epsilon$ dimension over the single parton subspaces leading to soft and collinear divergences

$$\sigma^{NLO}_{a,b} = \int_{m+1} [d\sigma^{R}_{a,b}|_{\epsilon=0} - d\sigma^{A}_{a,b}|_{\epsilon=0}] + \int_{m} [d\sigma^{V}_{a,b} + d\sigma^{C}_{a,b} + \int_{1} d\sigma^{A}_{a,b}]|_{\epsilon=0}$$

The cancelation of the divergences is guaranteed only for that jet observables which fulfill the condition

$$F_J^{(m+1)} \longrightarrow F_J^{(m)}$$

in the kinematically denegerate phase space regions (soft and collinear).

JET ALGORITHM: $k_\perp$ ALGORITHM (ELLIS-SOPER)

The algorithm starts with a list of the particles and the empty list of the jets.

1. For each particle (pseudo-particle) i in the list and for each pair (i,j) define

$$d_i = p_{T,i}^2 \quad , \qquad d_{ij} = \min(p_{T,i}^2, p_{T,j}^2)\frac{(y_i - y_j)^2 + (\phi_i - \phi_j)^2}{D^2} \quad ,$$

2. Find the minimum of all the d_i and d_{ij} and label it $d_{\min}$.

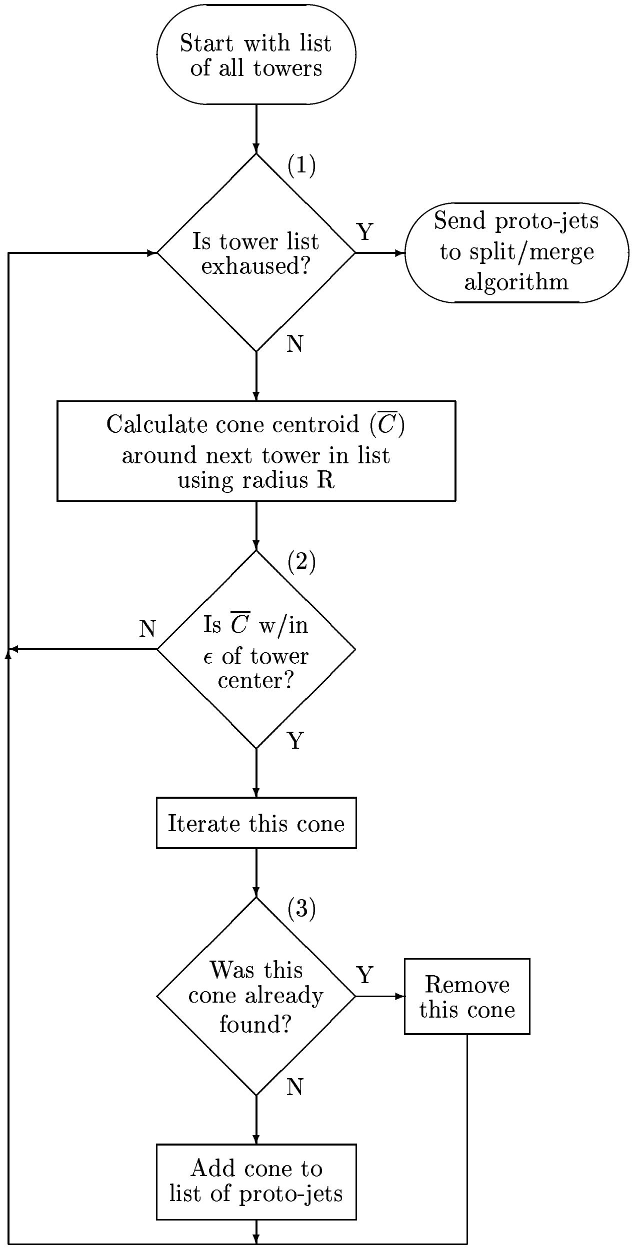
3. If $d_{\min} = d_{ij}$ then merge the two particles by the recombination scheme

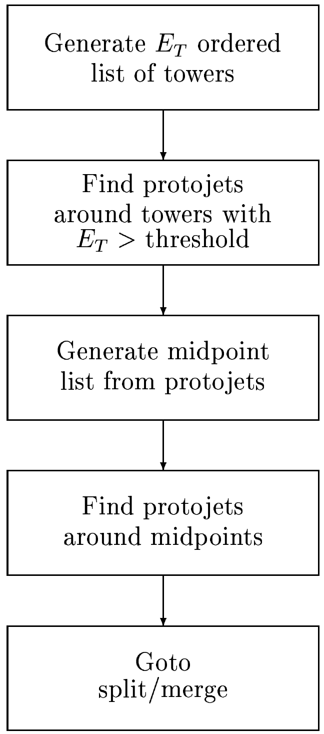
$$p_{(ij)} = p_i + p_j \quad .$$

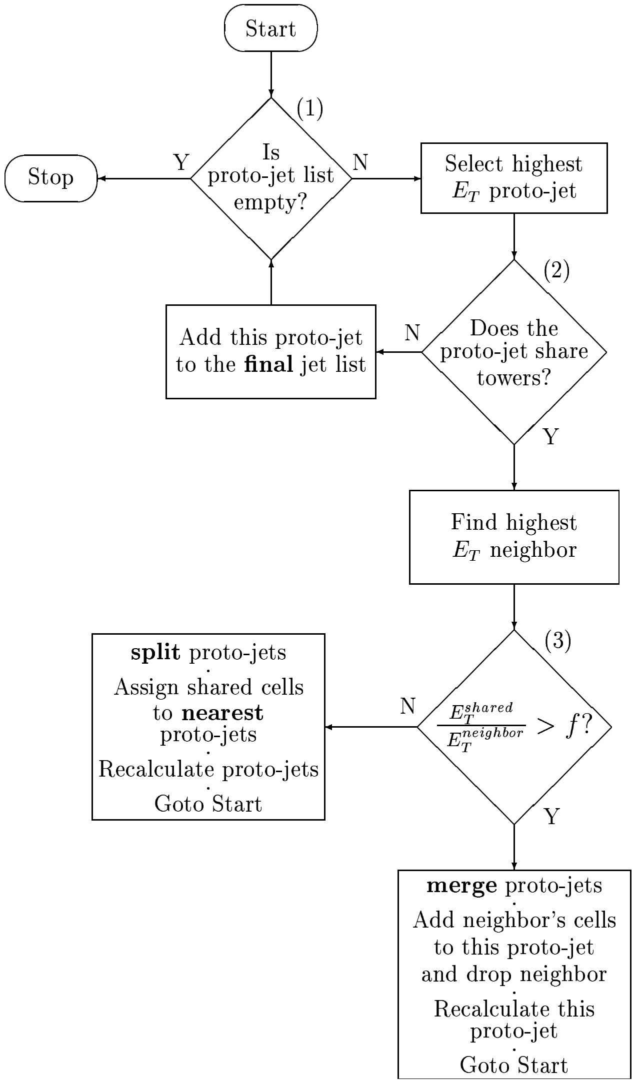
4. If $d_{\min} = d_i$, remove particle (pseudo-particle) i from the list of particle and add it to list of jets.

5. If any particles remain, go to step 1.

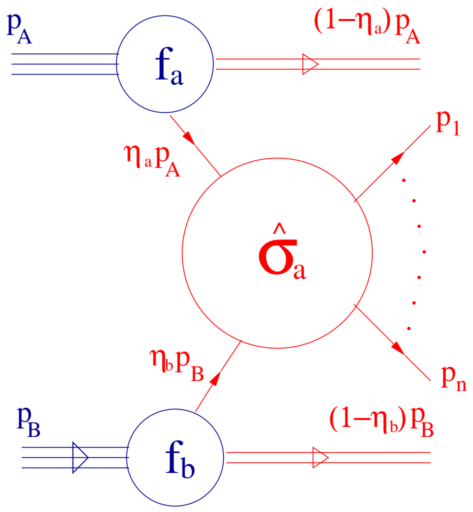
JET ALGORITHM: IMPROVED LEGACY CONE ALGORITHM







KINEMATICS OF THE HADRON-HADRON COLLISIONS



PDF and α_s:

LHAPDF in CTEQ6 mode, $\alpha_s(M_Z) = 0.118$

Matrix elements:

Six parton tree level: $0 \to gggggg$, $0 \to q\bar{q}gggg$, $0 \to q\bar{q}Q\bar{Q}gg$ and $0 \to q\bar{q}Q\bar{Q}r\bar{r}$

Gunion, Kunszt, Berends, Giele & Kuijf

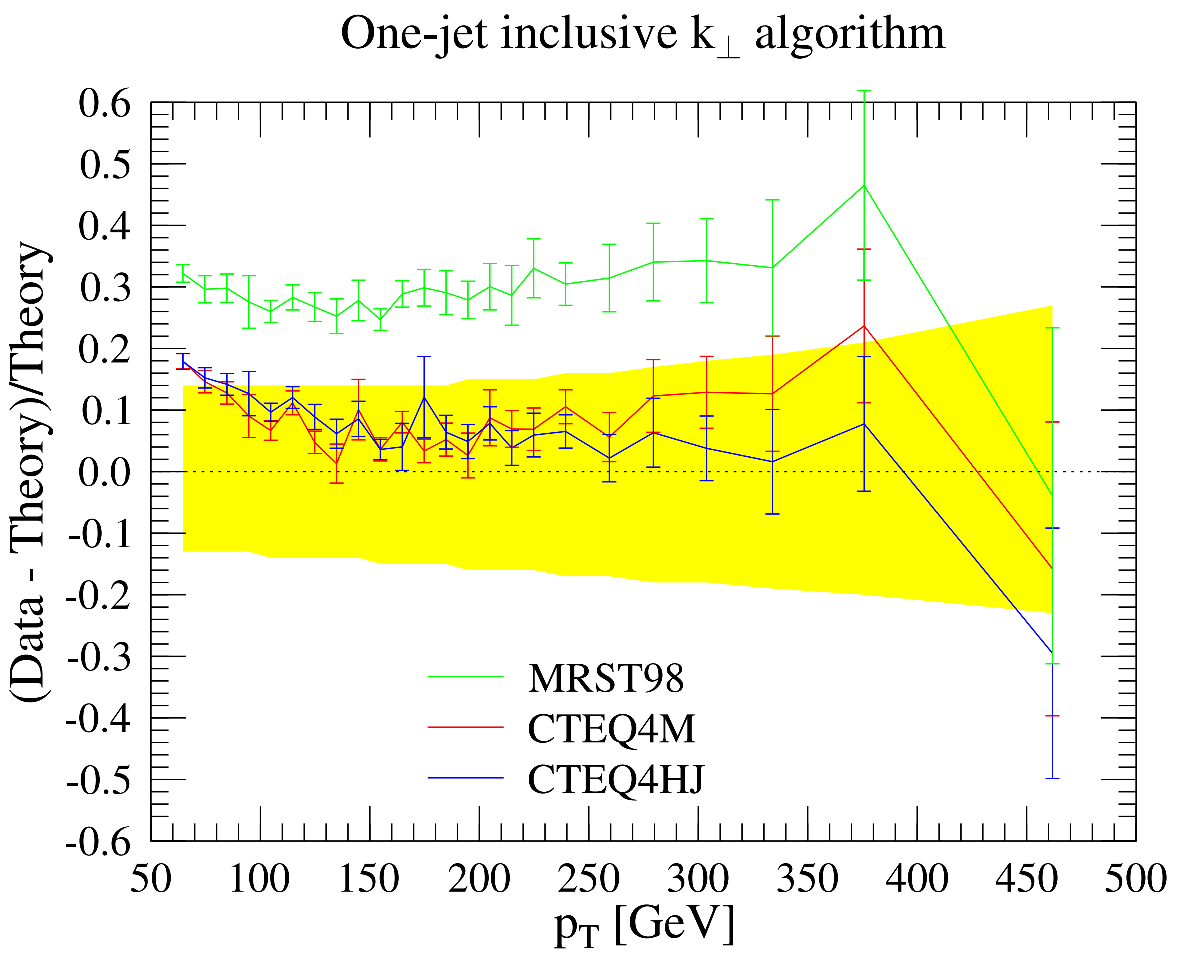
Five parton tree and 1-loop level: $0 \to ggggg$, $0 \to q\bar{q}ggg$

Bern, Dixon, Kosower

and $0 \to q\bar{q}Q\bar{Q}g$

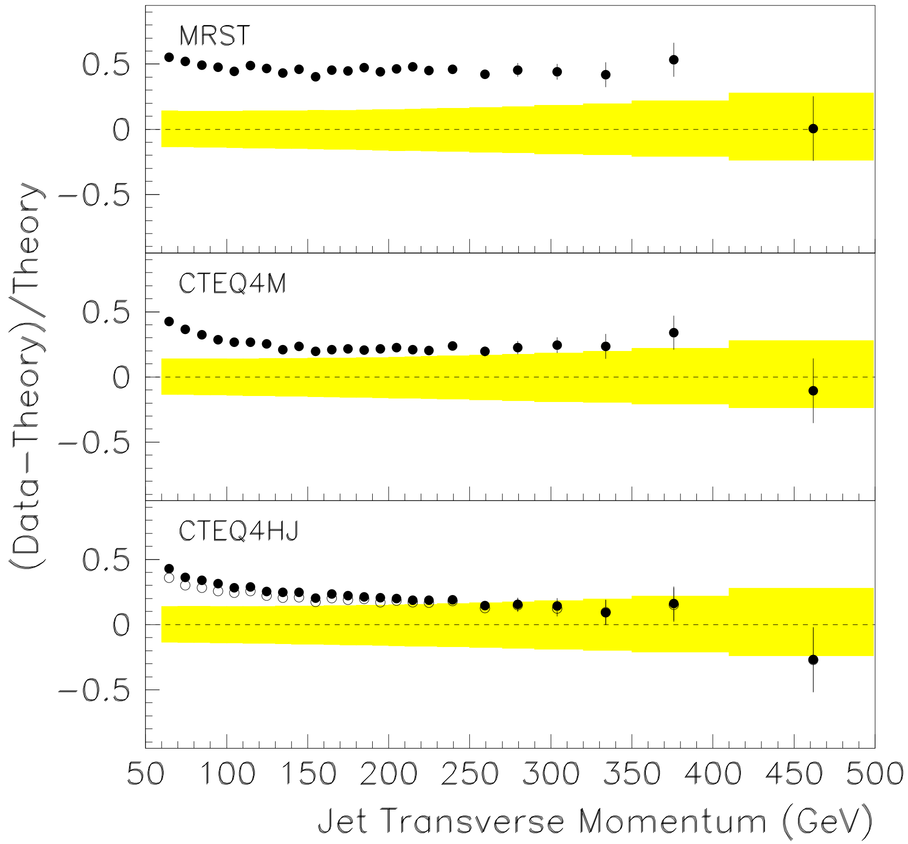
Kunszt, Signer, Trócsányi

ONE-JET INCLUSIVE CROSS SECTION AT NLO



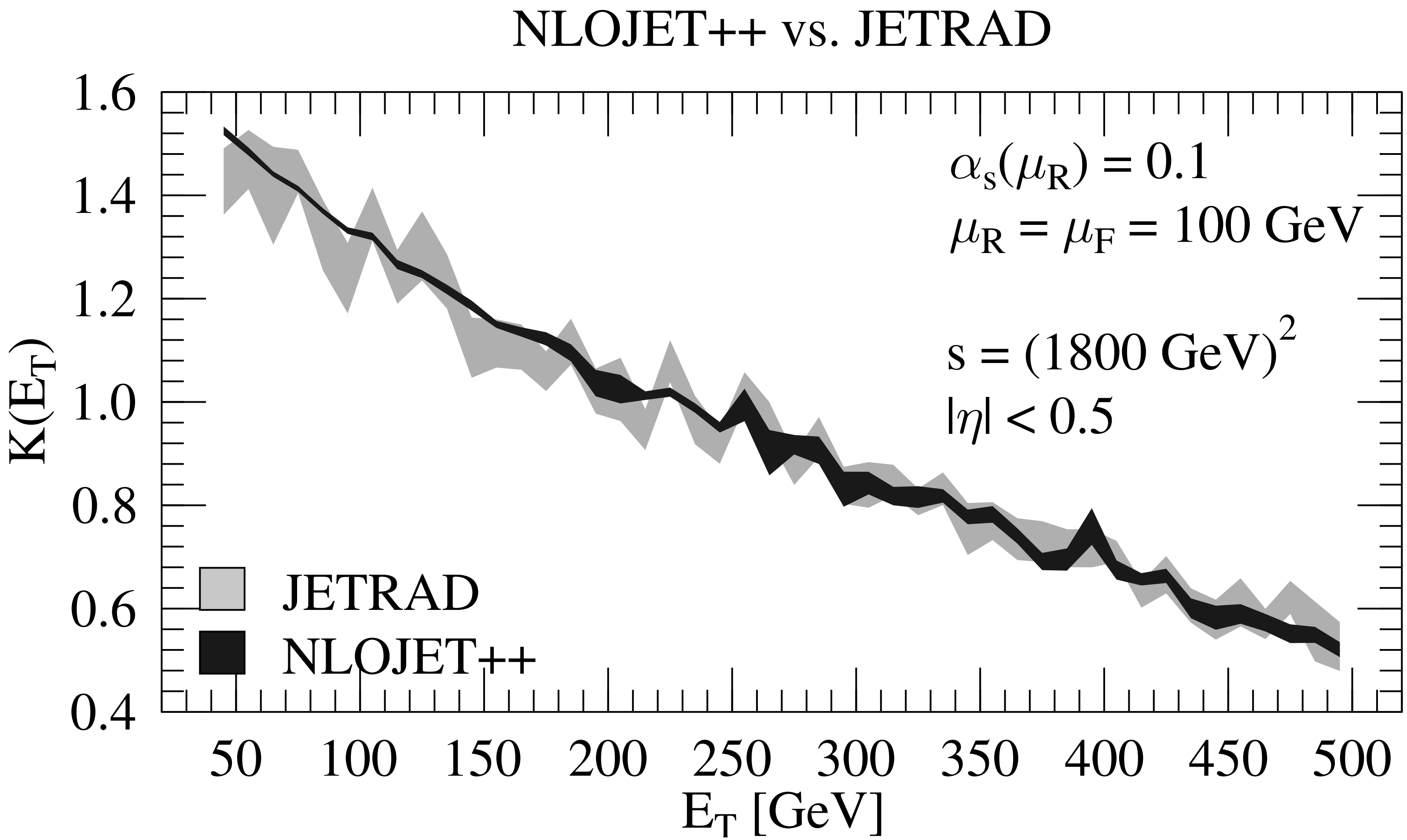
Difference between the data and the NLOJET++ prediction.

ONE-JET INCLUSIVE CROSS SECTION AT NLO



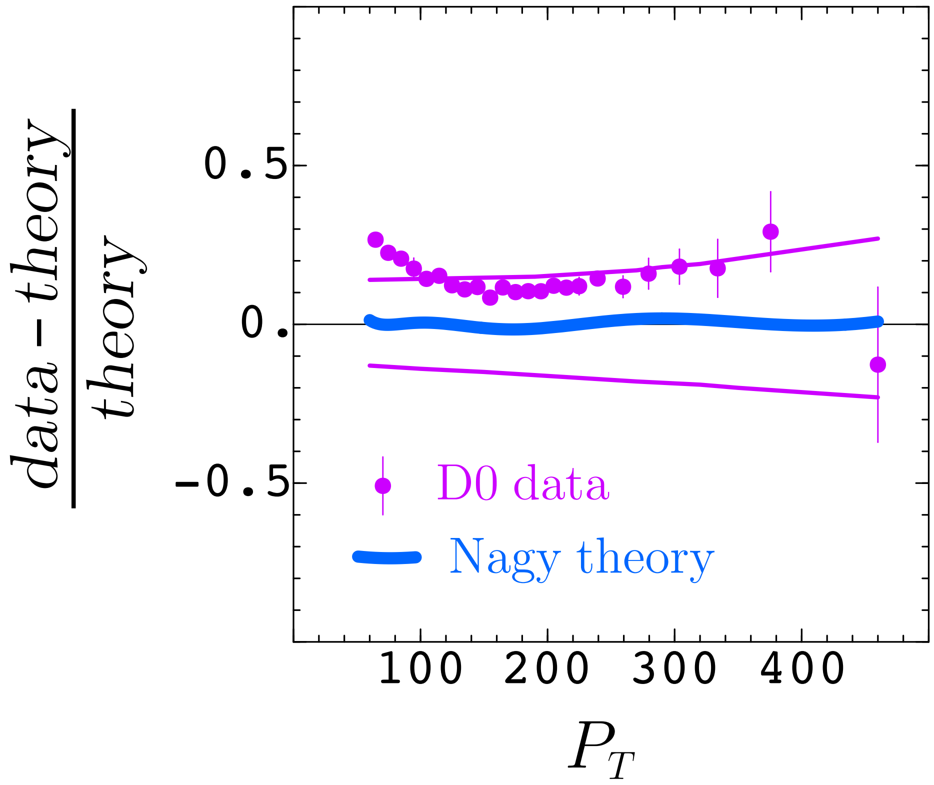
Difference between the data and the JETRAD prediction.

ONE-JET INCLUSIVE CROSS SECTION AT NLO



Comparison of the JETRAD and NLOJET++ programs using the MRSD'_- parton densities and $k_\perp$ algorithm.

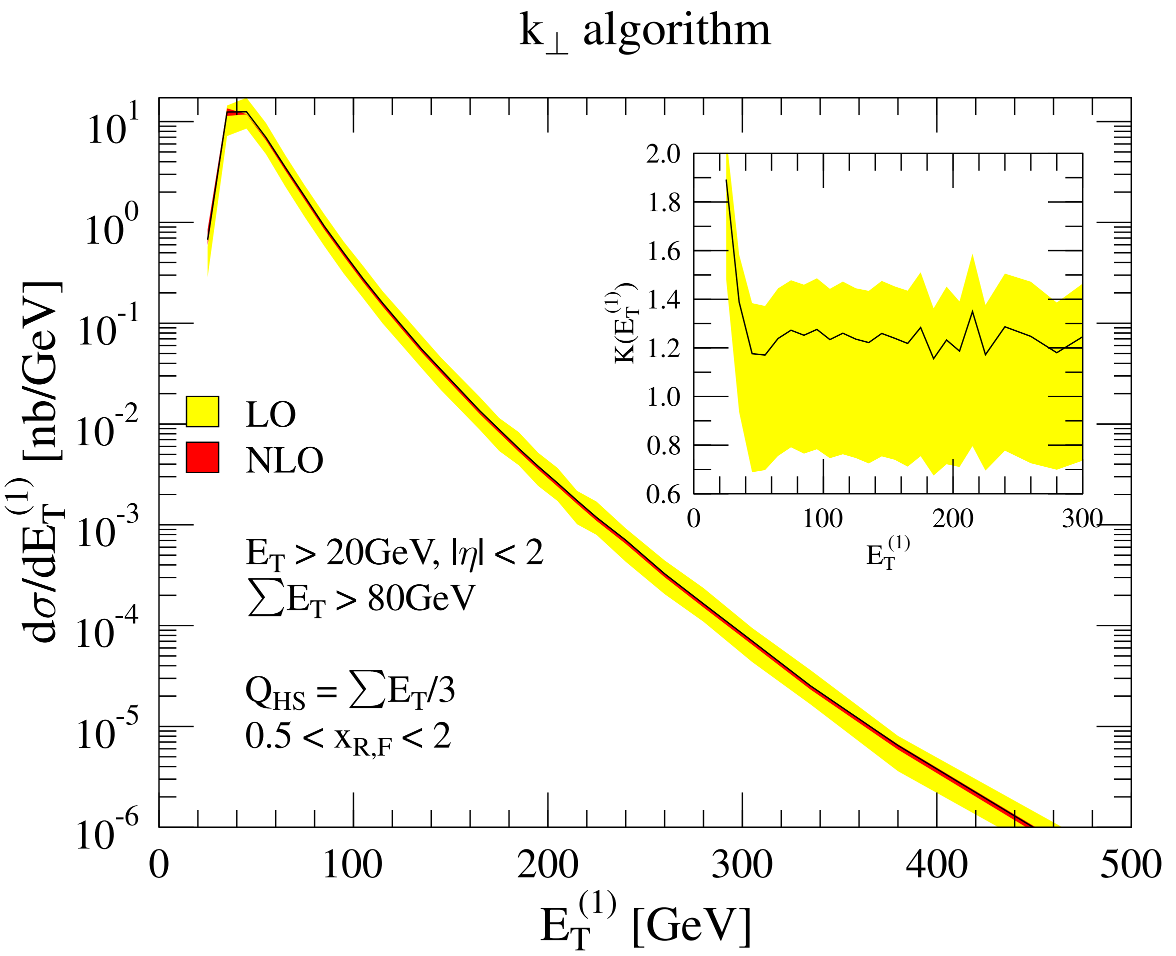
ONE-JET INCLUSIVE CROSS SECTION AT NLO



Base "theory" is EKS.
Partons are cteq4m.

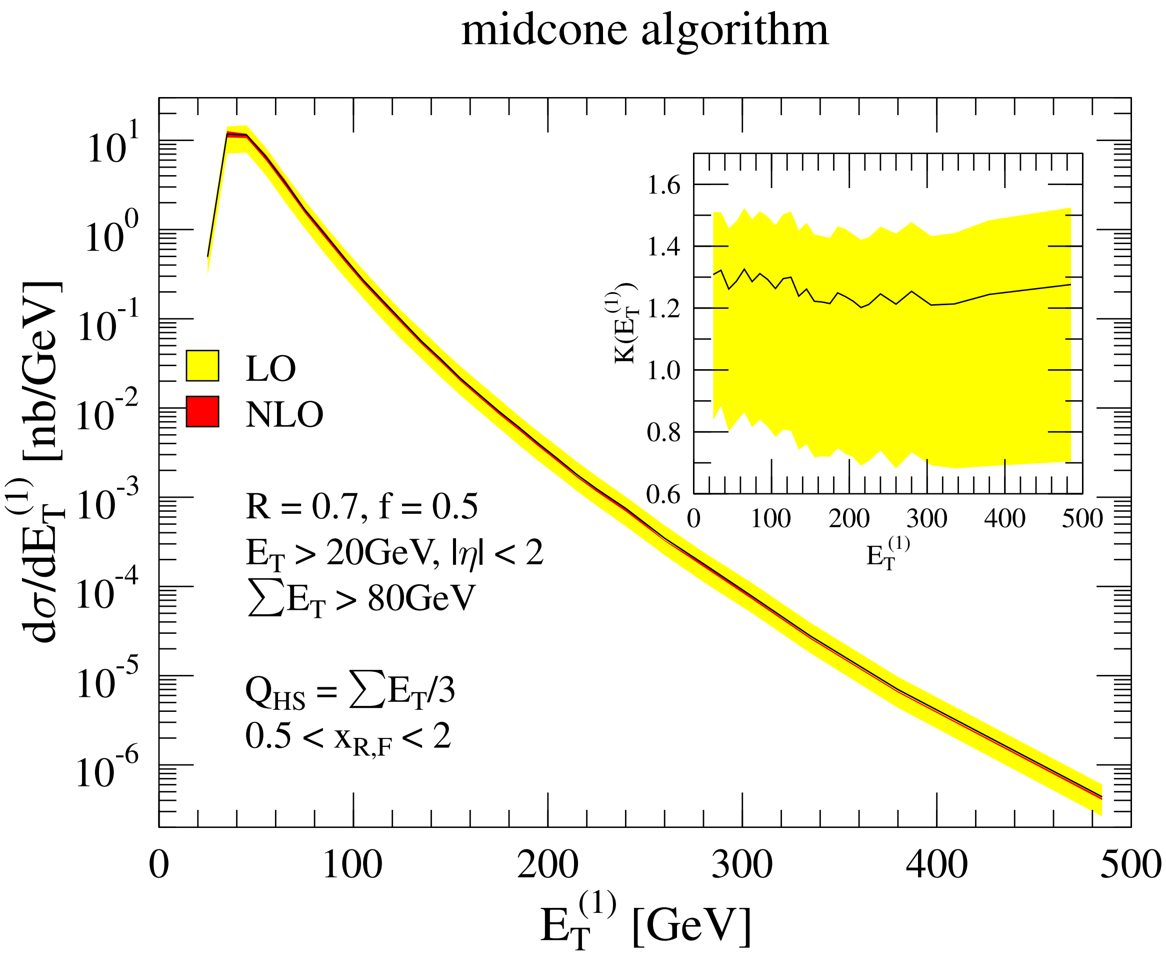
Comparison of the EKS and NLOJET++ programs using the CTEQ4M parton densities and $k_\perp$ algorithm.

THREE-JET IN HADRON HADRON COLLISIONS AT NLO



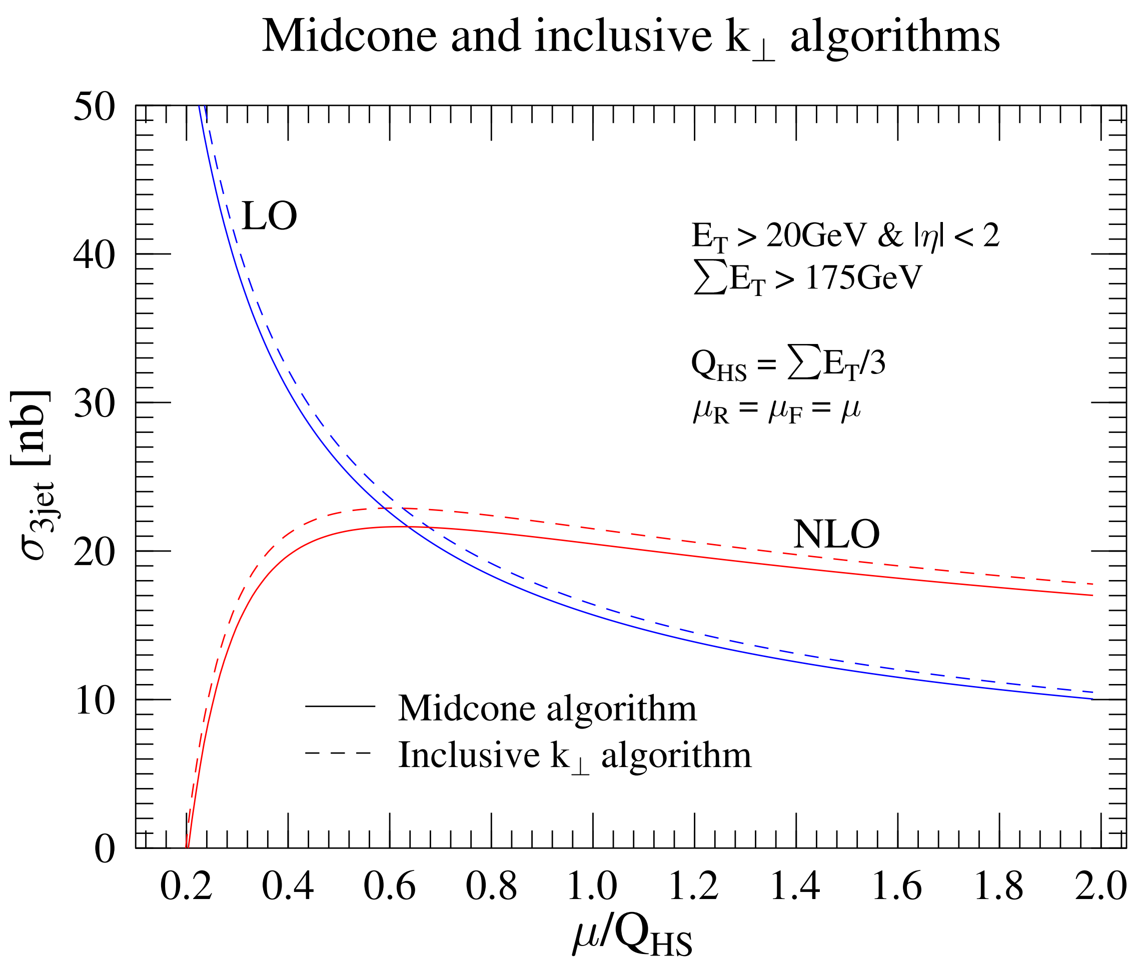
Differential 3-jet cross section as a function of tranverse energy of the leading jet

THREE-JET IN HADRON HADRON COLLISIONS AT NLO



Differential 3-jet cross section as a function of tranverse energy of the leading jet

THREE-JET IN HADRON HADRON COLLISIONS AT NLO



Scale dependence of the 3-jet cross section

DALITZ VARIABLES

The Daltiz variables are defined in the rest frame of the three leading (leading in transverse energy) jets

$$X_i = \frac{2E_i}{m_{3J}} \quad , \qquad i = 1, 2, 3 \quad ,$$

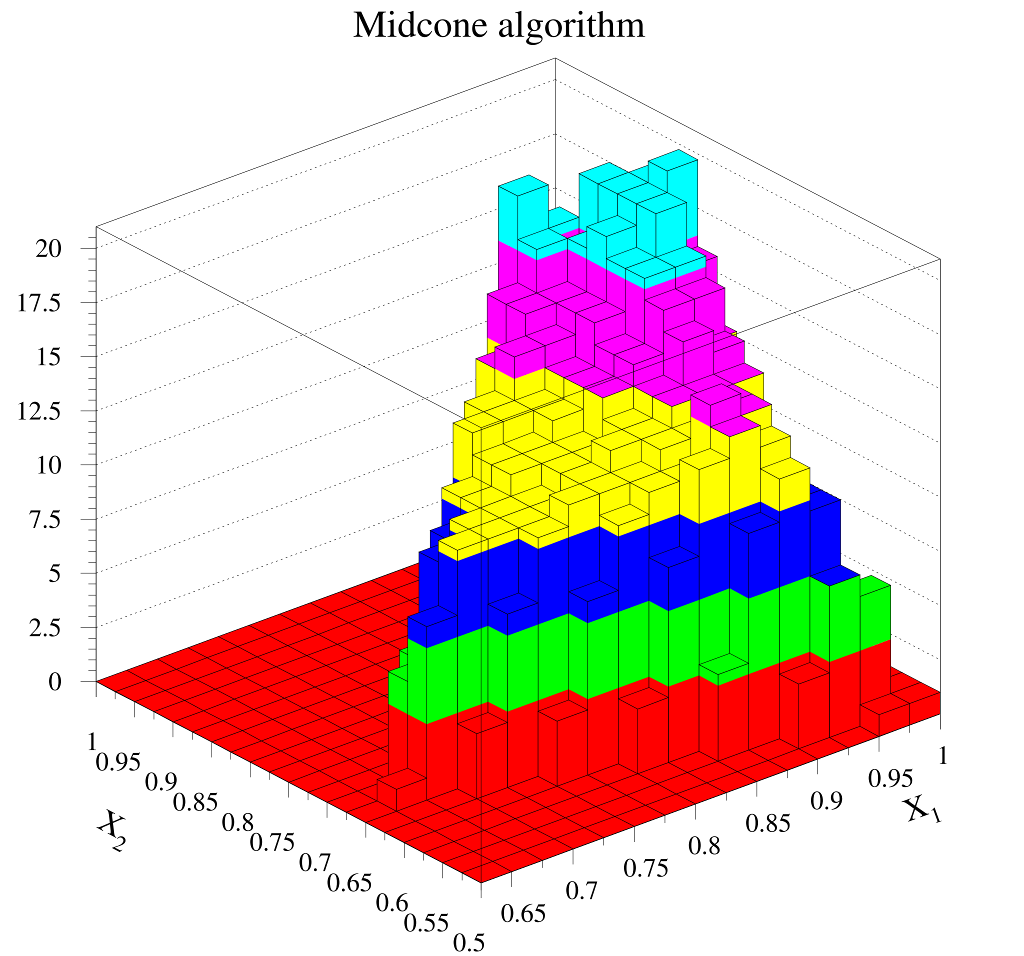
where the jets are labeled by $E_1 > E_2 > E_3$.

The jets are defined by using the inclusive $k_\perp$ and midcone algorithm.

Kinematical region:

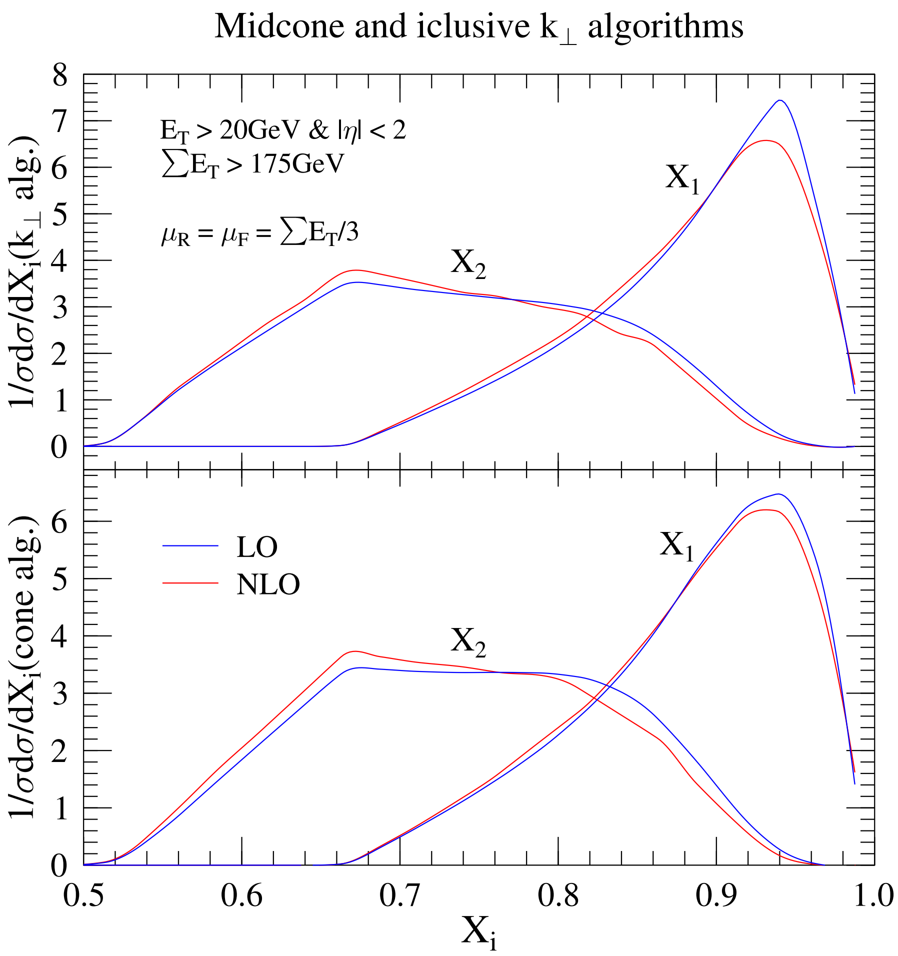
$$E_T > 20\text{GeV} \quad , \qquad |\eta| < 2 \quad , \qquad \sum E_T > 175\text{GeV} \quad .$$

DALITZ VARIABLES



Next-to-leading order perturbative prediction for normalized double differention distribution ($1/\sigma d\sigma/dX_1dX_2$) of the energy fraction variables X_1 and X_2 using the inclusive $k_\perp$ and midcone algorithm.

DALITZ VARIABLES



The energy fraction distribution of the leading (X_1) and second leading (X_2) jets. The upper figure is result with the inclusive $k_\perp$ algorithm and the lower figures shows the midcone result.

JET ALGORITHM: $k_\perp$ ALGORITHM (CATANI *et. al.*)

The algorithm starts with a list of the particles and the empty list of the jets.

1. For each particle (pseudo-particle) i in the list and for each pair (i, j) define d_i and d_{ij}.
2. Find the minimum of all the d_i and d_{ij} and label it $d_{\min}$.
3. If $d_{\min} = d_{ij}$ then merge the two particles by the recombination scheme.
4. If $d_{\min} = d_i$, remove particle (pseudo-particle) i from the list of particle and add it to list of beam jets.
5. Repeate this algorithm until all objects have d_i, d_{ij} larger than some stopping parameter d_{cut} .

To define the sub-jet structure we redifine the resolution variable $y_{cut} = Q_0/d_{cut} < 1$ and rerun the algorithm only for those partons which are assigned to any hard final state jet.

JET ALGORITHM: $k_\perp$ ALGORITHM (CATANI *et. al.*)

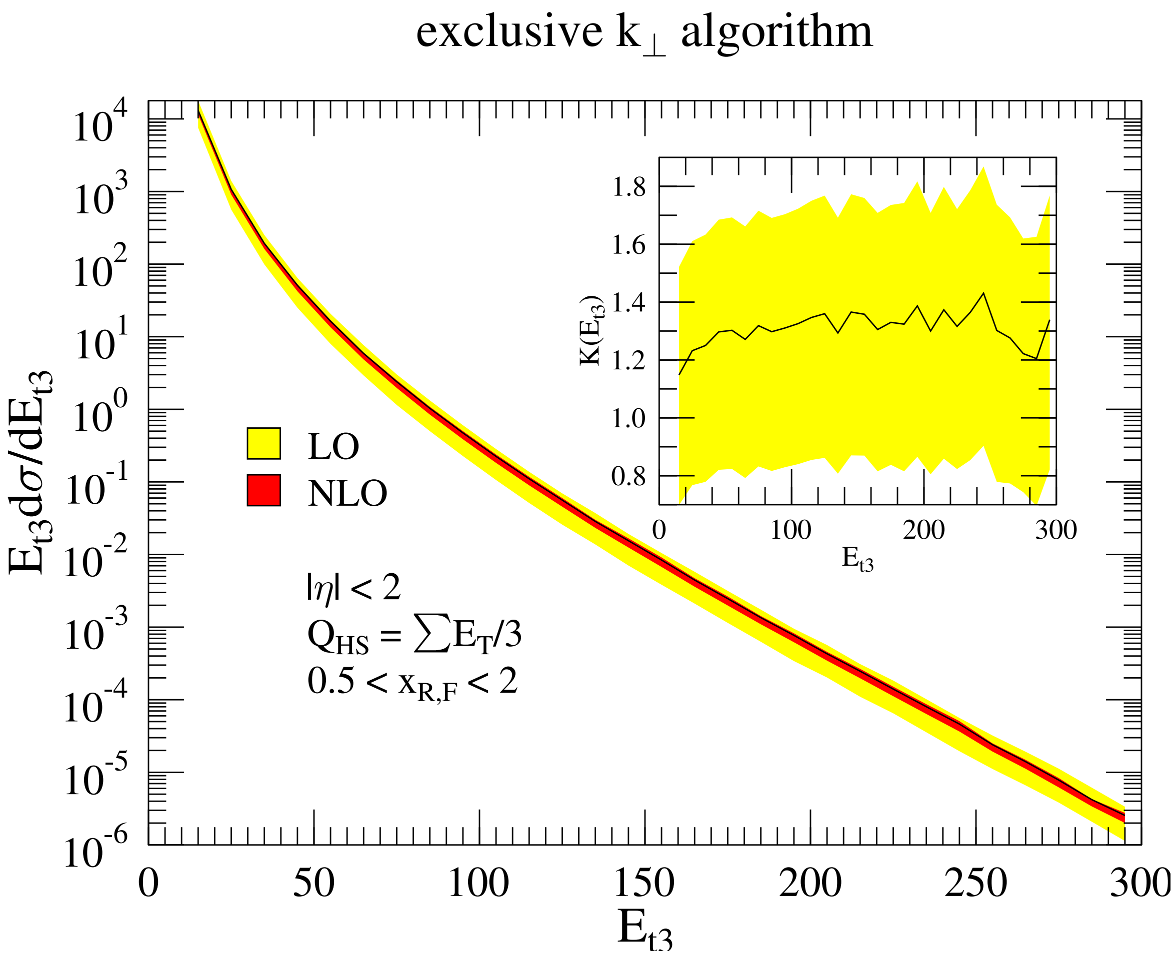
One can define event shape parameter E_{tn}. The variable E_{tn}^2 being the value of the smallest resolution variable when the event has n hard final state jets.

The n-jet exclusive cross section is given by

$$\sigma_{n-jet}(E_{cut}) = \int_{E_{cut}}^{\infty} dE_{tn} \frac{d\sigma}{dE_{tn}} - \int_{E_{cut}}^{\infty} dE_{tn+1} \frac{d\sigma}{dE_{tn+1}}$$

We can define event shape variable similary in the second step y_n.

E_{t3} EVENT SHAPE VARIABLE



Perturbative QCD prediction for E_{t3} distribution.

SUB-JET RATES & SUB-JET MULTIPLICITY

In this analysis we use two step jet algorithm:

1. Resolve two hard macro-jet in the final state. This step is characterized E_{t2} jet shape variable.
2. Resolve the subjet structure of these two macro-jets. This step is charecterized by the y_{cut} variable.

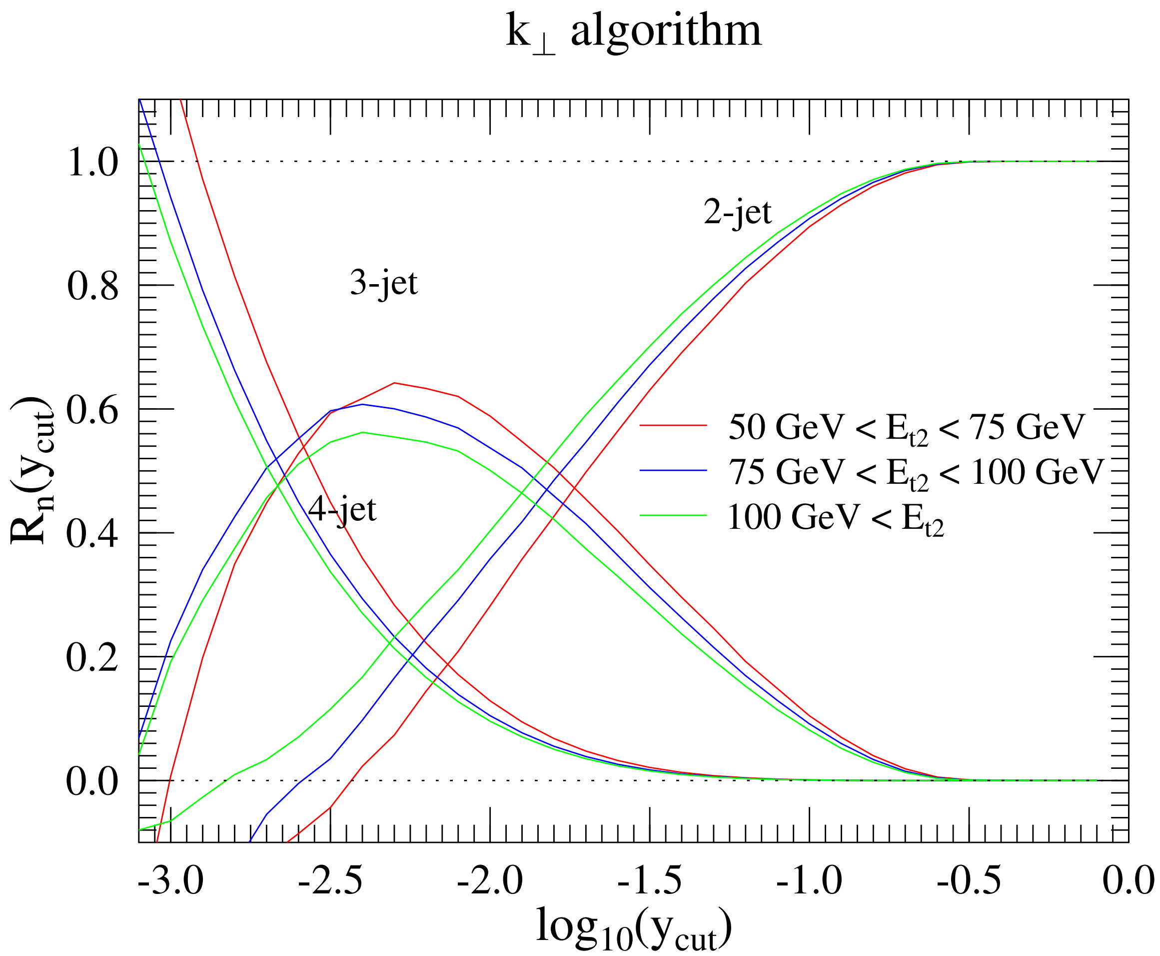
In the fix order calculation the sub-jet fraction is:

$$R_n(y_{cut}) = \frac{\sigma_n^{(0)}(y_{cut}) + \sigma_n^{(1)}(y_{cut}) + \cdots}{\sigma_0\left(1 + K_1 + \cdots\right)} \quad , \quad n \geq 2 \quad ,$$

and the sub-jet multiplicity

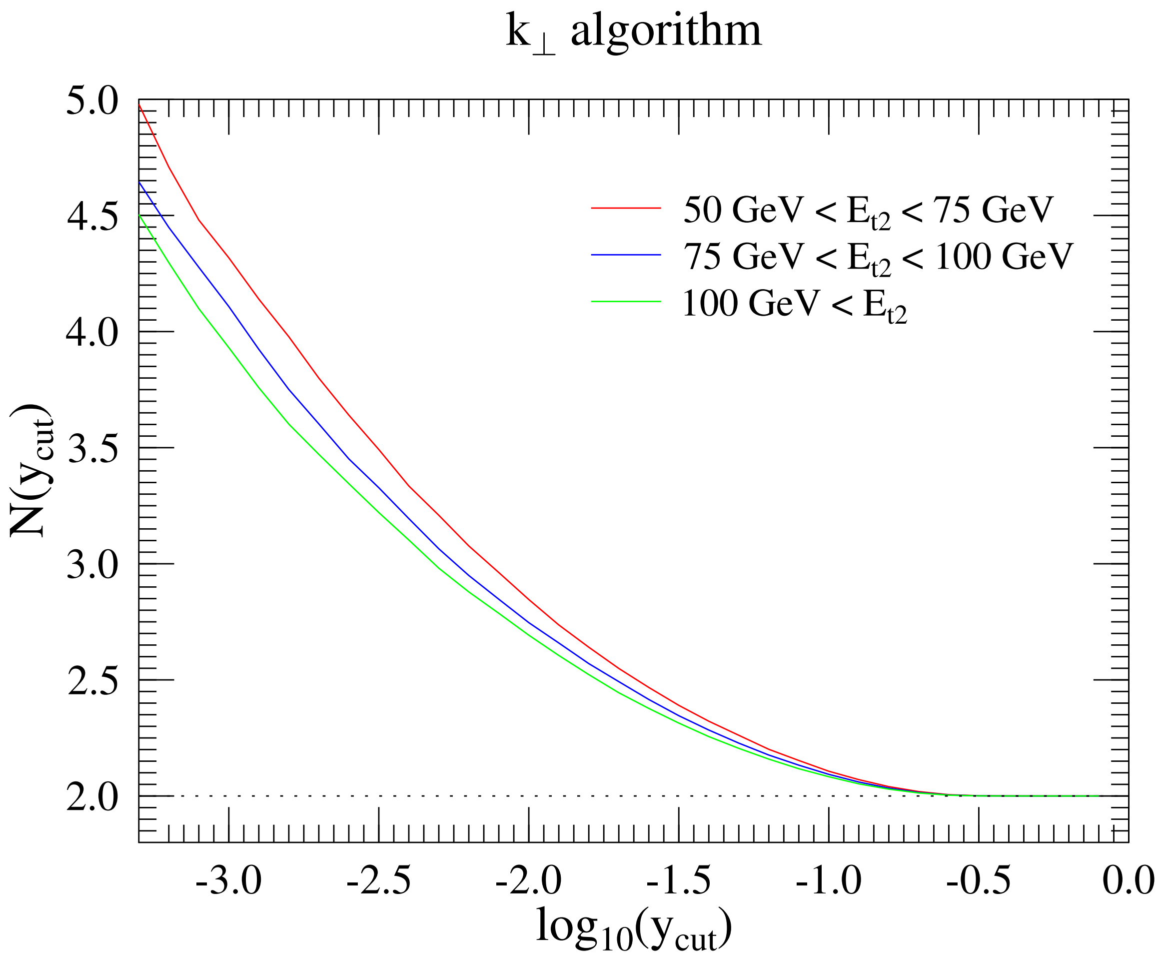
$$N(y_{cut}) = \sum_{n=2}^{\infty} n R_n(y_{cut}) \quad ,$$

SUB-JET RATES & SUB-JET MULTIPLICITY



Perturbative QCD prediction for the 2-, 3-, 4-subjet rates.

SUB-JET RATES & SUB-JET MULTIPLICITY



Perturbative QCD prediction for the sub-jet multiplicity.

EVENT SHAPES ON THE TRANSVERSE PLANE

One can define event shapes on the transverse plane. Important example is the transeverse thrust which is defined by

$$T_\perp = \max_{\vec{n}} \frac{\sum_{i\in C_N} |\vec{p}_{\perp,i}\cdot\vec{n}|}{\sum_{i\in C_N} |\vec{p}_{\perp,i}|} \quad ,$$

where C_N denote any well defined selection criteria

$$C_N = \{i \quad : \quad |\eta_i| < 1 \quad , \quad i = 1,\ldots,N\} \quad ,$$

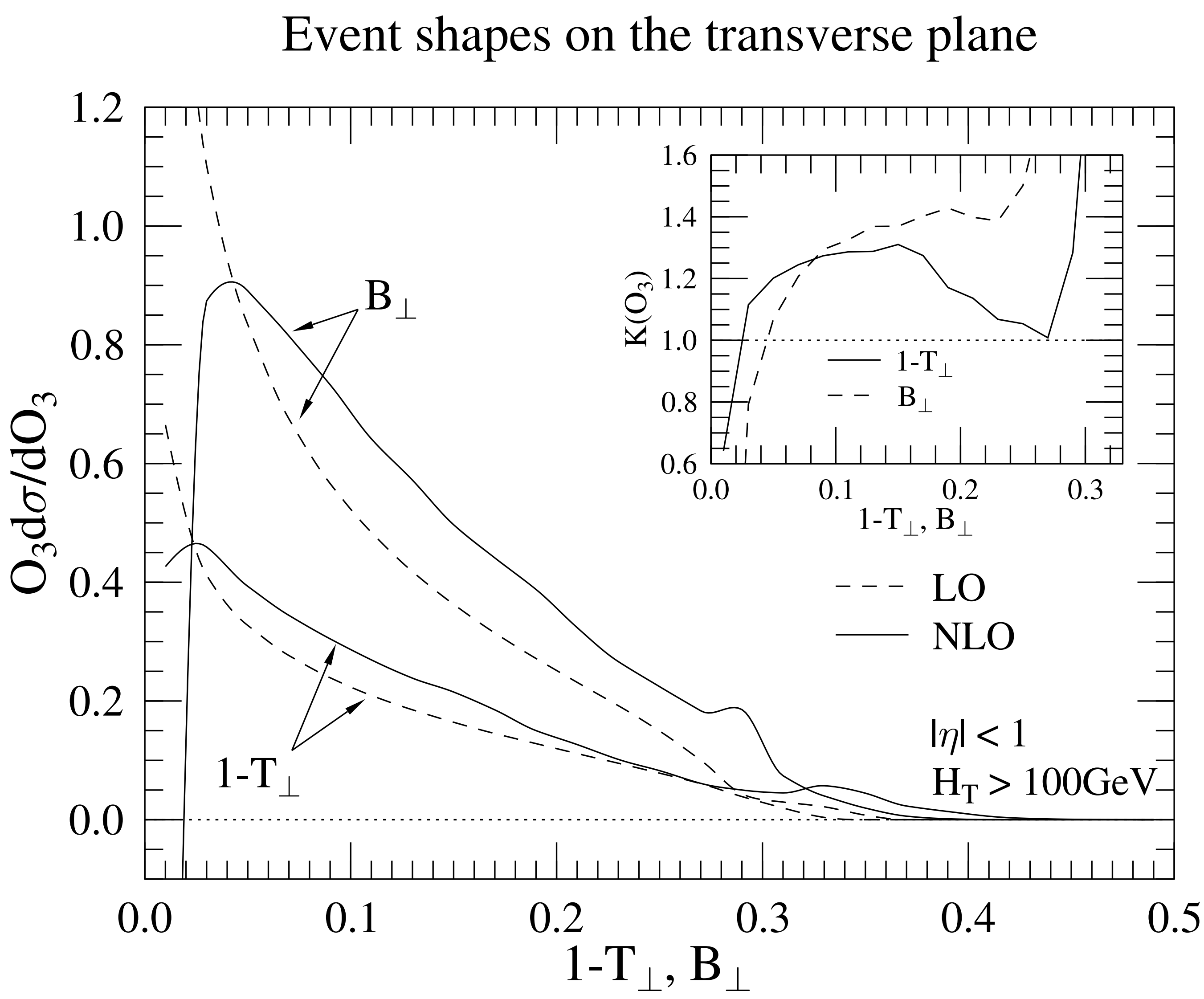
We can define similary the transverse jet broadening:

$$B_\perp = \frac{\sum_{i\in C_N} |\vec{p}_{\perp,i}\times\vec{n}|}{2\sum_{i\in C_N} |\vec{p}_{\perp,i}|} \quad .$$

we can calculate the differential distribution of the event shape or average values

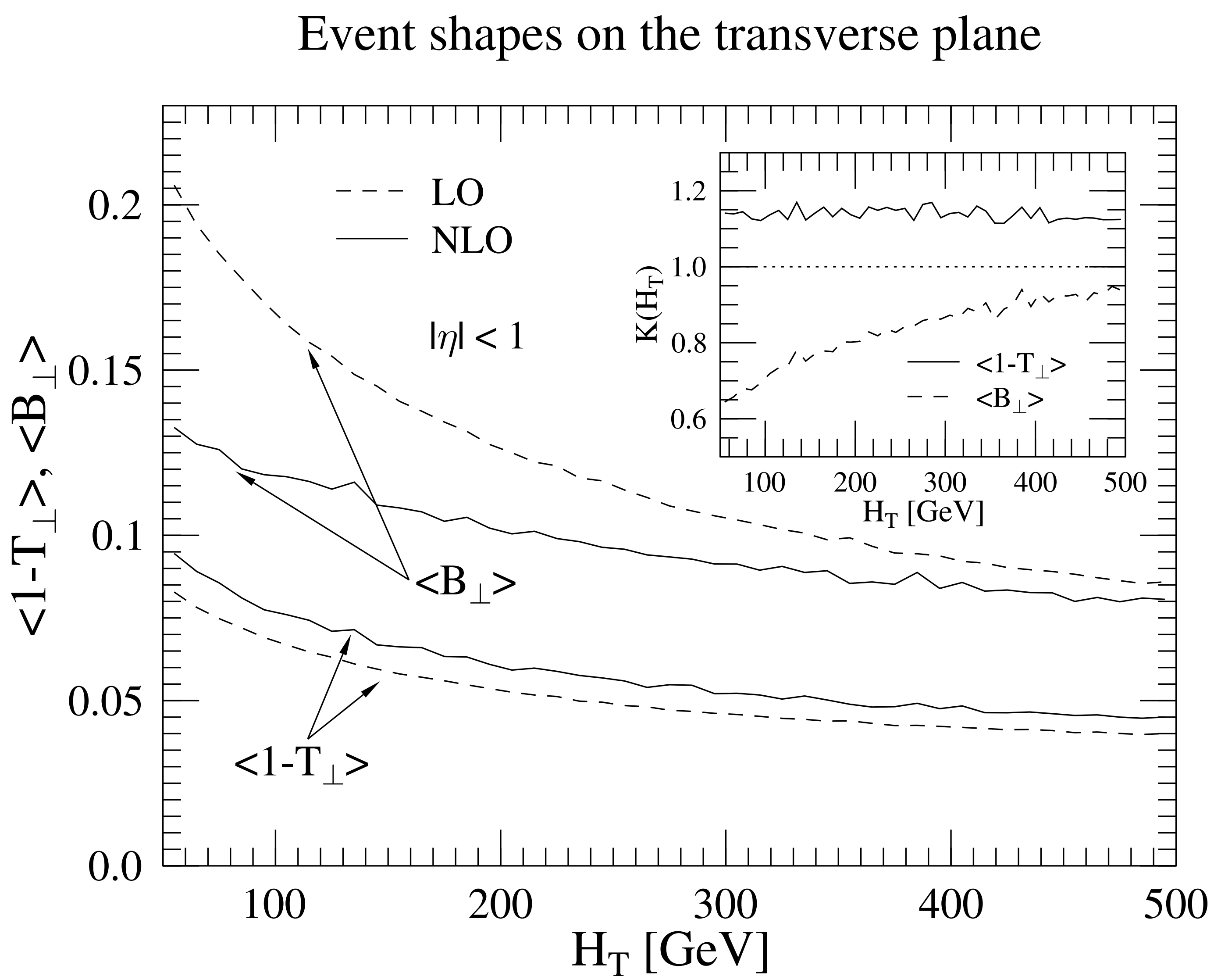
$$\Sigma(O_3) = \frac{O_3}{\sigma}\frac{d\sigma}{dO_3} \quad , \qquad \langle O_3\rangle = \int_0^1 dO_3 \Sigma(O_3)$$

EVENT SHAPES ON THE TRANSVERSE PLANE



NLOJET++ predictions for transverse thrust and jet broadening distribution.

EVENT SHAPES ON THE TRANSVERSE PLANE



NLOJET++ predictions for average value of transverse event shapes.

TRANSVERSE THRUST DISTRIBUTION

DØ measured the transverse thrust distribution of the two leading jets. The jets were defined by $k_\perp$ algorithm.

$$T_T = \max_{\vec{n}} \frac{\sum_a |\vec{p}_{T,a}| \cdot \vec{n}}{\sum_a p_{T,a}}$$

For tree partons in the final state : $\sqrt{3}/2 < T_{T_2} < 1$

For four partons in the final state : $\sqrt{2}/2 < T_{T_2} < 1$

Kinematical regions:

Pseudo-rapidity cuts : $|\eta^{\mathrm{jet}}_{1,2}| < 1$

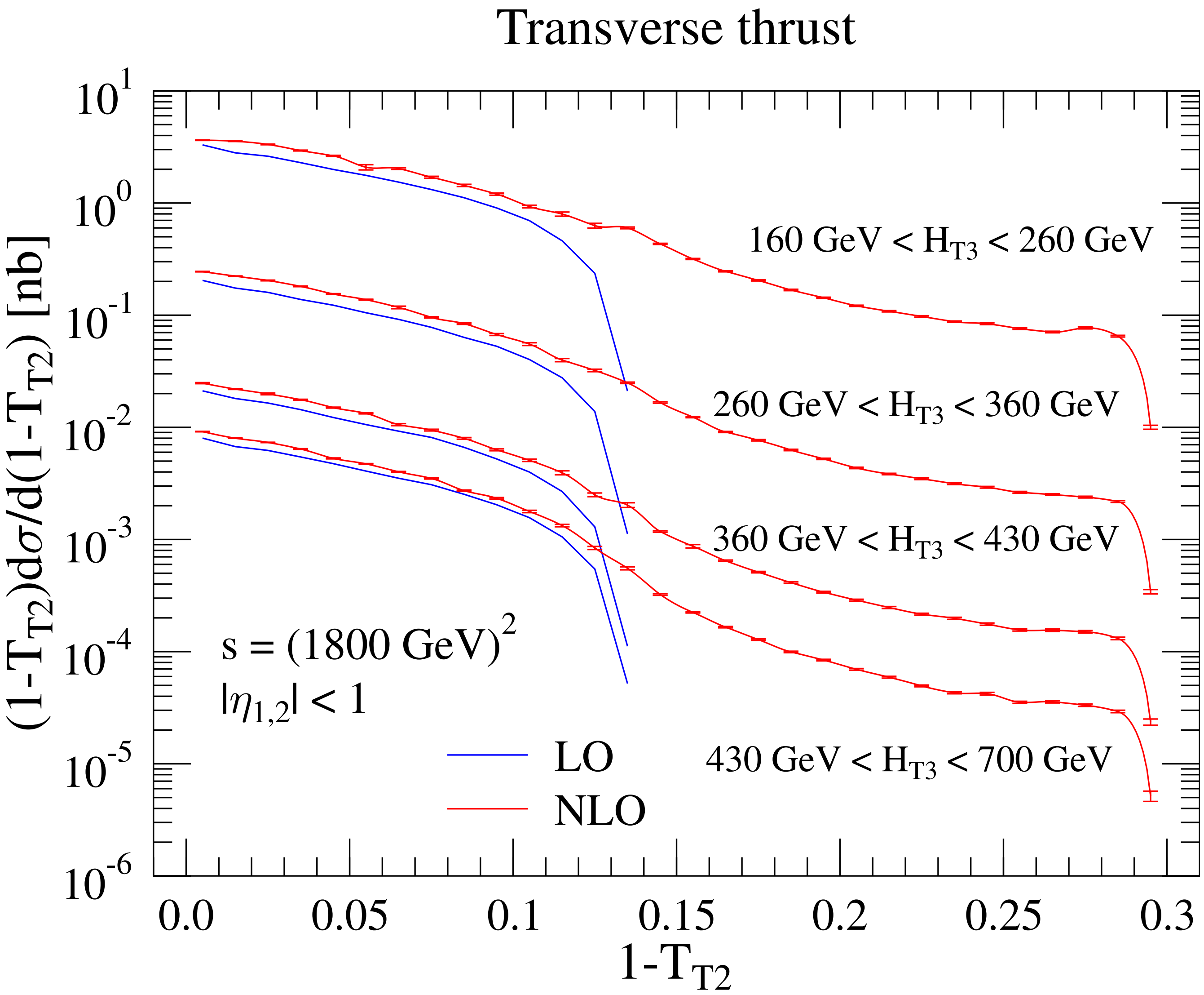
Transverse energy cuts : $H_T = E_T^{(1)} + E_T^{(2)} + E_T^{(3)}$

This distribution was measured over four H_T regions:

$160 < H_T < 260\,\mathrm{GeV}$, $260 < H_T < 360\,\mathrm{GeV}$,

$360 < H_T < 430\,\mathrm{GeV}$, $430 < H_T < 700\,\mathrm{GeV}$.

TRANSVERSE THRUST DISTRIBUTION



NLOJET++ prediction for transverse thrust distribution of the two leading jets at NLO.

PROGRAM: NLOJET++

A program for calculating next-to-leading order jet cross sections.

1. It knows several process:
   (a) 3,4-jet in e^+e^- at NLO and 5-jet at LO
   (b) 2,3-jet production in DIS at NLO and 4-jet at LO
   (c) 1,2,3-jet in hadron-hadron collision at NLO and 4-jet at LO
   (d) 2γ + 1-jet in hadron-hadron collision at NLO
   (e) 1,2,3-jet in photoproduction at NLO and 4-jet at LO
2. It implements the dipole method generally process independet way.
3. It's written in C++ but there are Fortran interfaces to the user defined parts.
4. This program could be a good starting point of a NNLO general program.
5. http://www.cpt.dur.ac.uk/˜nagyz/nlo++/

CPU TIME

All the presented distribution was calculated on a PC farm in the Theoretical Department in the Fermilab.

I used 24 processors (550MHz Pentium III) and every jobs run in the 1day queue.