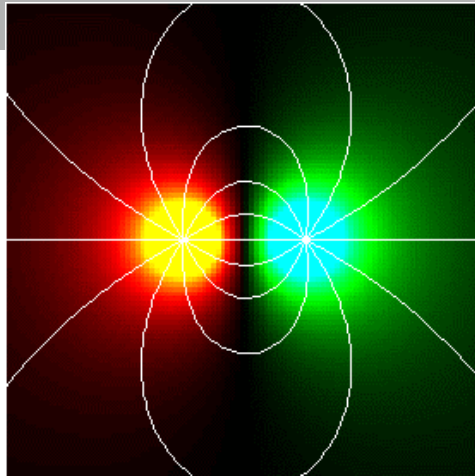


Electric forces & fields



PHY232 – Spring 2007

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<http://www.pa.msu.edu/~pumplin/PHY232>

(Ppt courtesy of Remco Zegers)

**MICHIGAN STATE
UNIVERSITY**

Electric Charges

Two types of charge in atom:

positive (carrier: **proton**)

negative (carrier: **electron**)

Nucleus consists of:

Protons (positive)

neutrons (neutral)

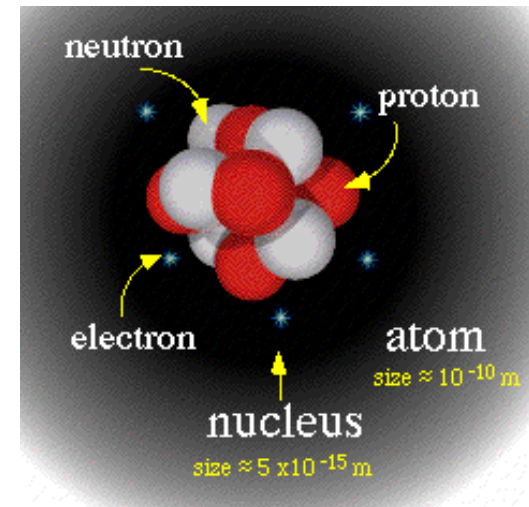
Nucleus is surrounded by cloud of **electrons (negative)**

If the atom is **not ionized, it is neutral.**

By removing electrons, it becomes ionized and positively charged, since there are more protons than electrons

Mass of the electron is much smaller than that of the proton or neutron

$$m_e = 9.109 \times 10^{-31} \text{ kg} \quad m_p = 1.6726 \times 10^{-27} \text{ kg}$$



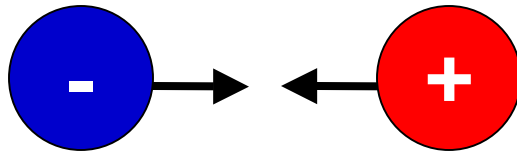
Question

A neutral atom has

- a) more neutrons than protons
- b) more protons than electrons
- c) the same number of neutrons and protons
- d) the same number of protons and electrons
- e) the same number of neutrons, electrons and protons

Electric Forces

Unlike charges attract each other.
(That's what keeps the electrons attached to their atom!)

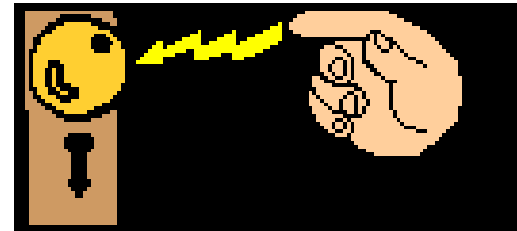


Like charges repel each other.
(A different “strong” force keeps the protons attached to their nucleus!)



Conservation of charge

In a closed system, **charge is conserved**. This means that charge is not 'created' but rather transferred from one object to another.

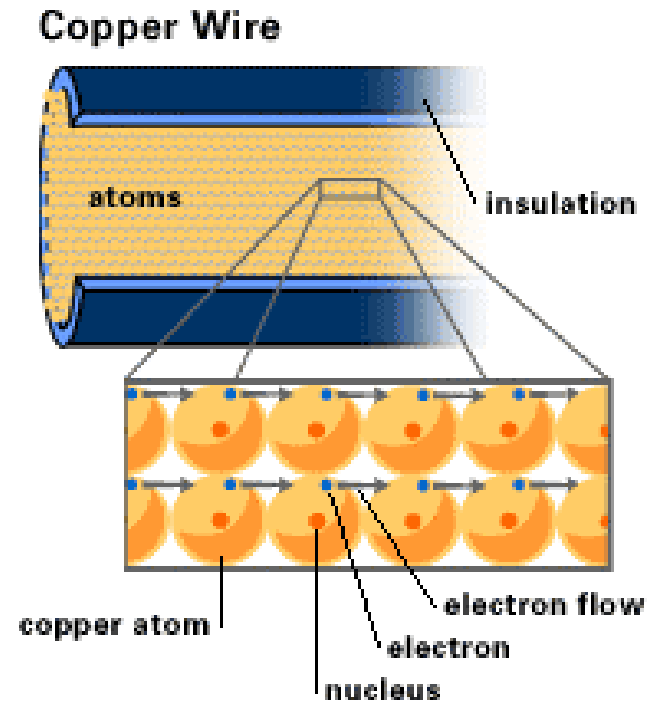
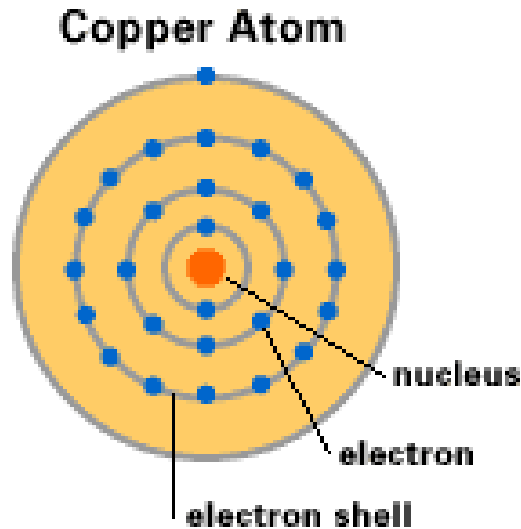


Charge is quantized; there are only discrete amounts of charge. The **electron carries one unit of negative charge (-e)** and the **proton carries one unit of positive charge (+e)**. $1e = 1.602 \times 10^{-19} \text{ C}$ (Coulomb)

conductors

In *conductors* (i.e., conducting materials) electric charge can move freely. The *resistance* to the flow of charge is very small.

Example: metals like Copper; one of the electrons from each atom can move freely.



Conductors, Insulators & Semiconductors

In *conductors*, charge can move freely - The *resistance* to flow of charge is very low.

In *insulators*, charge cannot move freely - The *resistance* to flow of charge is very high.

Semiconductors are materials whose properties are in between that of conductors and insulators (used in transistors).

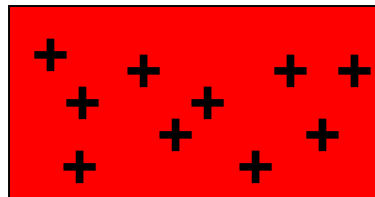
What makes a material a conductor or insulator or semiconductor?

It depends on the shell structure of the atoms involved.

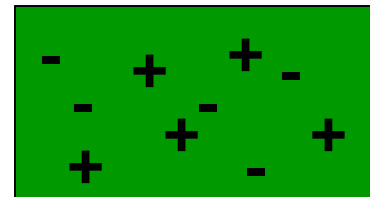
We will discuss this later in the course.

charging by conduction

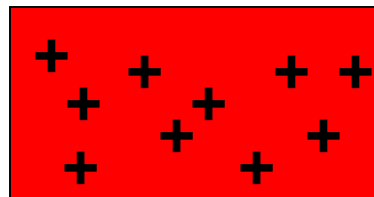
An object can be charged by conduction:



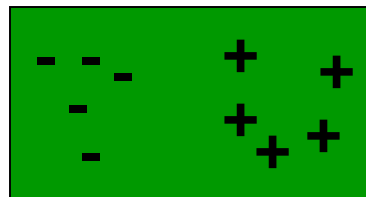
+ charged



neutral

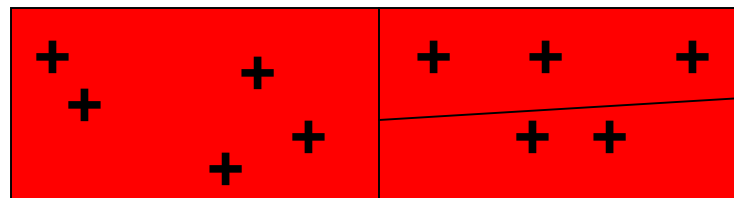


+ charged



neutral

charge is induced but object is still neutral

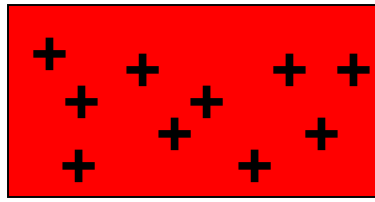


contact

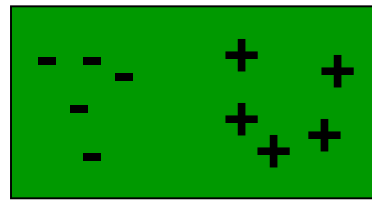
+ charged + charged

charge has moved by conduction

charging by induction

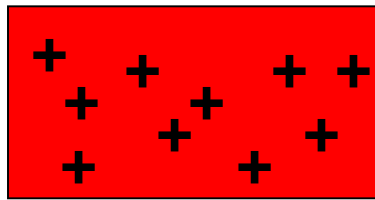


+ charged

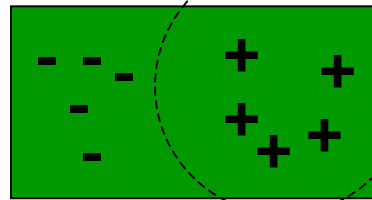


neutral

charge is induced:
object is still neutral but
polarized

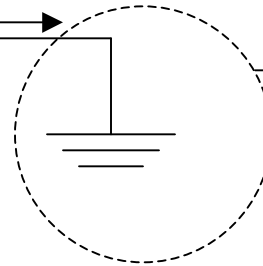


+ charged

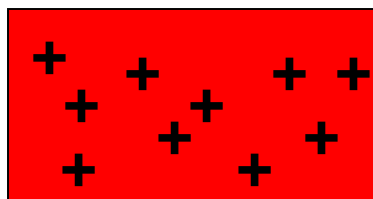


neutral

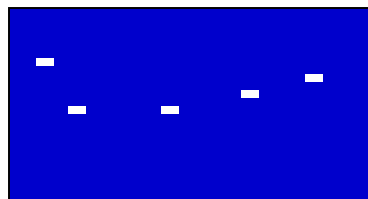
excess charge can escape



connected
to earth



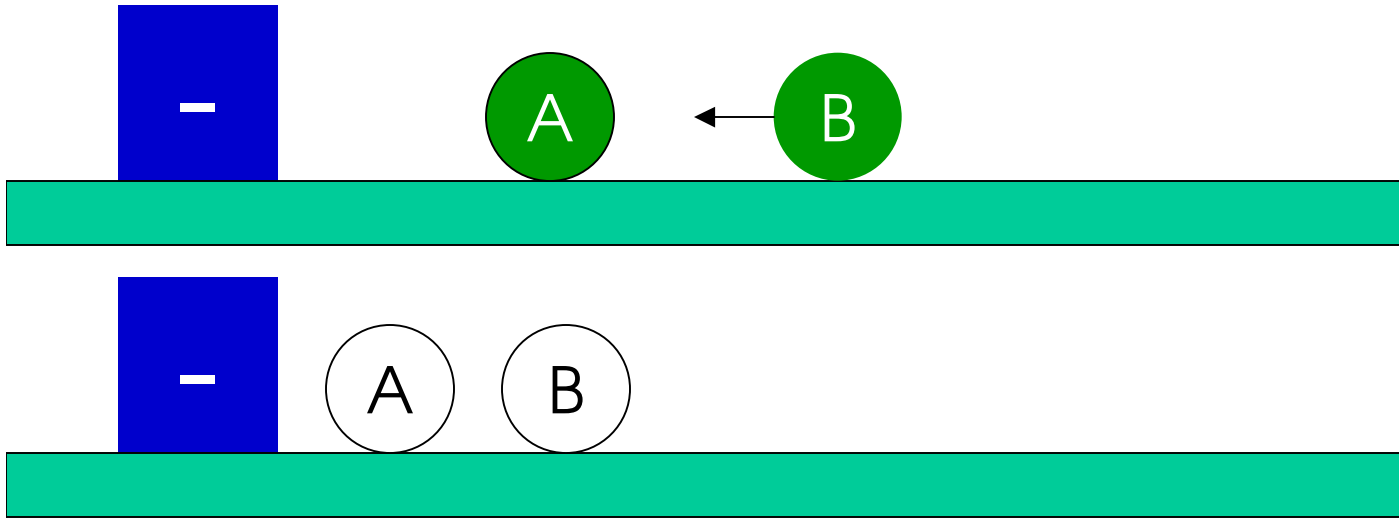
+ charged



- charged

The earth is an “infinite” sink/source of electrons

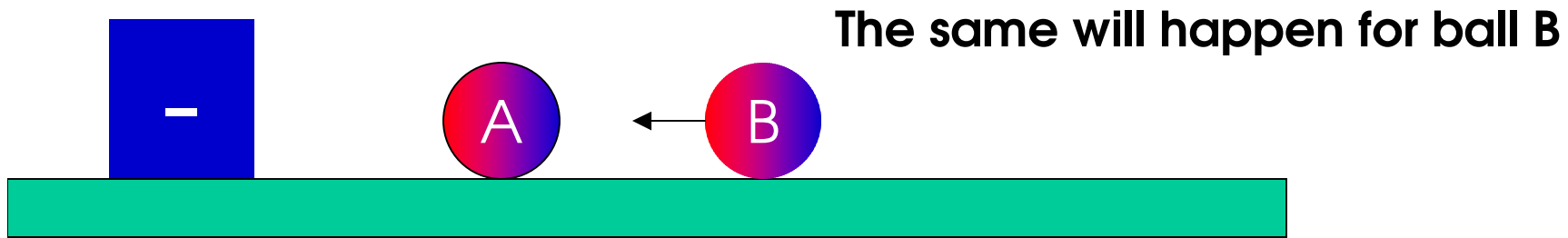
question



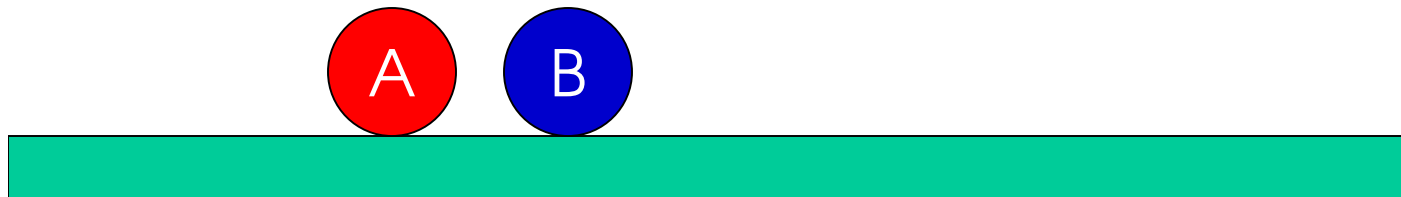
a large negatively charged block is placed on an insulated table. A neutral metal ball (A) is rolled towards it and stops before it hits the block. Then, a second neutral metal ball (B) is rolled towards ball (A). After the collision, ball A stops closer to the block (but without touching) and ball B stops further away from the block. The block is then removed. What is the final charge on balls A and B?

- a) Ball A is positive, ball B is negative
- b) Ball A is negative, ball B is positive
- c) Both ball remain neutral
- d) Both balls are positive

answer



Correct answer: A



Coulomb's law - 1785

$$F = k_e \frac{|q_1||q_2|}{r^2}$$

Coulomb's law:

directed along the line joining the two objects

is attractive if the charges have the opposite sign

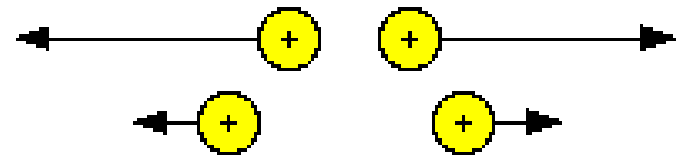
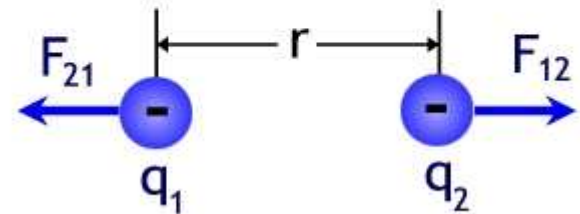
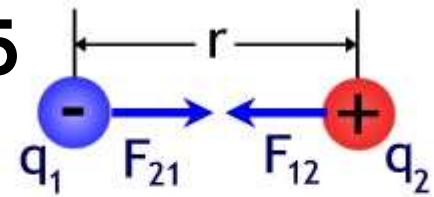
is repulsive if the charge if the same sign

k_e : Coulomb

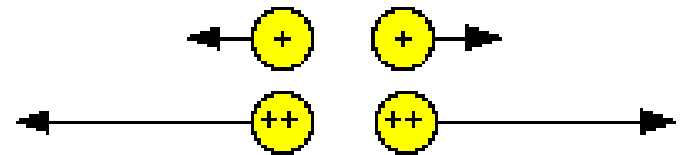
constant= $8.9875 \times 10^9 \text{ Nm}^2/\text{C}^2$

$\epsilon_0 = 1/(4 \pi k_e) = 8.85 \times 10^{-12} \text{ C}^2/(\text{Nm}^2)$

to be used later...



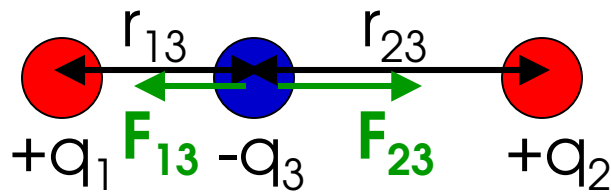
double the distance, force drops to 1/4



double the charge, force increases by factor of 4

Superposition Principle

When more than one charge acts on the charge of interest, each exerts an electric force. Each can be computed separately **and then added as vectors**



$$F_{13} = k_e \frac{|q_1||q_3|}{r_{13}^2} \quad F_{23} = k_e \frac{|q_2||q_3|}{r_{23}^2}$$

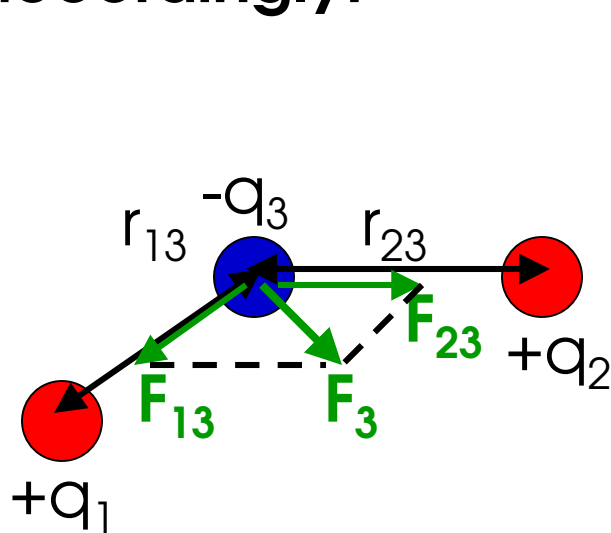
Add: $F_3 = F_{13} - F_{23}$

in this case F_{13} and F_{23} are along the same line and can be added as numbers, but be careful with the sign!

Choose a coordinate system and stick to it!

Superposition Principle II

Remember: forces are vectors, so treat them accordingly!



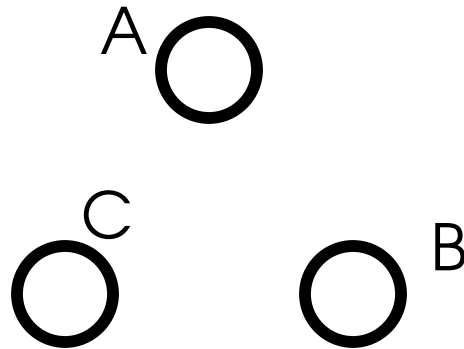
$$F_{13} = k_e \frac{|q_1||q_3|}{r_{13}^2}$$

$$F_{23} = k_e \frac{|q_2||q_3|}{r_{23}^2}$$

$$\vec{F}_3 = \vec{F}_{13} + \vec{F}_{23}$$

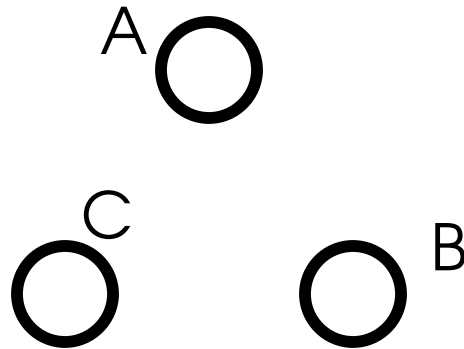
Add: In this case, you need to take into account the horizontal and vertical directions separately and then combine them to get the resultant force.

questions: true false



- a) if A and C are positive, B is pushed away from A and C
- b) if A is positive and B is positive, A and B will move further apart
- c) if A is neutral and C is positive, B will move along the line BC
- d) if A,B and C have the same charge, they will separate further

Answers to questions



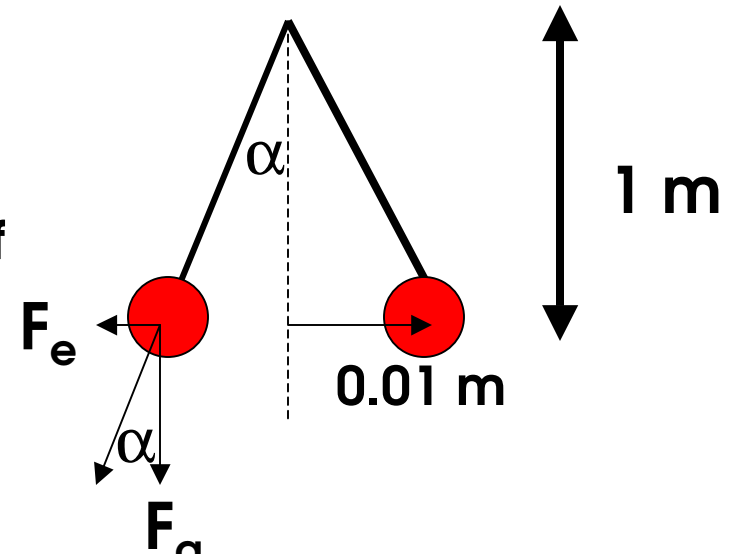
- a) if A and C are positive, B is pushed away from A and C
- b) if A is positive and B is positive, A and B will move further apart
- c) if A is neutral and C is positive, B will move along the line BC
- d) if A,B and C have the same charge, they will separate further

answers:

- a) false, if B is negative it will move towards A and C
- b) false, if C is negative and the absolute charge much larger than A and B, A and B could come closer
- c) false, B might be neutral and not move at all
- d) true, they will all feel an outward pointing force

A simple Electroscope

Two equal masses are charged positively (both $+1 \mu\text{C}$) and hung from massless ropes. They separate as shown in the figure. What is the mass of each?



$$\tan\alpha = 0.01/1 = F_e/F_g$$

$F_e = k_e q_1 q_2 / r_{12}^2$ (coulomb force) $F_g = mg = 9.81m$ (gravitational force)
with $q_1 = q_2 = q$ and $r_{12} = 2 * 0.01 = 0.02 \text{ m}$, $k = 8.99 \times 10^9 \text{ Nm}^2/\text{C}^2$

so: $m = F_e / (0.01g) = k_e q^2 / (0.01g r_{12}^2)$
 $m = 229 \text{ kg} !!$

The electric force is very strong compared to the grav. force! Compare:

$$F_g = Gm_1 m_2 / r_{12}^2 \text{ with } G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$$

$$F_e = k_e q_1 q_2 / r_{12}^2 \text{ with } k_e = 8.99 \times 10^9 \text{ Nm}^2/\text{C}^2$$

Electric Fields

Instead of a force acting on an object **A** by an object **B** magically over the distance between them, one can consider that **object A is situated in a field arising from the presence of object B.**

Because object **A** is in the **field** created by object **B**, it feels a **force**.

The electric field produced by a charge Q at the location of a small test charge q_0 is defined as:

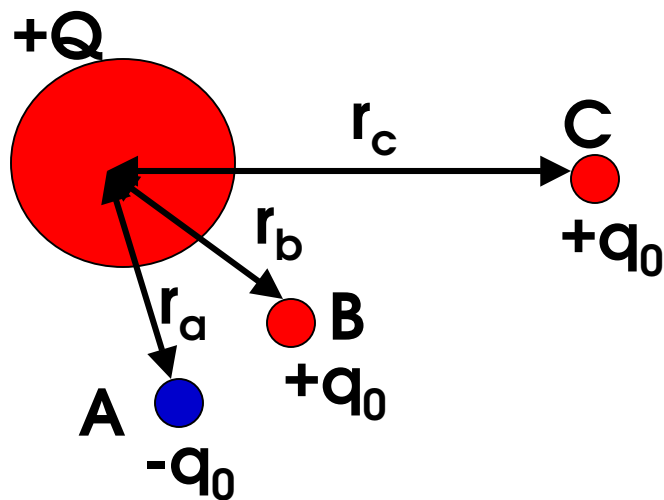
$$\vec{E} \equiv \frac{\vec{F}}{q_0} \quad \Rightarrow \quad \vec{F} = q_0 \vec{E}$$

$$|E| = k_e \frac{Q}{r^2}$$

The magnitude of E only depends on the charge of Q and not the sign and size of the test charge

electric fields II

electric fields and forces due to a charge $+Q$ on test charges of different charge and at different distances



$$r_a = r_b = r$$

$$r_c = 2 \times r$$

test charge	$ E $ direction*	$ F $ direction*
A	$k_e Q/r^2$ +	$k_e Qq_0/r^2$ -
B	$k_e Q/r^2$ +	$k_e Qq_0/r^2$ +
C	$k_e Q/(2r)^2$ +	$k_e Qq_0/(2r)^2$ +

*: '+' means pointing away from Q

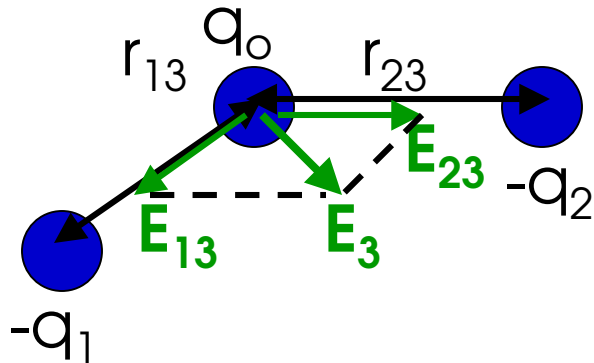
electric fields III

To determine the electric field at a certain point 3, due to the presence of two other charges 1 and 2, use the superposition principle.

$$E_{13} = k_e \frac{|q_1|}{r_{13}^2}$$

$$E_{23} = k_e \frac{|q_2|}{r_{23}^2}$$

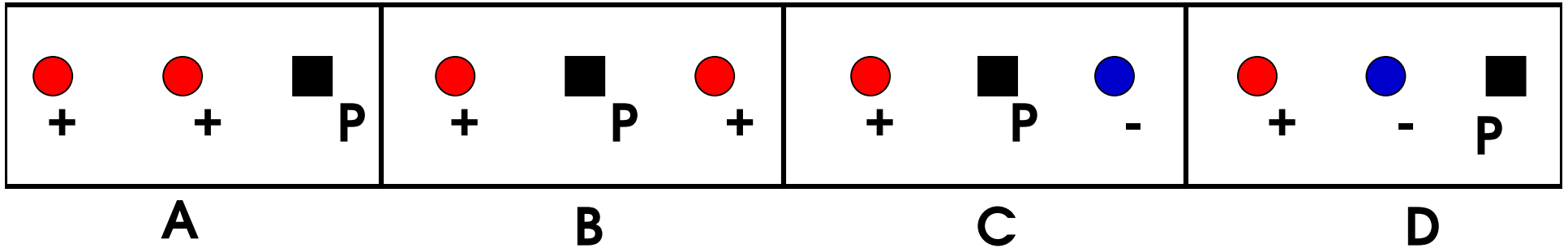
$$\vec{E}_3 = \vec{E}_{13} + \vec{E}_{23}$$



E_3 is independent of the test charge q_0

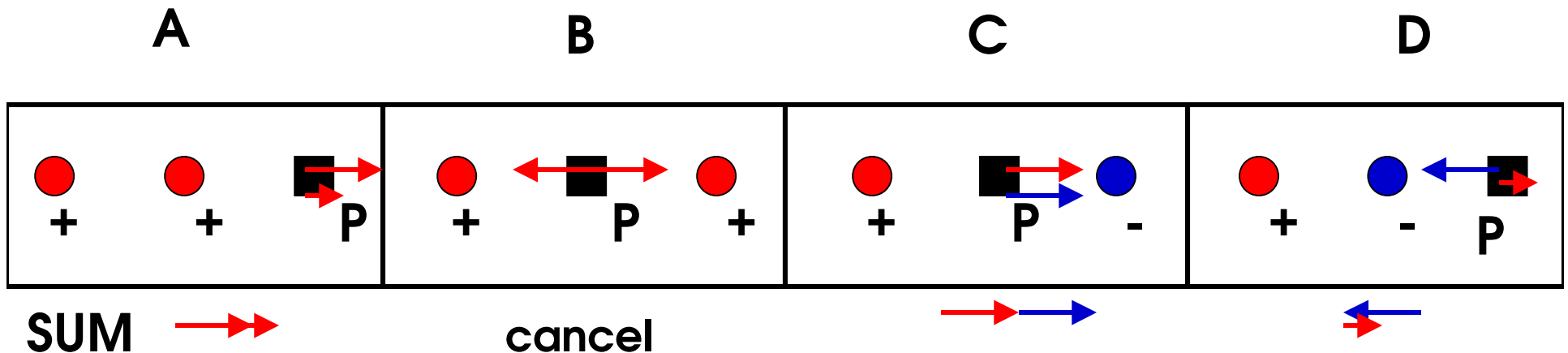
question

2 equal charges are lined up as shown in the figures. A third point (no charge) P is defined as well. In which case is the *magnitude* of the electric field at P largest? The distance between neighboring points is constant.



Answer to question

2 equal charges are lined up as shown in the figures. A third point (no charge) P is defined as well. In which case is the *magnitude* of the electric field at P largest? The distance between neighboring points is constant.



C is correct

electric field lines

To visualize electric fields, one can draw **field lines** that point in the direction of the field at any point following the following rules:

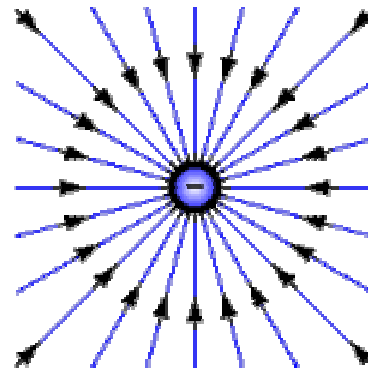
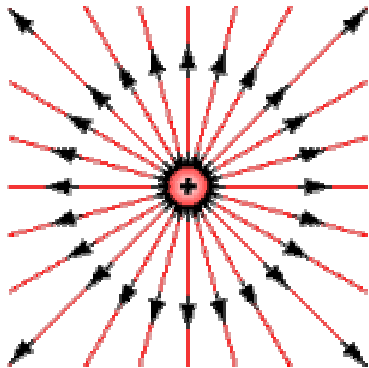
The electric field vector E is tangent to the electrical field lines at each point

The number of lines per unit area through a surface perpendicular to the lines is proportional to the field strength

field lines **start from a positive charge (or infinity)**

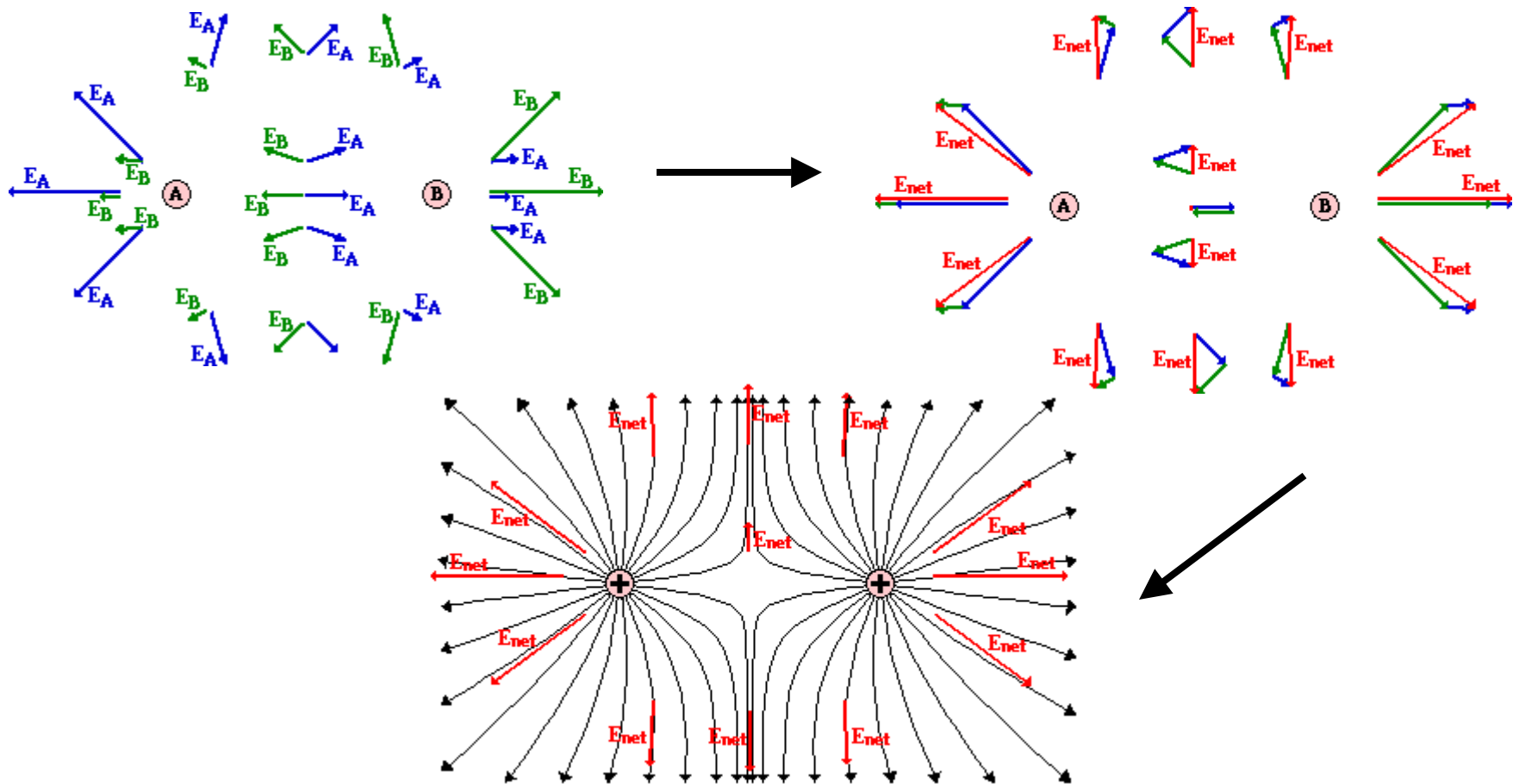
field lines **end at a negative charge (or infinity)**

field lines **never cross** (why not?)



electric field lines II

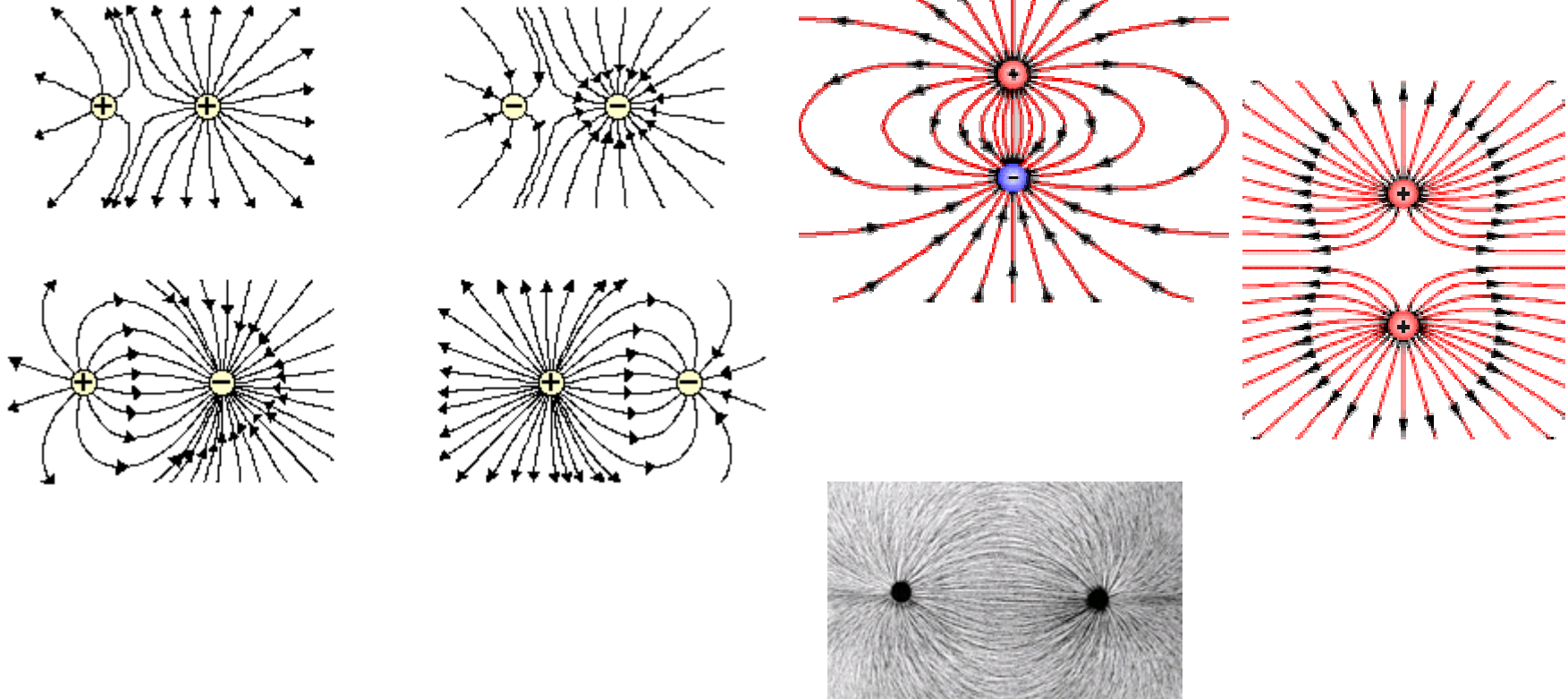
Following these rules one can draw the field lines for any system of charged objects



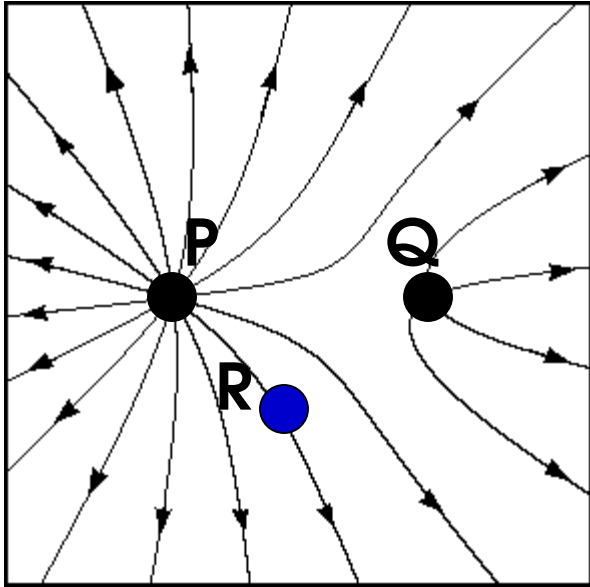
electric field lines II

examples

Electric Field Line Patterns for Objects with Unequal Amounts of Charge

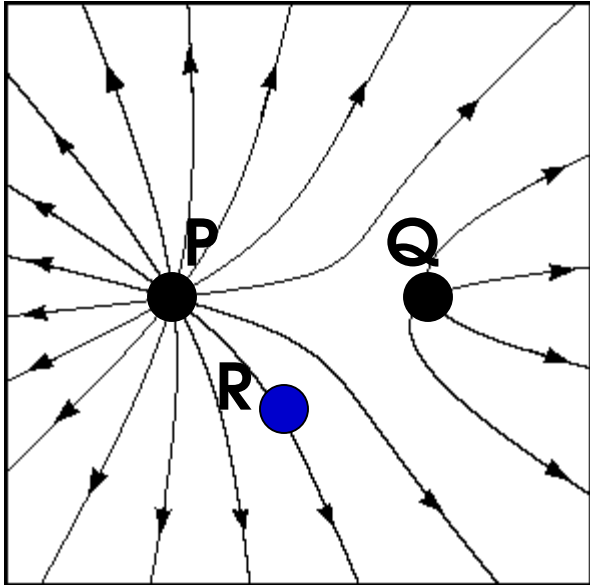


questions



- charge P is (a) positive or (b) negative
- charge Q is (a) positive or (b) negative
- charge P is (a) larger or (b) smaller than charge Q
- a negative charge at R would move
(a) toward P (b) away from P (c) toward Q
(d) none of the above

answers



- charge P is (a) positive
- charge Q is (a) positive
- charge P is (a) larger than charge Q
- a negative charge at R would move (d) toward a point between P and Q

conductors

In the absence of any external charges, an **insulated conductor** is in **equilibrium**, which means:

the **electric field is zero** everywhere **in the conductor**

since net field would result in motion

excess charge resides on the **surface**

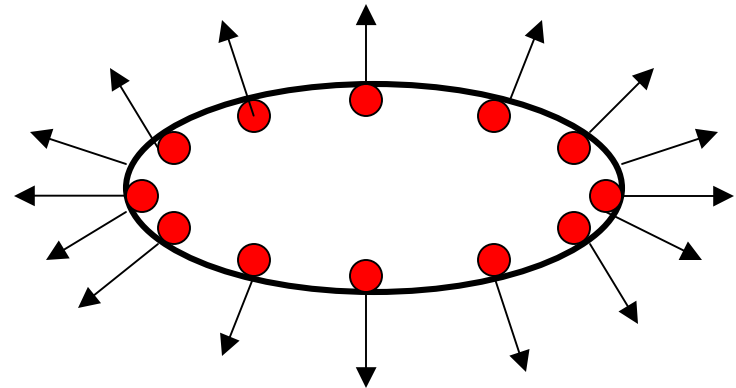
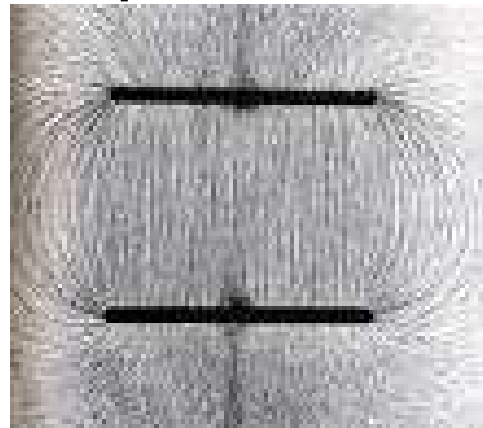
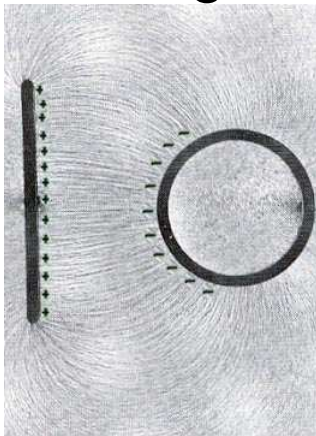
since electric force $\sim 1/r^2$ excess charge is repelled

the **field** just outside the conductor is **perpendicular to the surface**

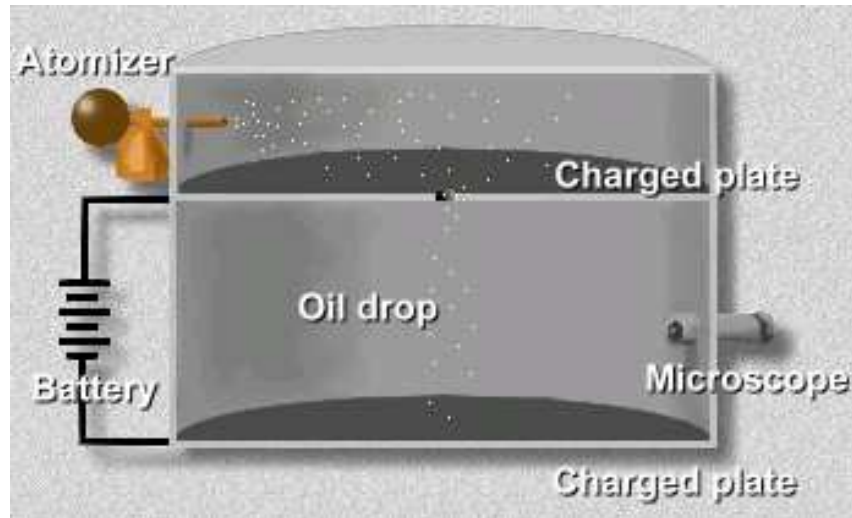
otherwise charge would move over the surface

charge accumulates where the **curvature** of the surface is **smallest**

charges move apart more at flatter surfaces



Millikan oil-drop experiment I



No E-field (battery off):

$$mg = kv$$

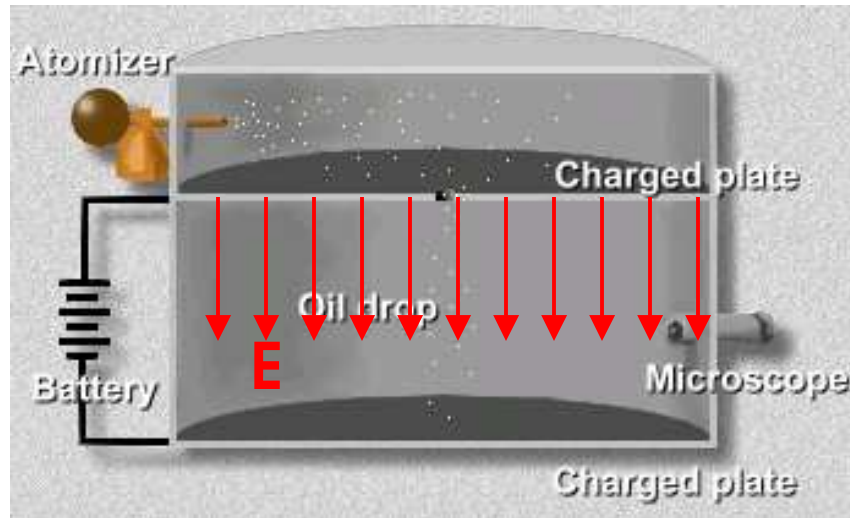
k: drag constant (known)

$$\text{so } m = kv/g$$

- Consider first the case where the battery was switched off.
- Oil droplets will fall and reach a constant velocity v which can be measured.
- At this velocity, the gravitational force balances the frictional (drag) force which equals $k_{\text{drag}} \times \text{velocity}$. From this the mass of the droplet can be determined.

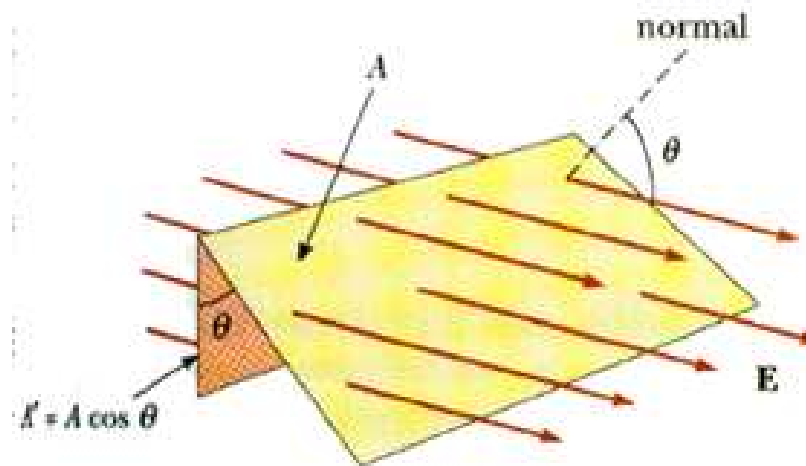
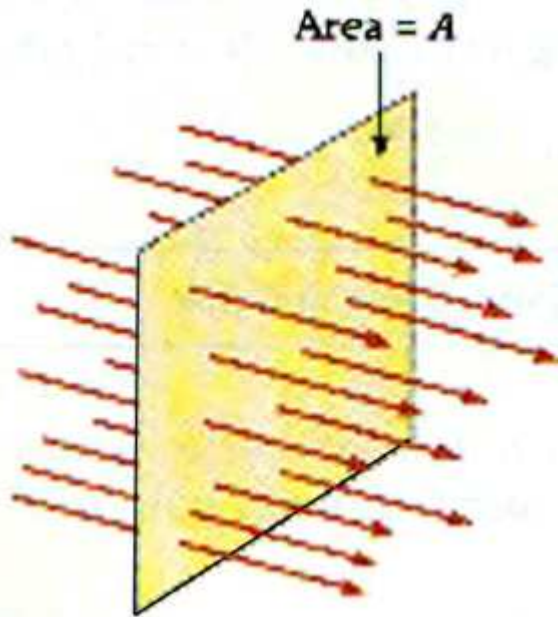
Now the E-field is switched on...

Millikan's oil drop experiment II



The droplets are negatively charged. By tuning E one can suspend them in air. If that happens the electrical force balances the gravitational force and $qE=mg$. Millikan found: $q=mg/E=n \cdot 1.6E-19$ and thus discovered that **charge was quantized**
Nobel prize 1923!

Electric flux



The number of field lines (N) through a surface (A) is proportional to the electric field $N \sim EA$:

The 'flux' $\Phi = EA$ (Nm^2/C)

If the field lines make an angle with the surface:

$\Phi = EA \cos \theta$ where θ is the angle between the field lines and the normal to the surface

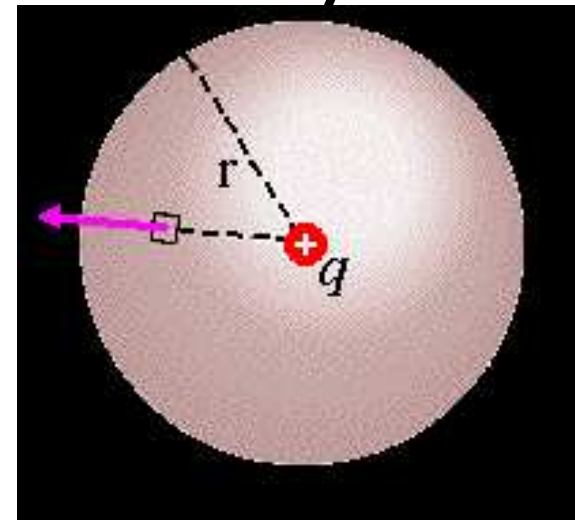
For field lines going through a closed surface (like a sphere), field lines entering the interior are negative and those leaving the interior are positive

Gauss' Law

Consider a point charge q . Imagine a sphere with radius r surrounding the charge. The E-field and flux anywhere on the sphere are:

$$E = k_e \frac{q}{r^2}$$

$$\Phi = EA = k_e \frac{q}{r^2} 4\pi r^2 = 4\pi k_e q = q / \epsilon_0$$



It can be proven that this holds for any closed surface:

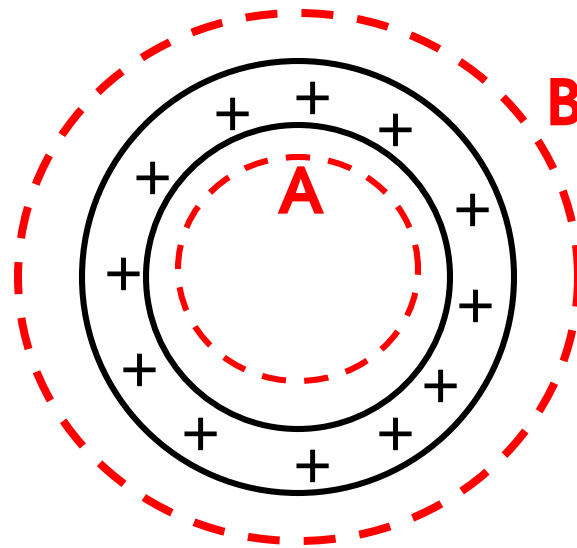
Gauss' Law:

$$\Phi = \frac{Q_{inside}}{\epsilon_0}$$

E-field inside/outside a sphere



Faraday's cage



charge on sphere: Q

consider **imaginary surface A**:

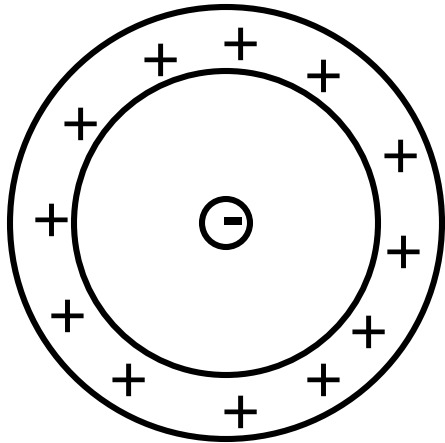
$$\Phi = Q_{\text{inside}} / \epsilon_0 = 0 = EA \text{ so, } E = 0 \text{ (no field inside charged sphere)}$$

consider **imaginary surface B**:

$$\Phi = Q_{\text{inside}} / \epsilon_0 = Q / \epsilon_0 = EA \text{ so,}$$

$$E = Q / (\epsilon_0 A) \text{ (net field outside charged sphere)}$$

question



A point charge $-q$ is located at the center of a spherical shell with radius a and charge $+q$ uniformly distributed over its surface. What is the E-field **a)** anywhere outside the shell and **b)** at a point inside the shell at distance r from the center.

$$\Phi = \frac{Q_{\text{inside}}}{\epsilon_0} = EA \quad \text{Gauss' Law}$$

- a) draw a Gaussian surface around the sphere and apply Gauss' Law: $E = Q_{\text{inside}} / (\epsilon A) = 0$ since $Q_{\text{inside}} = q - q = 0$
- b) draw a Gaussian surface around the $-q$ charge, but inside the shell at distance r from $-q$ and apply Gauss' Law:
 $E = Q_{\text{inside}} / (\epsilon A) = -q / (\epsilon_0 4\pi r^2) = -k_e q / r^2$ I.e. field points inward

question

A neutral object A is placed at a distance $r=0.01$ m away from a charge B of $+1\mu\text{C}$.

- What is the electric field at point A?
- What is the electric force on object A?
- What is the flux through the sphere around object B that has a radius $r=0.01$?
- A is replaced by a charge (object C) of $-1\mu\text{C}$. What is the force on C?
- What is the force on B?

- $E=kq_B/r_{AB}^2=8.99\times 10^9 \times 1\times 10^{-6} / 0.01^2=89.9\times 10^6 \text{ N/C}$
- $F=Eq_A=0$
- $\Phi=EA=89.9\times 10^6 \times (4\pi 0.01^2)=113\times 10^6 \text{ Nm}^2/\text{C}$
- $F=Eq_A=89.9\times 10^6 \times -1\times 10^{-6}=89.9 \text{ N (towards B)}$
- Same, but pointed towards A