

NLO QCD corrections to $pp \rightarrow t\bar{t}b\bar{b}$ at the LHC

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based on

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Physics at TeV colliders

Les Houches, June 8–17, 2009

Outline of the talk

- (1) **Introduction** - NLO corrections to multi-leg processes, $t\bar{t}b\bar{b}$ production
- (2) **Virtual corrections** - Feynman diagrams, tensor reduction, rational terms
- (3) **Real corrections** - Dipole subtraction
- (4) **Numerical results** - LHC cross section, CPU performance

(1) Introduction

Importance of **multi-leg processes** at the LHC

- huge W, Z and top-quark production rates + multiple jet emission
- multi-particle signatures with leptons and missing E
- serious backgrounds to Higgs and new-physics signals
(often not fully accessible to measurements)

Importance of **NLO QCD corrections** at the LHC

- reduce scale uncertainties (high powers of α_S !); improve description of jets
- systematics better than Tevatron; very high statistics

Technical problems for $2 \rightarrow 3, 4, \dots$ processes

- numerical instability of virtual corrections (Gram determinants)
- number and complexity of diagrams grow very fast

Challenges for NLO programs

- reliable predictions: **numerical stability**
- **sufficient speed**: distributions require > 1 event/sec!

Feynman diagrams have provided hadronic NLO cross sections for several nontrivial $2 \rightarrow 3$ processes

- $*pp \rightarrow t\bar{t}H, b\bar{b}H$ Beenakker/Dittmaier/Krämer/Plümper/Spira/Zerwas;
Dawson/Reina/Wackerroth/Orr/Jackson; Peng/Wen-Gan/Hong-Shen/Ren-You/Yi
- $pp \rightarrow HHH$ Plehn/Rauch; Binoth/Karg/Kauer/Rückl
- $pp \rightarrow Hjj$ Del Duca/ Kilgore/Oleari/Schmidt/Zeppenfeld; Campbell/Ellis/Zanderighi
Ciccolini/Denner/Dittmaier
- $pp \rightarrow jjj$ Bern/Dixon/Kosower; Kunst/Signer/Trocsanyi; Giele/Kilgore/Nagy
- $pp \rightarrow Vjj$ Bern/Dixon/Kosower; Ellis/Veseli; Campbell/Ellis;
- $pp \rightarrow VVj$ Dittmaier/Kallweit/Uwer; Campbell/Ellis/Zanderighi;
- $pp \rightarrow VVV$ Lazopoulos/Melnikov/Petriello; Hankele/Zeppenfeld;
Binoth/Ossola/Papadopoulos/Pittau
- $pp \rightarrow V + b\bar{b}$ Ferbes Cordero/Reina/Wackerroth
- $*pp \rightarrow t\bar{t}j$ Dittmaier/Uwer/Weinzierl
- $*pp \rightarrow t\bar{t}Z$ Lazopoulos/McElmurry/Melnikov/Petriello

$*NLO$ calculations related to $pp \rightarrow t\bar{t}H$ (like $pp \rightarrow t\bar{t}b\bar{b}$)

... and a few pioneering $2 \rightarrow 4$ results for e^+e^- and $\gamma\gamma$ colliders

- $e^+e^- \rightarrow 4f$ (EW) Denner/Dittmaier/Roth/Wieders '05
- $e^+e^- \rightarrow HH\nu\bar{\nu}$ (EW) Boudjema/Fujimoto/Ishikawa/Kaneko/Kurihara/Shimizu/Kato/Yasui '05
- $\gamma\gamma \rightarrow t\bar{t}b\bar{b}$ (QCD) Lei/Wen-Gan/Liang/Ren-You/Yi '07

Questions

- Can one repeat this success at hadron colliders?
- Should one switch to alternative methods like unitarity cuts?

Les Houches '05/'07 prioritized wishlist

	Reaction	background for	existing NLO cross sections
1.	VVj	$t\bar{t}H$, new physics	WWj: Dittmaier/Kallweit/Uwer '07; WWj: Campbell/Ellis/Zanderighi '07 WWj/ZZj: Binoth/Guillet/Karg/Kauer/Sanguinetti (in progress)
2.	$t\bar{t}b\bar{b}$	$t\bar{t}H$	complete NLO: Bredenstein/Denner/Dittmaier/P. '08-'09
3.	$t\bar{t}jj$	$t\bar{t}H$	\emptyset
4.	$VVb\bar{b}$	VBF \rightarrow H \rightarrow VV, $t\bar{t}$, NP	\emptyset
5.	$VVjj$	VBF \rightarrow H \rightarrow VV	VBF: Jäger/Oleari/Zeppenfeld '06 + Bozzi '07
6.	$Vjjj$	new physics	leading colour approximation: Ellis/Giele/Kunzt/Melnikov/Zanderighi '08; Ellis/Melnikov/Zanderighi '09; Berger/Bern/Dixon/Ferbes Cordero/Forde/Gleisberg/Ita/ Kosower/Maitre '09
7.	VVV	new physics	ZZZ: Lazopoulos/Melnikov/Petriello '07; WWZ: Hankele/Zeppenfeld '07; VVV: Binoth/Ossola/Papadopoulos/Pittau '07
8.	$b\bar{b}b\bar{b}$	Higgs and new physics	qq channel (virtual): Binoth/Guffanti/Guillet/Heinrich/Karg/ Kauer/Mertsch/Reiter/Reuter/Sanguinetti

virtual amplitudes for several $2 \rightarrow 4$ processes van Hameren/Papadopoulos/Pittau '09

Technical motivation for $t\bar{t}b\bar{b}$: excellent benchmark calculation

Validate/compare NLO algorithms by solving a non-trivial problem

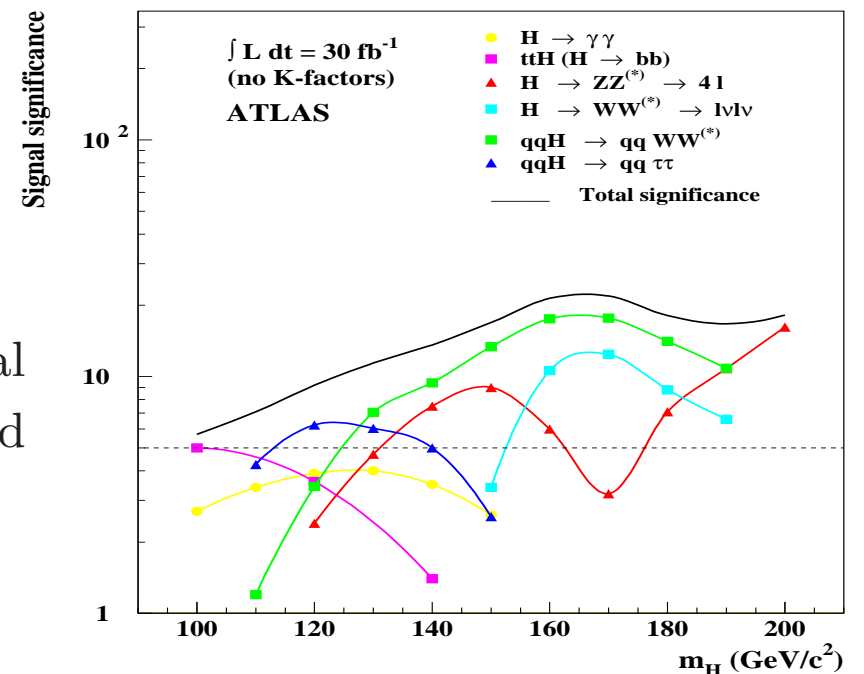
- $2 \rightarrow 4$ process with $\mathcal{O}(10^3)$ diagrams involving hexagons up to rank 4
- 6 coloured legs, massless and massive quarks, gluons

Phenomenological motivation for $t\bar{t}b\bar{b}$: irreducible background to $t\bar{t}H(H \rightarrow b\bar{b})$

Associated $t\bar{t}H(H \rightarrow b\bar{b})$ production

- opportunity to observe $H \rightarrow b\bar{b}$ channel and exploit dominance of its branching ratio for $M_H < 135 \text{ GeV}$
- measurement of top Yukawa coupling
- ATLAS TDR indicated discovery potential (disappeared after more reliable background estimates)
- the background has a dramatic impact

Early ATLAS studies of Higgs discovery potential (ATLAS '03)



Idea of $t\bar{t}H(H \rightarrow b\bar{b})$ analysis

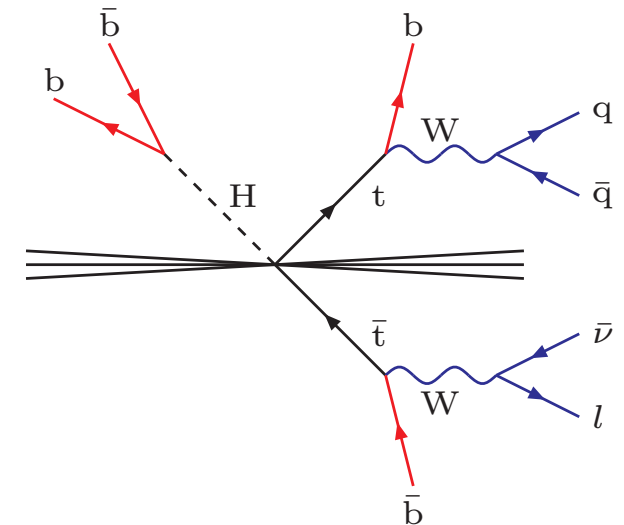
- consider semileptonic decay channel: $b\bar{b}b\bar{b}jjl\nu$ final state with 4 b-quarks!
- **identify $b\bar{b}$ pair from Higgs decay**
- observe resonance in $m_{b\bar{b}}$ distribution

Main problem: b-quark combinatorics

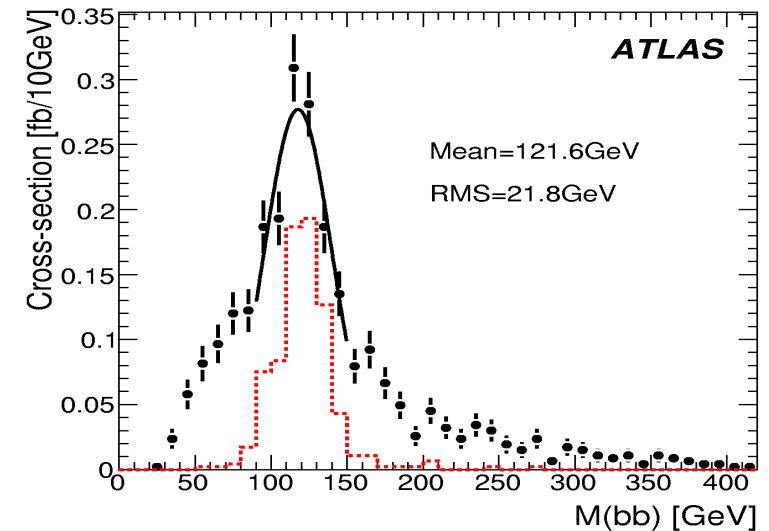
- only 1/6 possible b-quark pairs is correct!
- perform full $t\bar{t}$ reconstruction to identify b-quarks from top (and Higgs) decay
- very difficult due to presence of ≥ 6 jets and bottom/light-jet missidentification
- **rate of correct b-pairings only 1/3!**

Consequences

- **dilution of Higgs resonance**
- **increase of background in resonance region**



dilution of Higgs resonance in $t\bar{t}H$



Background and systematic uncertainty

Backgrounds (ATLAS analysis)

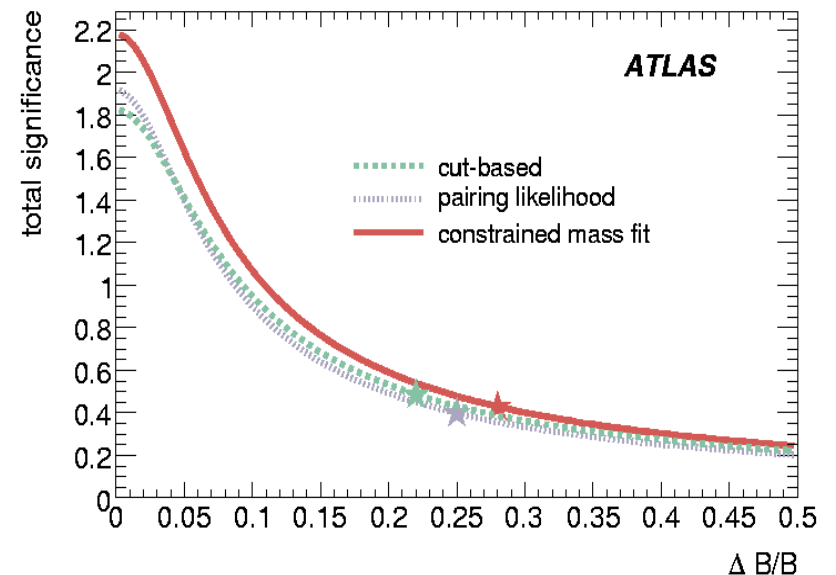
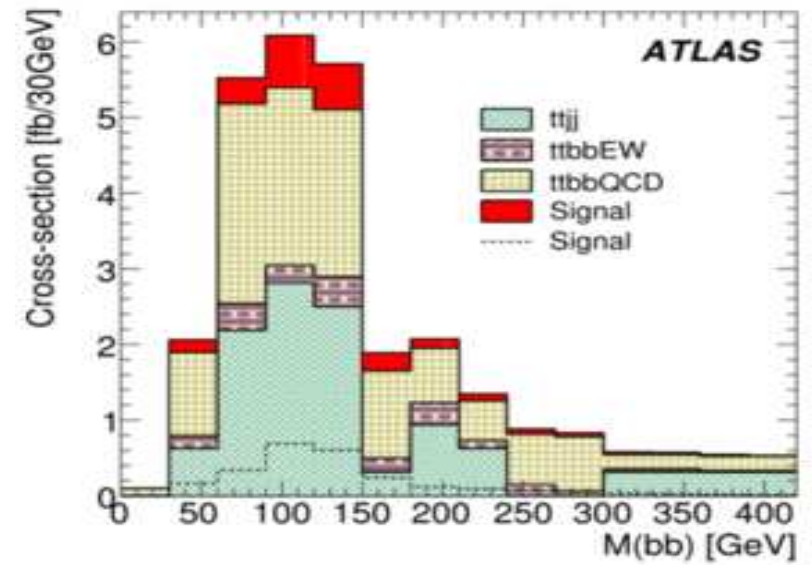
- $t\bar{t}b\bar{b}$ (AcerMC, $\mu_{\text{QCD}} = m_t + m_{\bar{b}b}/2$)
- $t\bar{t}jj$ (MC@NLO, $\mu_{\text{QCD}}^2 = m_t^2 + \langle p_{\text{T},t}^2 \rangle$)

Statistics and systematics (30 fb^{-1})

- $S/\sqrt{B} \simeq 2$ sufficient for measurement
- $S/B \simeq 0.1$ implies that $\Delta B/B$ systematic uncertainty of $\mathcal{O}(10\%)$ kills measurement!

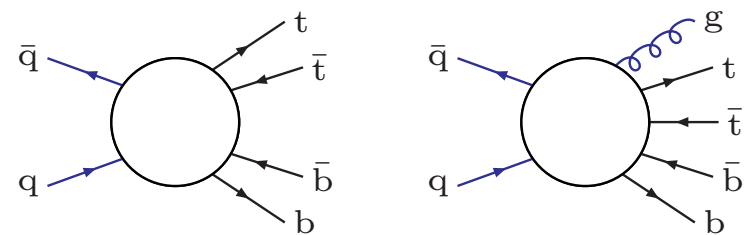
Strategy for precise determination of B

- similar shape of S and B prevents direct extraction of B from data
- measure B normalization in signal-free region and extrapolate to signal-rich region using precise shape predictions
- **impossible without $t\bar{t}b\bar{b}$ and $t\bar{t}jj$ at NLO!**

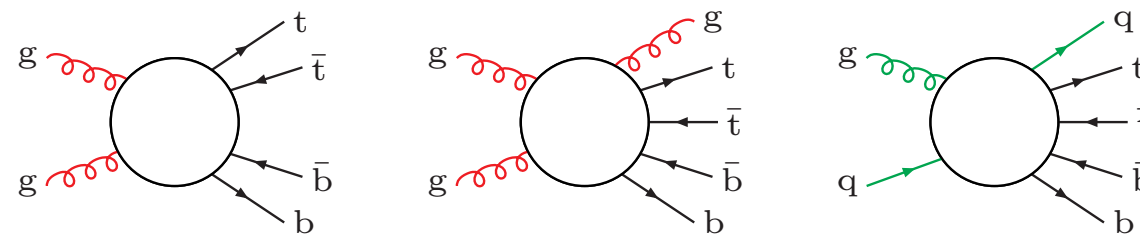


2 → 4 and 2 → 5 Feynman diagrams

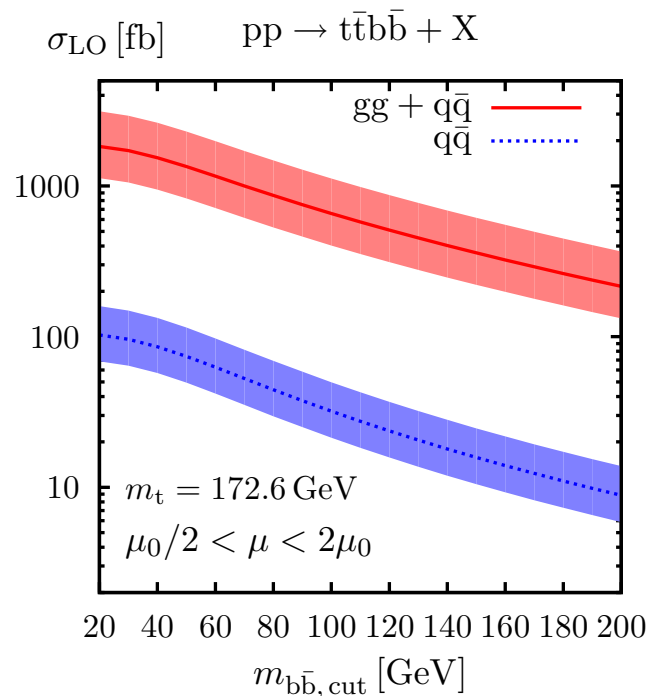
Quark-antiquark and gluon induced processes



arXiv:0807.1248



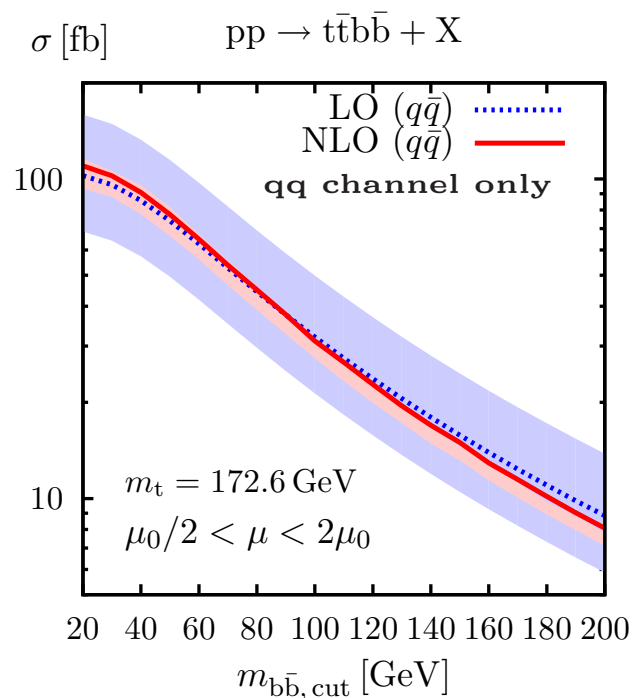
arXiv:0905.0110



diagrams and impact on σ_{tot}

	$q\bar{q}$	gg	qg
LO	7	36	
virtual	188	1003	
real	64	341	64
$(\sigma/\sigma_{\text{tot}})_{\text{LO}}$	5%	95%	
$(\sigma/\sigma_{\text{tot}})_{\text{NLO}}$	3%	92%	5%

Step 1: quark anti-quark channel



Motivation

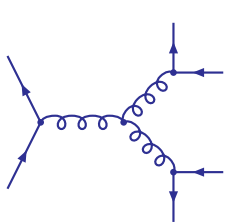
- 6-fermion process related to $e^+e^- \rightarrow 4f$
- proof of principle of feasibility of calculation

Main results [[arXiv:0807.1248](https://arxiv.org/abs/0807.1248)]

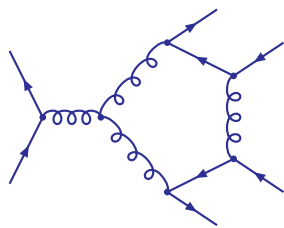
- small NLO correction ($K=1.03$)
- strong reduction of scale dependence
- very high CPU performance of 13 ms/PS-point

(2) Tree and one-loop contributions to $q\bar{q}/gg \rightarrow t\bar{t}b\bar{b}$

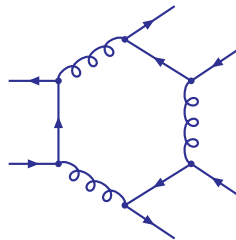
Tree and one-loop sample diagrams in the $q\bar{q}$ and gg channels



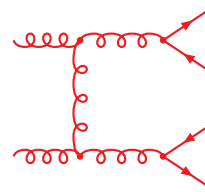
7 trees



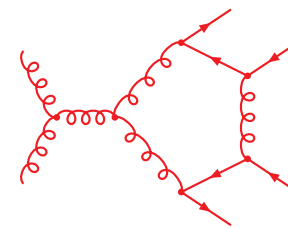
24 pentagons



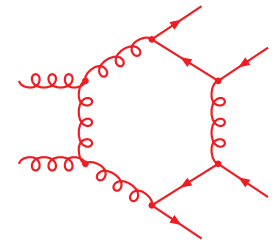
8 hexagons



36 trees



114 pentagons



40 hexagons

Two independent calculations

- diagrams generated with `FeynArts 1.0 / 3.2` [[Külbeck/Böhm/Denner '90](#); [Hahn '01](#)]
- one calculation uses `FormCalc 5.2` [[Hahn '06](#)] for preliminary algebraic manipulations (Dirac algebra, covariant decomposition)
- bulk of reduction with two in-house `MATHEMATICA` programs
- numerics with two independent `Fortran77` codes
(two libraries for tensor integrals)

Top quarks massive and bottom quarks massless

Structure of the calculation

Standard matrix elements and colour structures for **individual diagrams**

$$\mathcal{D} = \underbrace{\left[\sum_k a_k \mathcal{C}_k(\{c\}) \right]}_{\text{factorized colour structure}} \sum_i \mathcal{F}_i \underbrace{\mathcal{S}_i(\{p\}, \{\lambda\})}_{\text{standard matrix elements}}$$

Form factors \mathcal{F}_i in terms of tensor integrals

$$\mathcal{F}_i = \sum_{j_1 \dots j_R} \mathcal{K}_{i,j_1 \dots j_R} \underbrace{T_{j_1 \dots j_R}(\{p\})}_{\text{tensor loop coefficients}}$$

computed numerically diagram by diagram (no analytic reduction to scalar integrals)

Main goals

- reduction to small set of standard matrix elements \mathcal{S}_i
- fast and stable numerical evaluation of tensor integrals $T_{j_1 \dots j_R}$

Colour structure

- colour basis in the $q\bar{q}$ and gg channels

$$\underbrace{\delta_{i_1 i_2} \delta_{i_3 i_4} \delta_{i_5 i_6}, \quad \delta_{i_1 i_2} T_{i_3 i_4}^a T_{i_5 i_6}^a, \quad \dots}_{6 \text{ elements}} \quad \underbrace{\delta^{a_1 a_2} \delta_{i_3 i_4} \delta_{i_5 i_6}, \quad T_{i_3 i_4}^{a_1} T_{i_5 i_6}^{a_2}, \quad \dots}_{14 \text{ elements}}$$

- colour factorization for individual diagrams \Rightarrow CPU time doesn't scale with the number of colour structures

Rational terms originating from UV $1/(D-4)$ poles of tensor loop integrals

$$\mathcal{K}_{j_1 \dots j_R}(D) \underbrace{T_{j_1 \dots j_R}}_{\frac{R_{j_1 \dots j_R}}{(D-4)} + \text{UV-finite part}} = \mathcal{K}_{j_1 \dots j_R}(4) T_{j_1 \dots j_R} + \mathcal{K}'_{j_1 \dots j_R}(4) R_{j_1 \dots j_R} + \mathcal{O}(D-4)$$

- residues $R_{j_1 \dots j_R}$ of UV poles of tensor integrals explicitly available
- after $(D-4)$ -expansion continue calculation in $D=4$
- **rational terms from IR poles** appear in intermediate expressions but **cancel at amplitude level and can thus be ignored** (proven for any scattering amplitude in App. A of [JHEP 0808, 108 \(2008\) \[arXiv:0807.1248\]](#))

Cancellation of rational terms originating from IR poles

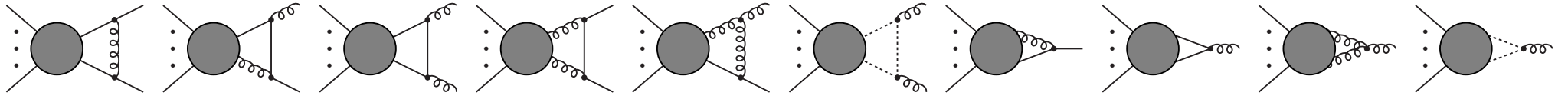
Explicit calculations in terms of tensor and/or scalar integrals contain "rational" terms arising from double and single **IR poles** of tensor loop integrals

$$\mathcal{K}_{j_1 \dots j_R}(D) \underbrace{T_{j_1 \dots j_R}} \Rightarrow \mathcal{K}'_{j_1 \dots j_R}(4) R_{1,j_1 \dots j_R} + \frac{1}{2} \mathcal{K}''_{j_1 \dots j_R}(4) R_{2,j_1 \dots j_R} + \dots$$
$$\frac{R_{2;j_1 \dots j_R}}{(D-4)^2} + \frac{R_{1;j_1 \dots j_R}}{(D-4)} + \text{IR-finite part}$$

- can require lot of algebraic work (second-order expansion)
 - cancel completely in truncated (!) one-loop amplitudes (observed in various calculations)
 - explicit proof based on the following two observations
- (1) **Tensor-reduction identities are free from rational terms of IR type**
- Only $g^{\mu\nu}$ -type components of tensor integrals ($T_{00\dots}$) receive D -dependent factors in reduction identities
 - while soft and collinear regions ($q^\mu \rightarrow xp^\mu$) yield only $p_i^\mu p_j^\mu \dots$ tensor structures

(B) IR-divergent part of (individual) loop diagrams can be expressed in terms of integrals with D -independent prefactors

(B.1) consider all diagrams involving IR-divergent integrals



(B.2) look for trace-like contractions, $g_{\nu\lambda}\Gamma^{\nu\lambda}$, which can produce D -dep. coefficients:

$$g_{\nu\lambda} g^{\nu\lambda} = D, \quad g_{\nu\lambda} \gamma^\nu \not{p} \gamma^\lambda = (2 - D)\not{p}, \dots \quad (1)$$

(B.3) Manipulating the IR-divergent part of the integrand in D dim (using appropriate loop parametrizations and taking into account double-counting subtleties)

$$= \int d^D q \epsilon^{\mu*} \frac{(q + 2p)_\lambda g_{\mu\nu} - (2q + p)_\mu g_{\nu\lambda} + (q - p)_\nu g_{\lambda\mu}}{q^2 (q + p)^2} \Gamma^{\nu\lambda}(q)$$

we find that the IR-singular part is free from trace-like contractions (due to the collinear limit $\epsilon^{\mu*} (2q + p)_\mu \rightarrow 0$ in the above example)

Practical consequence

- "rational" terms from IR poles can be systematically neglected
- subtlety related to scaleless 2-point functions

$$B_0(0, 0, 0) = \left(\frac{2}{D-4} \right)_{\text{IR}} - \left(\frac{2}{D-4} \right)_{\text{UV}} = 0$$
$$(D-4)B_0(0, 0, 0) \Rightarrow (D-4) \left(\frac{-2}{D-4} \right)_{\text{UV}} = -2$$

Generality and applicability

- cancellation applies to any **truncated** QCD amplitude and is independent from reduction algorithm (OPP, unitarity, PV, ...)
- proven for Feynman t'Hooft or similar gauges (power-1 propagators)
- δZ factors contain rational terms of IR origin

Reduction of Standard Matrix Elements (SMEs): strategy/simple examples

(A) **q \bar{q} channel**: after standard D -dim manipulations, $\mathcal{O}(10^3)$ Dirac struct. of type

$$\underbrace{\left[\bar{v}(p_1) \dots \gamma^\mu \gamma^\nu \not{p}_3 \dots u(p_2) \right]}_{\text{q}\bar{\text{q}} \text{ chain}} \quad \underbrace{\left[\bar{v}(p_3) \dots \gamma^\mu \gamma^\nu \gamma^\rho \not{p}_6 \dots u(p_4) \right]}_{\text{t}\bar{\text{t}} \text{ chain}} \quad \underbrace{\left[\bar{v}(p_5) \dots \gamma^\rho \not{p}_2 \not{p}_3 \dots u(p_6) \right]}_{\text{b}\bar{\text{b}} \text{ chain}}$$

Needs additional identities in order to eliminate \not{p} -terms and γ -contractions

- without introducing new denominators that might spoil numerical stability

After cancellation of $1/(D-4)$ poles we work in 4 dimensions and introduce

$$\omega_\pm = \frac{1}{2}(1 \pm \gamma^5) \quad \bar{v}(p_i)\Gamma u(p_j) \Rightarrow \sum_{\lambda=\pm} \bar{v}(p_i)\Gamma \omega_\lambda u(p_j)$$

This permits to use the **Chisholm identity**

$$i\varepsilon^{\alpha\beta\gamma\delta} \gamma_\delta \gamma^5 = \gamma^\alpha \gamma^\beta \gamma^\gamma - g^{\alpha\beta} \gamma^\gamma + g^{\alpha\gamma} \gamma^\beta + g^{\beta\gamma} \gamma^\alpha$$

Contracting with $\otimes \gamma_\gamma \gamma^5$ and symmetrizing \Rightarrow eliminates the ε -tensor and yields

$$i\varepsilon^{\alpha\beta\gamma\delta} \left[\gamma_\delta \gamma^5 \otimes \gamma_\gamma \gamma^5 + (\delta \leftrightarrow \gamma) \right] = 0 \quad \Rightarrow \quad \gamma^\mu \gamma^\alpha \gamma^\beta \omega_\pm \otimes \gamma_\mu \omega_\mp = \gamma^\mu \omega_\pm \otimes \gamma^\alpha \gamma^\beta \gamma_\mu \omega_\mp$$

and similar identities that permit to exchange $\gamma^\alpha \gamma^\beta$ between γ -contracted chains

Other identities involving doubly γ -contracted chains

$$\gamma^\mu \gamma^\alpha \gamma^\nu \omega_\pm \otimes \gamma_\mu \gamma^\beta \gamma_\nu \omega_\mp = 4\gamma^\beta \omega_\pm \otimes \gamma^\alpha \omega_\mp \quad \text{etc.}$$

can be used (for instance)

- to reduce number of γ -contractions
- or (in the inverse direction) to shift \not{p} -terms and then exploit Dirac equation

$$\begin{aligned} 4 \left[\bar{v}(p_1) \dots \not{p}_4 \omega_\pm u(p_2) \right] \left[\bar{v}(p_3) \dots \not{p}_2 \omega_\mp u(p_4) \right] &= \left[\bar{v}(p_1) \dots \gamma^\mu \not{p}_2 \gamma^\nu \omega_\pm u(p_2) \right] \left[\bar{v}(p_3) \dots \gamma_\mu \not{p}_4 \gamma_\nu \omega_\mp u(p_4) \right] \\ &= 4(p_2 p_4) \left[\bar{v}(p_1) \dots \gamma^\mu \omega_\pm u(p_2) \right] \left[\bar{v}(p_3) \dots \gamma_\mu \omega_\mp u(p_4) \right] + \text{mass-terms} \end{aligned}$$

In practice

- Chisolm identity + Dirac equation + momentum conservation (+ a lot of patience)
- yields several useful relations
- construct a sophisticated algorithm to reduce # of γ -contractions and \not{p} -terms

At the end of the day **25 types** of standard matrix elements

- 10 of "massless" type: one Dirac matrix per chain

$$\begin{aligned} & \left[\bar{v}(p_1) \not{p}_i \omega_\alpha u(p_2) \right] \left[\bar{v}(p_3) \gamma^\mu \omega_\beta u(p_4) \right] \left[\bar{v}(p_5) \gamma^\mu \omega_\rho u(p_6) \right] \\ & \left[\bar{v}(p_1) \not{p}_i \omega_\alpha u(p_2) \right] \left[\bar{v}(p_3) \not{p}_j \omega_\beta u(p_4) \right] \left[\bar{v}(p_5) \not{p}_k \omega_\rho u(p_6) \right] \end{aligned}$$

- 15 of "massive" type: 2/0 Dirac matrices inside the $t\bar{t}$ chain

$$\begin{aligned} & \left[\bar{v}(p_1) \not{p}_i \omega_\alpha u(p_2) \right] \left[\bar{v}(p_3) \not{p}_j \gamma^\mu \omega_\beta u(p_4) \right] \left[\bar{v}(p_5) \not{p}_k \omega_\rho u(p_6) \right] \\ & \left[\bar{v}(p_1) \gamma^\mu \omega_\alpha u(p_2) \right] \left[\bar{v}(p_3) \not{p}_j \not{p}_j \omega_\beta u(p_4) \right] \left[\bar{v}(p_5) \gamma^\mu \omega_\rho u(p_6) \right] \\ & \left[\bar{v}(p_1) \gamma^\mu \omega_\alpha u(p_2) \right] \left[\bar{v}(p_3) \gamma^\mu \gamma^\nu \omega_\beta u(p_4) \right] \left[\bar{v}(p_5) \gamma^\nu \omega_\rho u(p_6) \right] \\ & \left[\bar{v}(p_1) \gamma^\mu \omega_\alpha u(p_2) \right] \left[\bar{v}(p_3) \omega_\beta u(p_4) \right] \left[\bar{v}(p_5) \gamma^\mu \omega_\rho u(p_6) \right] \\ & \left[\bar{v}(p_1) \not{p}_i \omega_\alpha u(p_2) \right] \left[\bar{v}(p_3) \omega_\beta u(p_4) \right] \left[\bar{v}(p_5) \not{p}_k \omega_\rho u(p_6) \right] \end{aligned}$$

25×8 chiral structures $(\omega_\alpha \otimes \omega_\beta \otimes \omega_\rho) \Rightarrow$ **200 SMEs** for the $q\bar{q}$ channel

(B) Reduction strategy for gg channel

Standard D -dim manipulations for quark chains and gluon-helicity vectors
 $(\epsilon_1 p_1 = \epsilon_2 p_2 = \epsilon_1 p_2 = \epsilon_2 p_1 = 0)$ yield structures of type

$$\underbrace{\left\{ \epsilon_1^\mu \epsilon_2^\nu, (\epsilon_1 \epsilon_2) p_2^\mu p_4^\nu, (\epsilon_1 p_4)(\epsilon_2 p_3) g^{\mu\nu}, \dots \right\}}_{\text{gluon polarization vectors}} \underbrace{\left[\bar{v}(p_3) \dots \gamma_\mu \gamma_\rho \not{p}_6 \dots u(p_4) \right]}_{\text{t}\bar{\text{t}} \text{ chain}} \underbrace{\left[\bar{v}(p_5) \dots \gamma^\rho \gamma_\nu \not{p}_2 \not{p}_3 \dots u(p_6) \right]}_{\text{b}\bar{\text{b}} \text{ chain}}$$

Strategy B.1: employ q \bar{q} -channel method (γ^5 and Chisolm-based identities) to reduce two fermion chains \Rightarrow **502 SMEs**

Strategy B.2: less-sophisticated 4-dim reduction based only on a relation of type

$$\gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} \gamma^{\mu_4} \gamma^{\mu_5} = g^{\mu_1 \mu_2} \gamma^{\mu_3} \gamma^{\mu_4} \gamma^{\mu_5} + \dots + g^{\mu_1 \mu_2} g^{\mu_3 \mu_4} \gamma^{\mu_5} + \dots$$

- can be derived from Chisolm identity but does not involve γ^5
- permits to eliminate all γ -products with more than 3 terms \Rightarrow **970 SMEs**
- factor 2 more SMEs but completely process-independent reduction

Surprising result: speed of codes based on B.1 and B.2 reduction almost identical.
CPU efficiency not due to highly sophisticated process-dependent manipulations!

Stable reduction of tensor loop integrals (developed for $e^+e^- \rightarrow 4f$) [Denner/Dittmaier '05]

6- and 5-point tensor integrals (exploit linear dependence of $4 + n$ momenta)

- reduction to lower-point integrals [Melrose '65; Denner/Dittmaier '02]
- simultaneous rank-reduction [Binoth/Guillet/Heinrich/Pilon/Schubert '05; Denner/Dittmaier '05]
- no Gram determinants \Rightarrow no numerical problems in practice

4- and 3-point tensor integrals

- Passarino–Veltman reduction [Passarino/Veltman '79]
- alternative methods for **small Gram determinants** [Denner/Dittmaier '05]
(analogies with techniques proposed by Ferrogli/Passera/Passarino/Uccirati '03;
Binoth/Guillet/Heinrich/Pilon/Schubert '05; Ellis/Giele/Zanderighi '06)

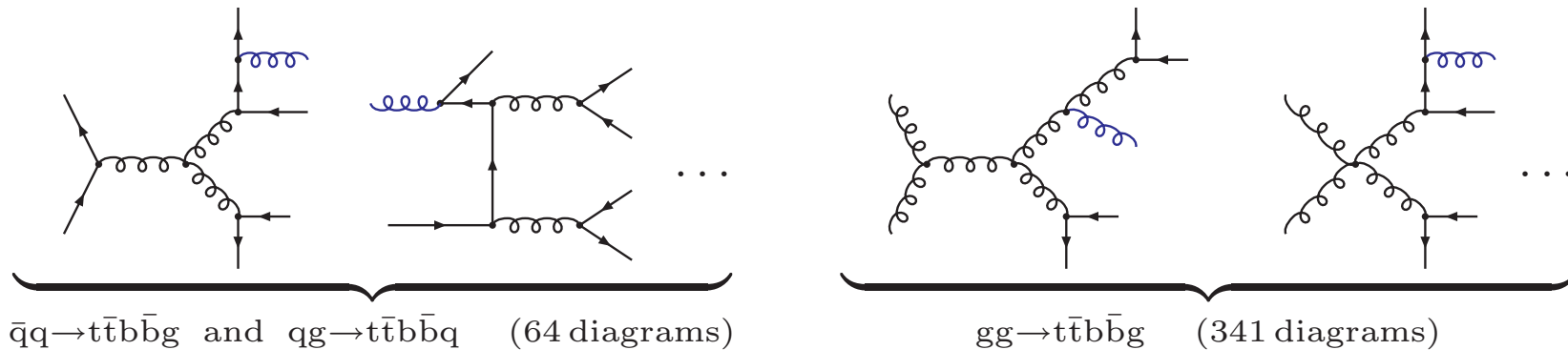
2- and 1-point tensor integrals

- numerically stable analytic representations [Passarino/Veltman '79; Denner/Dittmaier '05]

(3) Real corrections (qq/gg/qg channels)

- Also for real corrections: 2 independent calculations

Two types of matrix elements



- **Madgraph 4.1.33** [Alwall/Demin/deVisscher/Frederix/Herquet/Maltoni/Plehn/Rainwater/Stelzer'07] for all channels
- analytical calculation with **Weyl–van der Waerden spinors** [Dittmaier '98] for qq/qg channels
- in-house numerical algorithm based on **off-shell recursions** [Berends/Giele '88; Caravaglios/Moretti '95; Draggiotis/Kleiss/Papadopoulos '98] for gg channel

Treatment of soft and collinear singularities with **dipole subtraction**

Catani/Seymour '96; Dittmaier '99; Catani/Dittmaier/Seymour/Trócsányi '02

$$\int d\sigma_{2\rightarrow 5} = \int \left[d\sigma_{2\rightarrow 5} - \sum_{\substack{i,j=1 \\ i \neq j}}^6 d\sigma_{2\rightarrow 5}^{\text{dipole},ij} \right] + \sum_{\substack{i,j=1 \\ i \neq j}}^6 \mathcal{F}_{ij} \otimes d\sigma_{2\rightarrow 4}$$

- numerically stable/efficient but non-trivial: 30 qq/gg (10 qg) subtraction terms
- **in-house dipoles** checked against **MadDipole** [Frederix/Gehrmann/Greiner '08] (gg/qg) and **PS slicing** [Giele/Glover '92; Giele et al. '93; Keller/Laenen '98; Harris/Owens '01] (qq)
- initial-state collinear singularities cancelled by $\overline{\text{MS}}$ -redefinition of PDFs

Phase-space integration

- **adaptive multi-channel Monte Carlo** [Berends/Kleiss/Pittau '94; Kleiss/Pittau '94] as in **RACONWW** [Denner/Dittmaier/Roth/Wackerath '99] / **PROFECY4f** [Bredenstein/Denner/Dittmaier/Weber '06]
- $\mathcal{O}(1400)$ channels to map all peaks from propagators (300) and dipoles (1100)

11-dimensional phase space, many channels and dipoles \Rightarrow CPU-time! (see later)

Numerical checks

- (A) **LO checked against SHERPA** [Gleisberg/Hoche/Krauss/Schalicke/Schumann/Winter '03]
- (B) **Precision checks for individual NLO components in single PS points**
(typical precision: 10 to 14 digits)

Virtual corrections

- UV, soft and collinear cancellations
- agreement between 2 independent implementations

Real emission

- agreement of 2 \rightarrow 5 matrix elements
- agreement between two dipole implementations
- cancellations in soft and collinear regions

(C) **Integrated NLO cross section**

- qq channel: agreement between 2 dipole implementations and PS slicing
- gg/qg channels: two independent calculations based on dipole subtraction agree at 1-2 sigma level with statistical accuracy at $2 \times 10^{-3} \times \sigma_{\text{NLO}}$ level

(4) NLO results for the LHC

Parton masses

- $m_t = 172.6 \text{ GeV}$
- g, q and b massless \Rightarrow recombination!

k_T -Jet-Algorithm [Run II Jet physics group: Blazey et al. [hep-ex/0005012](#)]

- Select partons with $|\eta| < 5$
- Reconstruct jets with $\sqrt{\Delta\phi^2 + \Delta y^2} > D = 0.8$

Cuts for b-jets

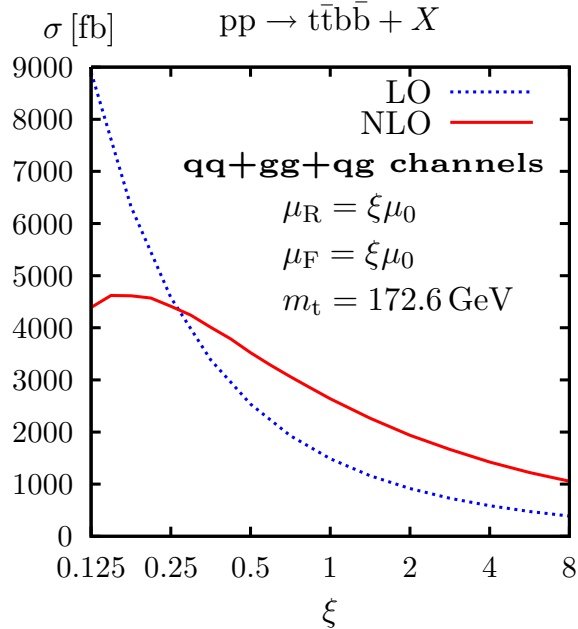
- require two b-jets with $p_{T,j} > 20 \text{ GeV}$ and $y_j < 2.5$
- $b\bar{b}$ invariant mass: $m_{b\bar{b}} > m_{b\bar{b},\text{cut}}$

Strong coupling, PDFs and central scale*

- CTEQ6M with $\alpha_S(M_Z) = 0.118$
- 2-loop running to $\mu = \mu_R = \mu_F$
- central scale $\mu_0 = m_t + m_{b\bar{b},\text{cut}}/2$

* LO obtained with LO α_S , LO PDFs and 1-loop running

LO and NLO scale dependence (qq+gg+qg channels)

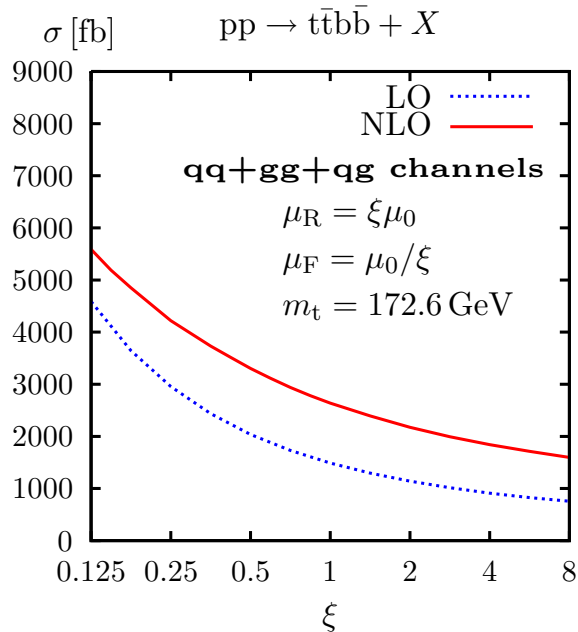


Reduction of scale dependence for σ_{tot}

- rescaling $\mu_{R,F} = m_t$ by common factor $\xi \in [0.5, 2]$
- 70% dependence at LO
- 34% dependence at NLO

Rescaling μ_F by $1/\xi$ (lower plot)

- qualitatively similar behaviour
- dominant dependence from $\alpha_S(\mu_R)^4$



Very large NLO correction

- LO and NLO curves do not cross around $\xi = 1$
- $K = 1.77$ at $\xi = 1$ (central scale too high!)
- completely different wrt $q\bar{q}$ channel ($K = 1.03$)
- bad news: strong $t\bar{t}H$ -background enhancement!

How to reduce large NLO contribution?

Idea

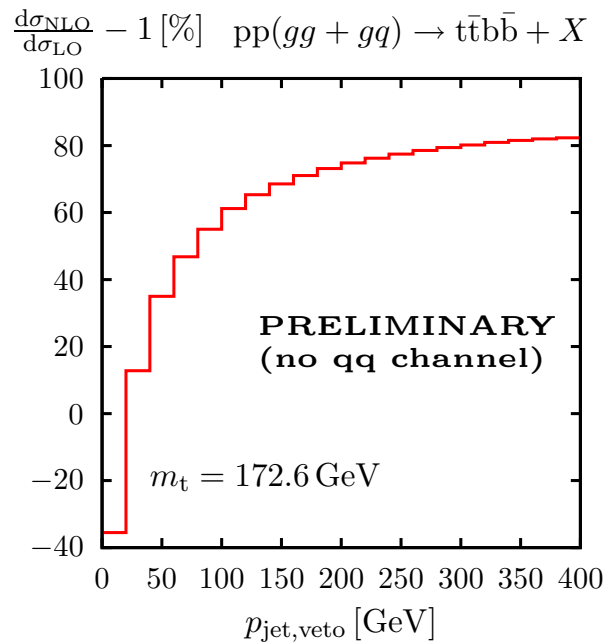
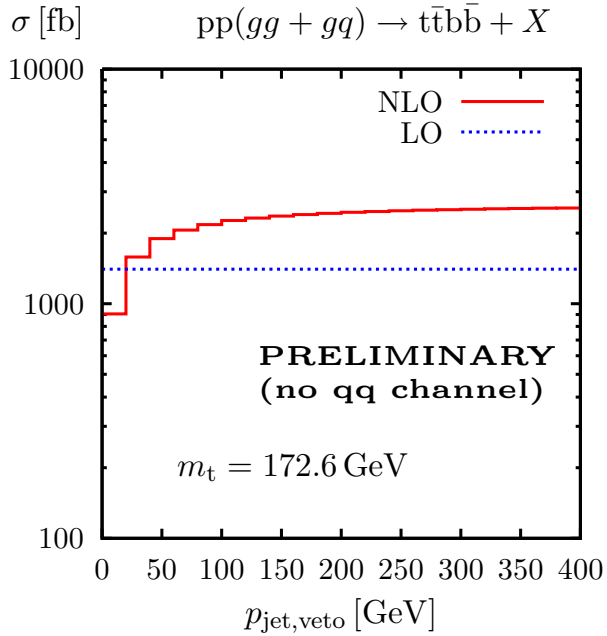
- large K -factor due to hard jet emission
- underestimated by PDF evolution (collinear approximation)

Introduce a veto against jets with $p_T^{\text{jet}} \geq p_T^{\text{veto}}$

- realistic choice $p_T^{\text{veto}} \sim 50 - 100$ GeV
- $\sigma_{\text{NLO}}^{\text{tot}}$ quite sensitive to p_T^{veto} in this region
- can reduce K -factor up to roughly 1.2

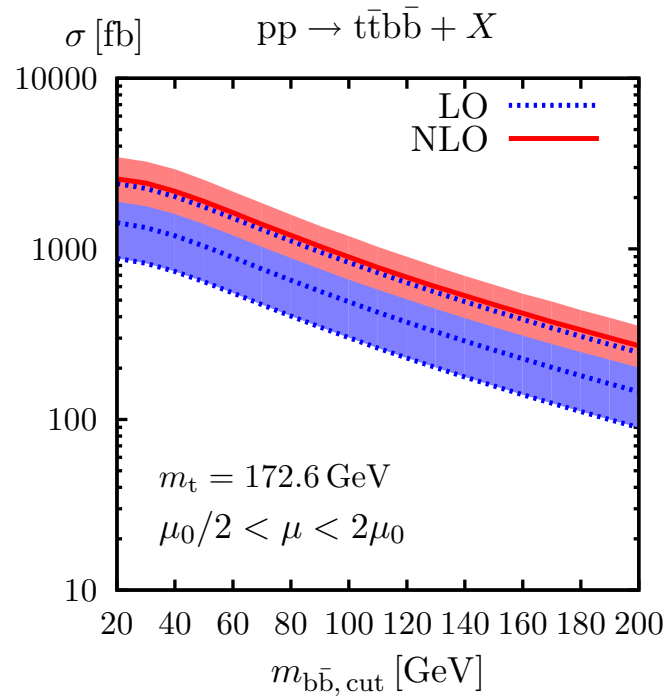
Important issues remain to be studied

- impact of p_T^{veto} on scale dependence?
- resulting systematic uncertainties?
- impact on distributions?



$m_{b\bar{b}}$ -dependence of σ_{LO} and σ_{NLO}

Plotted curve



- $m_{b\bar{b}}$ -distribution integrated over $m_{b\bar{b}} > m_{b\bar{b},\text{cut}}$
- no jet veto
- LO and NLO with uncertainty bands around $\mu_0 = m_t + m_{b\bar{b},\text{cut}}/2$

Similar behaviour as for σ_{tot}

- scale dependence reduced by factor 2
- bands overlap but NLO central value slightly above LO band (large K -factor)

K -factor quite insensitive to $m_{b\bar{b}}$

Statistical precision and speed of the calculation

Single 3GHz Intel Xeon processor & pgf77 Portland compiler

	$\sigma/\sigma_{\text{LO}}$	# events (after cuts)	$(\Delta\sigma)_{\text{stat}}/\sigma$	runtime	time/event
NLOtree (gg)	85%	5.8×10^6	0.4×10^{-3}	2h	1.4ms
virtual (gg)	10%	0.46×10^6	0.7×10^{-3}	20h	160ms
real + dipoles (gg/qg)	87%	16.5×10^6	2.6×10^{-3}	47h	10ms

- couple of days on single CPU $\Rightarrow \mathcal{O}(10^7)$ events and $\mathcal{O}(10^{-3})$ stat. accuracy
- for same precision $(\Delta\sigma)_{\text{stat}}$ **virtual corrections require less CPU-time than real corrections** (scale-dependent statement!)
- **speed of virtual corrections is remarkably high: 160 ms/event** (including colour and polarization sums!)

Some (process-dependent) remarks about CPU efficiency

- Speed of diagrammatic method **in striking contrast to pessimistic expectations** based on generic arguments (factorial complexity of Feynman diagrams)
- **Is it possible to speed up the virtual corrections beyond 160ms/event?**
 - ⇒ Comparison with unitarity-based algorithms would be very instructive
 - ⇒ Looking at method-independent (and simple) ingredient:

CPU-time for master integrals ~ 10 ms/event

suggests that there is not much room for further dramatic improvement

Conclusions

NLO QCD calculation for $pp \rightarrow t\bar{t}b\bar{b}$ at the LHC

- very important for $t\bar{t}H$ measurement
- $2 \rightarrow 4$ reaction with highest priority in the 2005 Les Houches wish list

Phenomenological result

- strong enhancement of $t\bar{t}H$ -background ($K \simeq 1.8$)
- might be reduced with a jet veto (to be studied)

Technical considerations

- First complete $2 \rightarrow 4$ NLO calculation at hadron colliders
- Excellent testing ground for NLO multi-leg methods
- Remarkably high CPU efficiency of diagrammatic tensor-reduction approach