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In[2]:= (* This is how you make a comment in Mathematica *)

In[3]:= (* In mathematica, you type a formula,
         then hit [shift]+[enter] to have mathematica evaluate the expression *)

In[4]:= 2 + 2 (* [shift] + [enter] *)

Out[4]= 4

In[5]:= (* you can enter several expressions, and then evaluate them all at once *)
         (* use [return] to start a new line for each command *)
         a = 3
         b = 7
         c = a + b

Out[5]= 210

In[6]:= (* mathematica will evaluate an
         expression quietly if you put a semicolon at the end *)
         a = 4;
         b = 9;
         c = a + b

Out[8]= 13

In[9]:= (* to unset the value of a variable use "Clear" *)

In[10]:= a

Out[10]= 4

In[11]:= Clear[a]

In[12]:= a

Out[12]= a

In[13]:= b
         c

Out[13]= 9

Out[14]= 13

In[15]:= Clear[b, c]

In[16]:= b
         c

Out[16]= b

Out[17]= c
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In[18]:= (* Built-in functions start with a capital letter. Square
          brackets enclose the arguments passed to a function *)
          Sin[2.0]
          Exp[8.1]

Out[18]= 0.909297

Out[19]= 3294.47

In[20]:= (* Defining a function *)
          f[x_] := Exp[-x^2]
          g[x_, y_] := Sin[Sqrt[x^2 + y^2]]

In[22]:= f[x]
          g[a, b]

Out[22]= e-x2

Out[23]= Sin[ $\sqrt{a^2 + b^2}$ ]

In[24]:= f[2]
          g[3, z]

Out[24]=  $\frac{1}{e^4}$ 

Out[25]= Sin[ $\sqrt{9 + z^2}$ ]

In[26]:= (* Use a decimal point on at least one argument to get a numerical output *)
          f[2.]
          g[3., 2]

Out[26]= 0.0183156

Out[27]= -0.447492

In[28]:= (* Some characters have special meanings *)
          Log[E]
          I^2
          Sin[ $\pi$ ]

Out[28]= 1

Out[29]= -1

Out[30]= 0

In[31]:= (* Mathematica is case-sensitive *)
          Log[e]
          i^2
          Sin[ $\Pi$ ]

Out[31]= Log[e]

Out[32]= i2

Out[33]= Sin[ $\Pi$ ]
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In[34]:= (* For powers use [^] *)
          3^2
          3.^-1
```

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Out[34]= 9
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Out[35]= 0.333333
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In[36]:= (* For multiplication use [*] or a space *)
          2*3
          2 3
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Out[36]= 6
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```
Out[37]= 6
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In[38]:= (* For division use [/] or [ctrl]+[/] *)
          6/2
           $\frac{6}{2}$ 
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Out[38]= 3
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Out[39]= 3
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In[40]:= (* Differentiation *)
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In[41]:= D[f[x]]
          D[f[x], {x, 2}]
          D[g[x, y], {y, 1}]
          D[g[x, y], {x, 1}, {y, 1}]
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Out[41]= e-x2
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Out[42]= -2 e-x2 + 4 e-x2 x2
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Out[43]=  $\frac{y \operatorname{Cos}[\sqrt{x^2 + y^2}]}{\sqrt{x^2 + y^2}}$ 
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Out[44]=  $-\frac{x y \operatorname{Cos}[\sqrt{x^2 + y^2}]}{(x^2 + y^2)^{3/2}} - \frac{x y \operatorname{Sin}[\sqrt{x^2 + y^2}]}{x^2 + y^2}$ 
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In[45]:= (* Integration *)
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In[46]:= Integrate[f[x], x]
          Integrate[f[x], {x, -∞, ∞}]
```

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Out[46]=  $\frac{1}{2} \sqrt{\pi} \operatorname{Erf}[x]$ 
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Out[47]=  $\sqrt{\pi}$ 
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```
In[48]:= (* to make special characters, use [Esc] then [text] then [Esc] *)
          (* [Esc][p][Esc] gives π *)
          (* [Esc][Y][Esc] gives Ξ *)
          (* [Esc][inf][Esc] gives ∞ *)
```

```
In[49]:= (* Define a 2x2 matrix *)
         m = {{1, 2}, {3, 4}}

Out[49]= {{1, 2}, {3, 4}}

In[50]:= MatrixForm[m]

Out[50]//MatrixForm=
      ( 1 2 )
      ( 3 4 )

In[51]:= (* extracting elements *)

In[52]:= m[[1]]

Out[52]= {1, 2}

In[53]:= m[[2]]

Out[53]= {3, 4}

In[54]:= m[[1, 1]]

Out[54]= 1

In[55]:= m[[2, 1]]

Out[55]= 3

In[56]:= (* define a 2-component vector *)
         x = {a, b}

Out[56]= {a, b}

In[57]:= MatrixForm[x]

Out[57]//MatrixForm=
      ( a )
      ( b )

In[58]:= (* MatrixMultiplication *)
         y = m.x;
         MatrixForm[y]

Out[59]//MatrixForm=
      ( a + 2 b )
      ( 3 a + 4 b )

In[60]:= (* Finding eigenvalues and eigenvectors *)
         y = Eigensystem[m]

Out[60]= {{ $\frac{1}{2} (5 + \sqrt{33})$ },  $\frac{1}{2} (5 - \sqrt{33})$ }, {{ $-\frac{4}{3} + \frac{1}{6} (5 + \sqrt{33})$ , 1}, { $-\frac{4}{3} + \frac{1}{6} (5 - \sqrt{33})$ , 1}}}
```

```
In[61]:= (* to extract the eigenvalues *)
 $\omega_1 = y[[1, 1]]$ 
 $\omega_2 = y[[1, 2]]$ 
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$$\text{Out}[61] = \frac{1}{2} (5 + \sqrt{33})$$

$$\text{Out}[62] = \frac{1}{2} (5 - \sqrt{33})$$

```
In[63]:= (* to extract the eigenvectors *)
 $v_1 = y[[2, 1]]$ 
 $v_2 = y[[2, 2]]$ 
```

$$\text{Out}[63] = \left\{ -\frac{4}{3} + \frac{1}{6} (5 + \sqrt{33}), 1 \right\}$$

$$\text{Out}[64] = \left\{ -\frac{4}{3} + \frac{1}{6} (5 - \sqrt{33}), 1 \right\}$$

```
In[65]:= (* normalize these vectors *)
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In[66]:=  $e_1 = \frac{v_1}{\text{Sqrt}[v_1.v_1]}$ 
 $e_2 = \frac{v_2}{\text{Sqrt}[v_2.v_2]}$ 
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$$\text{Out}[66] = \left\{ \frac{-\frac{4}{3} + \frac{1}{6} (5 + \sqrt{33})}{\sqrt{1 + \left(-\frac{4}{3} + \frac{1}{6} (5 + \sqrt{33})\right)^2}}, \frac{1}{\sqrt{1 + \left(-\frac{4}{3} + \frac{1}{6} (5 + \sqrt{33})\right)^2}} \right\}$$

$$\text{Out}[67] = \left\{ \frac{-\frac{4}{3} + \frac{1}{6} (5 - \sqrt{33})}{\sqrt{1 + \left(-\frac{4}{3} + \frac{1}{6} (5 - \sqrt{33})\right)^2}}, \frac{1}{\sqrt{1 + \left(-\frac{4}{3} + \frac{1}{6} (5 - \sqrt{33})\right)^2}} \right\}$$

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In[68]:= (* to evaluate numerically *)
 $N[e_1]$ 
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$$\text{Out}[68] = \{0.415974, 0.909377\}$$

```
In[69]:= (* verify results *)
 $(m - \omega_1 \text{IdentityMatrix}[2]).v_1$ 
```

$$\text{Out}[69] = \left\{ 2 + \left(1 + \frac{1}{2} (-5 - \sqrt{33})\right) \left(-\frac{4}{3} + \frac{1}{6} (5 + \sqrt{33})\right), 4 + \frac{1}{2} (-5 - \sqrt{33}) + 3 \left(-\frac{4}{3} + \frac{1}{6} (5 + \sqrt{33})\right) \right\}$$

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In[70]:= (* we wanted to get zero, let's try asking Mathematica to simplify the expression *)
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In[71]:=  $\text{FullSimplify}[(m - \omega_1 \text{IdentityMatrix}[2]).v_1]$ 
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$$\text{Out}[71] = \{0, 0\}$$

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In[72]:=  $\text{FullSimplify}[(m - \omega_2 \text{IdentityMatrix}[2]).v_2]$ 
```

$$\text{Out}[72] = \{0, 0\}$$

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In[73]:= (* Invert a matrix *)
 $\text{Inverse}[m]$ 
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$$\text{Out}[73] = \left\{ \{-2, 1\}, \left\{ \frac{3}{2}, -\frac{1}{2} \right\} \right\}$$

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In[74]:= m.Inverse[m]
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Out[74]= {{1, 0}, {0, 1}}
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In[75]:= (* to exponentiate a matrix *)
MatrixExp[m]
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Out[75]= {{-1/22 e^{5/2 - sqrt(33)/2} (-11 - sqrt(33) - 11 e^{sqrt(33)} + sqrt(33) e^{sqrt(33)}), 2 e^{5/2 - sqrt(33)/2} (-1 + e^{sqrt(33)})/sqrt(33)},
           {sqrt(3/11) e^{5/2 - sqrt(33)/2} (-1 + e^{sqrt(33)}), 1/22 e^{5/2 - sqrt(33)/2} (11 - sqrt(33) + 11 e^{sqrt(33)} + sqrt(33) e^{sqrt(33)})}}
```

```
In[76]:= (* Exp[m] will just exponentiate each element *)
w = {{a, b}, {c, d}};
Exp[w]
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Out[77]= {{e^a, e^b}, {e^c, e^d}}
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```
In[78]:= (* Raising a matrix to a power, e.g. w^2 *)
MatrixPower[w, 2]
MatrixPower[m, 2.]
m.m
```

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Out[78]= {{a^2 + b c, a b + b d}, {a c + c d, b c + d^2}}
```

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Out[79]= {{7., 10.}, {15., 22.}}
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```
Out[80]= {{7, 10}, {15, 22}}
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In[81]:= (* w^2 just squares each element *)
w^2
w.w
```

```
Out[81]= {{a^2, b^2}, {c^2, d^2}}
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```
Out[82]= {{a^2 + b c, a b + b d}, {a c + c d, b c + d^2}}
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In[83]:=
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In[84]:=
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In[85]:=
```

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In[86]:=
```

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In[87]:=
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