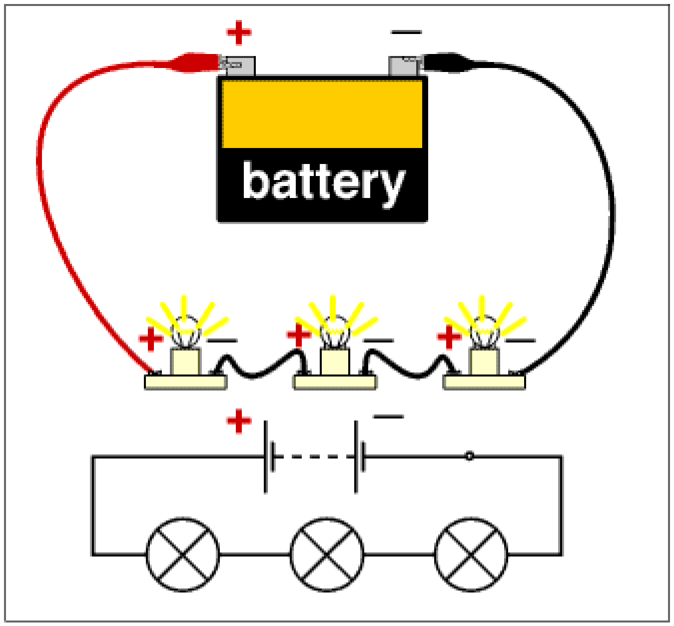
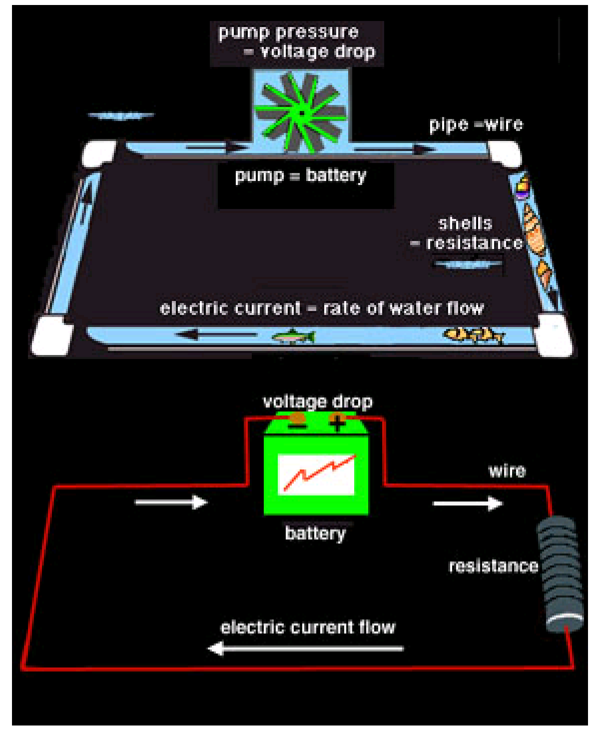
Direct-current Circuits





PHY232 – Spring 2007
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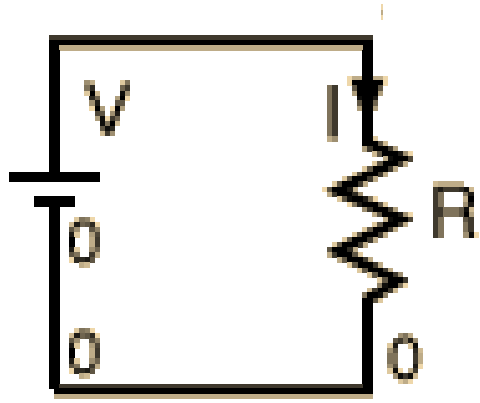
http://www.pa.msu.edu/~pumplin/PHY232
(Ppt courtesy of Remco Zegers)

MICHIGAN STATE UNIVERSITY

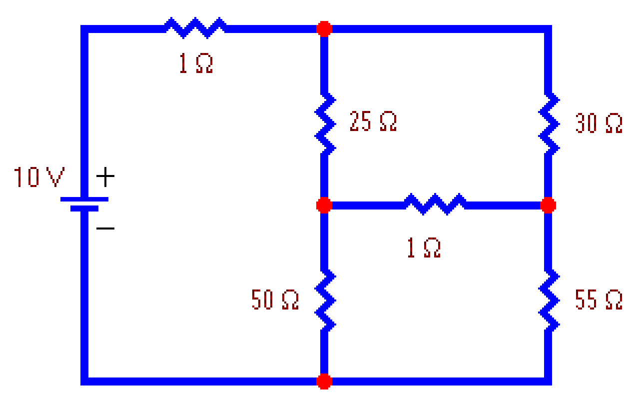
So far, we have looked at systems with only one resistor



for “ohmic” resistor:

$$V = IR \qquad I = \frac{V}{R} \qquad R = \frac{V}{I}$$

Now look at systems with multiple resistors, which are placed in series, parallel or in series and parallel.

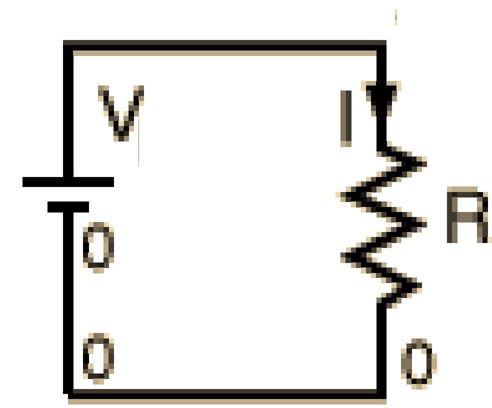


quiz

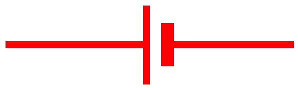
- At V=10V someone measures a current of 1A through the below circuit. When she raises the voltage to 25V, the current becomes 2 A. Is the resistor Ohmic?

  a) YES

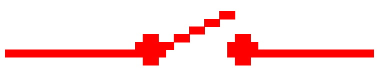
  b) NO



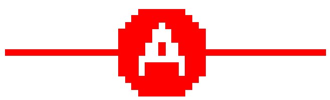
building blocks



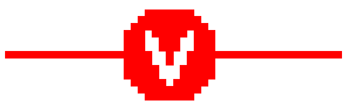
battery or other potential source: Provides emf (electromotive force) to the circuit



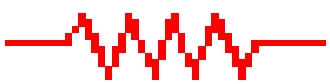
switch: allows current to flow is closed



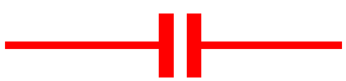
ampere meter: measures current



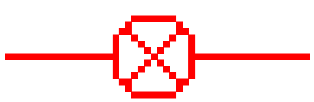
volt meter: measures voltage



resistor



capacitance



lightbulb (I usually show a realistic picture or resistor instead)

light bulb

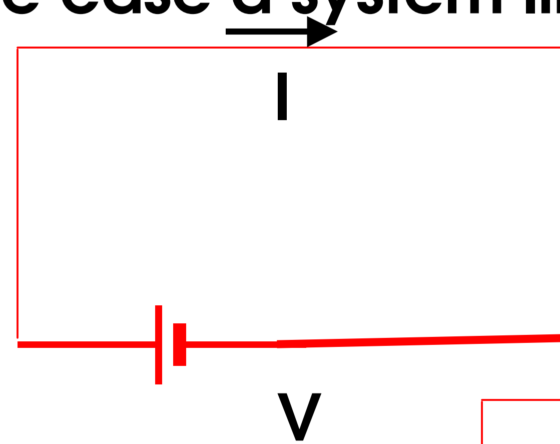
made of tungsten: $\alpha=4.8\times10^{-3}$ 1/K
temperature of filament: ~2800 K
so $R=R_0(1+\alpha(T-T_0))=13R_0$!!!

consequences:
1) A hot lightbulb has a much higher resistance
2) A light bulb usually fails just when switched on because the resistance is small and the current high, and thus the power delivered high ($P=I^2R$)

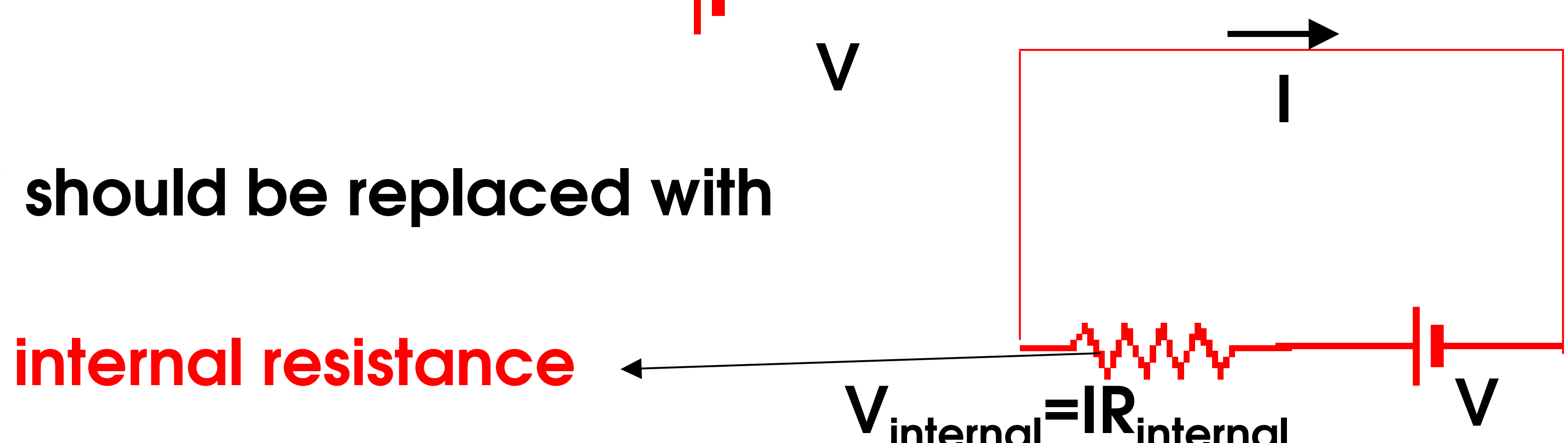
In the demos shown in this lecture, all lightbulbs have the same resistance if at the same temperature, but depending on the current through them, the temperature will be different and thus their resistances

assumptions I

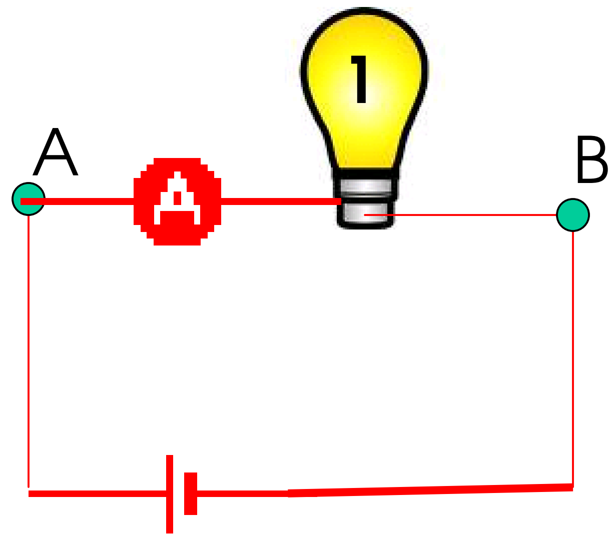
- 1) The internal resistance of a battery or other voltage source is zero. This is not really true (notice that a battery becomes warm after being used for a while)
- if this were not the case a system like this:



- should be replaced with

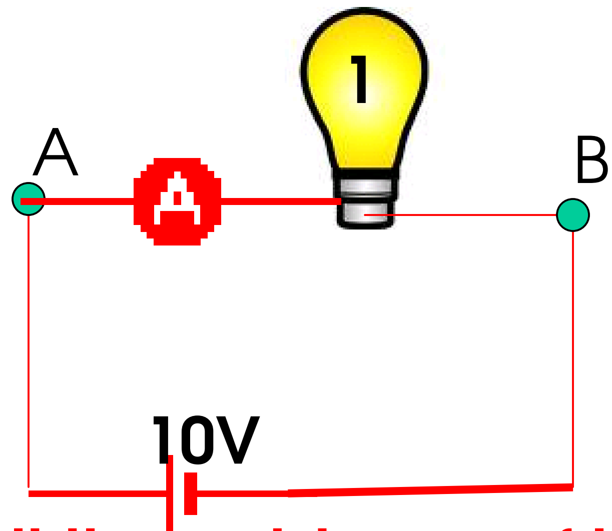


assumptions II



- An ampere meter (current meter) has a negligible internal resistance, so that the voltage drop over the meter $V_A=IR_A$ is negligible as well
- usually, we do not even draw the ampere meter even though we try to find the current through a certain line
- remember that an ampere meter must be placed in series with the device we want to measure the current through

question

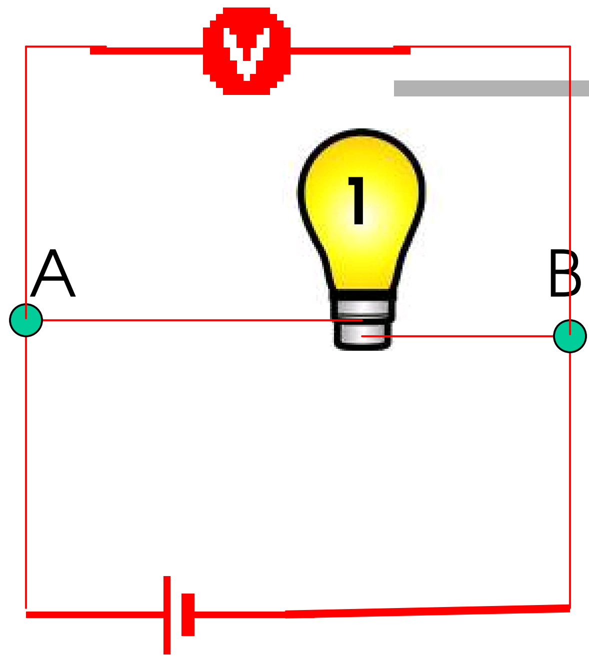


If in the above circuit the resistance of the Ampere meter is not zero, it will not measure the right current that would be present if the meter were not present.

a) true, the total current will change and thus also the current in the Ampere meter

b) not true, current cannot get stuck in the line and thus the measurement will not be affected

assumptions III



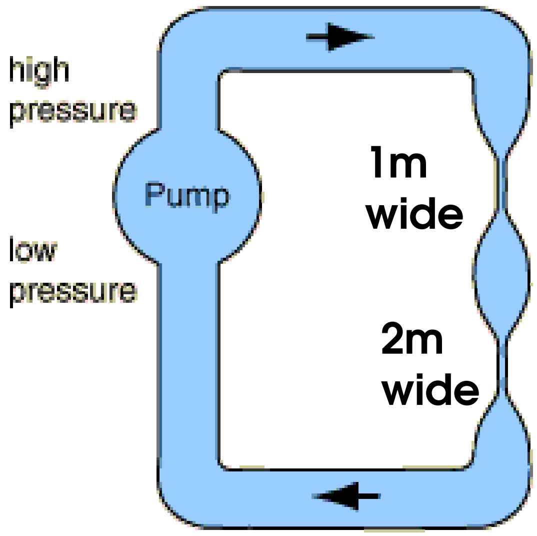
- a volt meter has an infinite internal resistance, so that no current will flow through it.
- usually, we do not even draw the volt meter even though we try the potential over a certain branch in the circuit
- remember that a volt meter must be placed in parallel with the device we want to measure the voltage over

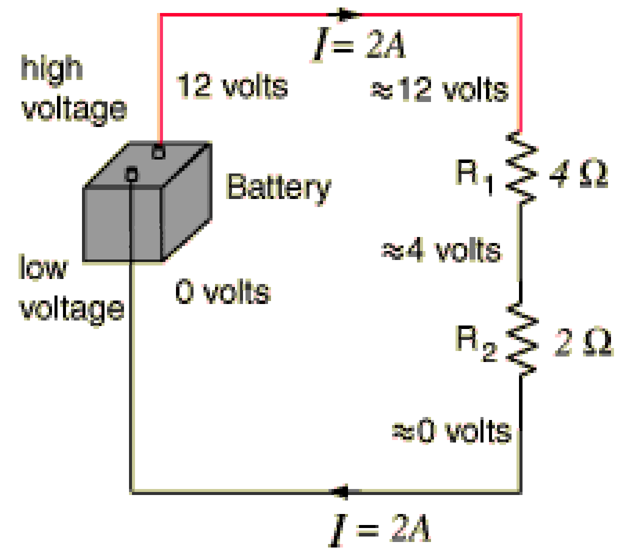
assumptions IV

- We can neglect the resistance of wires that connect the various devices in our circuit. This is true as long as the resistance of the device is much larger than that of the wires

basic building blocks: two resistors in series

Poiseuille:
Flow~ΔPr⁴/l





- The water flow (m^3/s) through the two narrow pipes must be equal (else water gets stuck), so the pressure drop is larger over the narrowest of the two. The total pressure drop is equal to the sum of the two pressure drops over both narrow pipes
- The current (I) through the two resistors must be equal (else electrons would get stuck), so the voltage drop is larger over the highest of the two. The total voltage drop is equal to the sum of the two voltage drops over the resistors.

demo
2 light in series

resistors in series II

The voltage over R_1 and R_2:

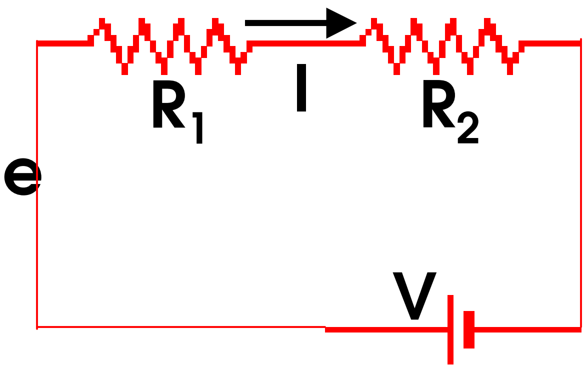
1) $V = IR_1 + IR_2 = I(R_1 + R_2)$

if we want to replace R_1,R_2 with one equivalent R:

2) $V = IR_{eq}$

and by combining 1) and 2)

$R_{eq} = R_1 + R_2$

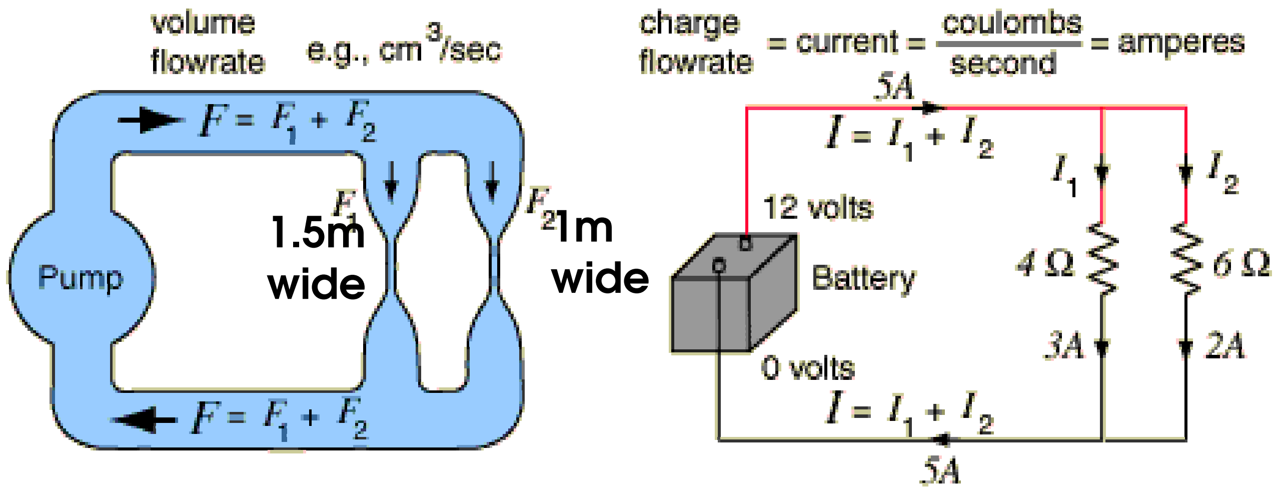


- **For n resistors placed in series in a circuit:**

$$R_{eq} = R_1 + R_2 + \ldots + R_n$$

Note: $R_{eq} > R_i$ I=1,2…n the equivalent R is always larger than each of the separate resistors

second building block: resistors in parallel



- The pressure drop over the two narrow pipes must be equal (before and after the pipes the pressure is the same), but the water prefers to flow through the wider canal, i.e, the flow (m^3/s) is higher through the wider canal.
- The voltage drop over the two resistors must be equal (before and after the resistors the voltage is the same), but the electrons prefer to go through the smaller resistor, i.e, the current (A) is higher through the smaller resistor.

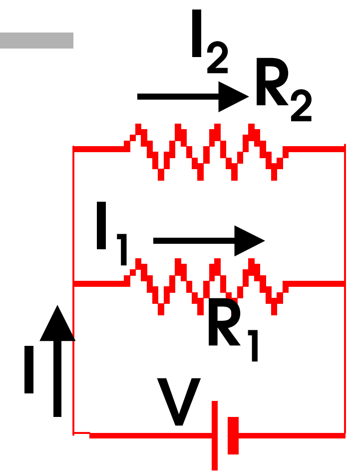
demo

2 light in parallel

resistors in parallel II

For the current through the circuit:

1) $$I = I_1 + I_2 = \frac{V}{R_1} + \frac{V}{R_2}$$



if we want to replace R_1,R_2 with one equivalent R:

2) $$I = \frac{V}{R_{eq}}$$

and by combining 1) and 2): $$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

Ø For n resistors placed in parallel in a circuit:

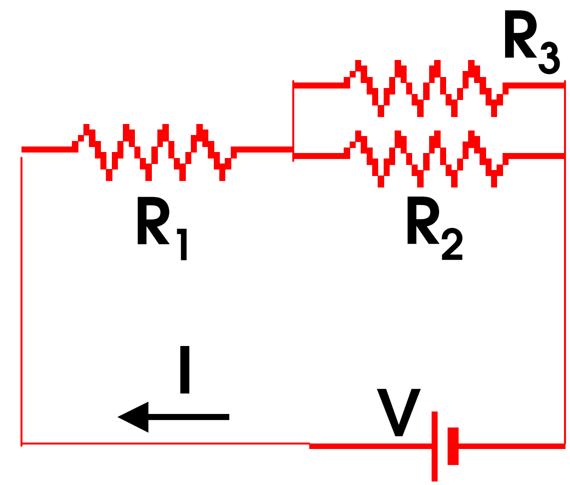
$$1/R_{eq} = 1/R_1+1/R_2+\ldots+1/R_n$$

Note: $R_{eq}<R_i$ with I=1,2…n R_{eq} is always smaller than each of the separate resistors

question

- what is the equivalent resistance of all resistors as placed in the below circuit? If V=12V, what is the current I?

R_1=3 Ohm
R_2=3 Ohm
R_3=3 Ohm
V=12V



R_2 & R_3 are in parallel

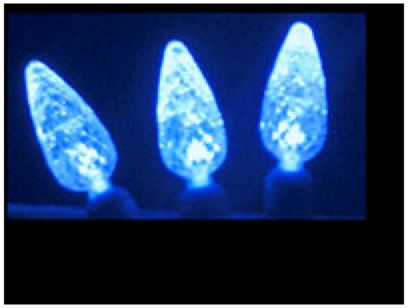
$1/R_{23}=1/R_2+1/R_3=1/3+1/3=2/3$

$R_{23}=3/2$ Ohm

R_1 is in series with R_{23}

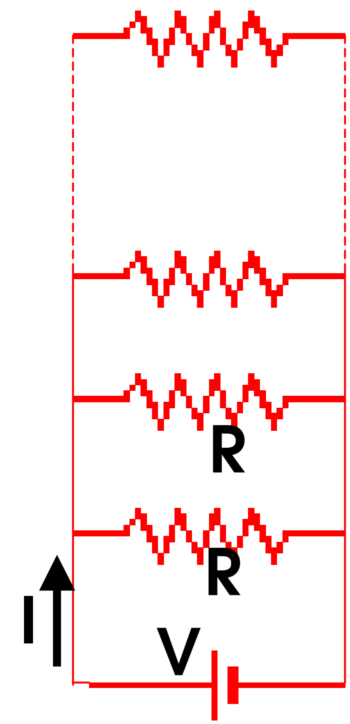
$R_{123}=R_1+R_{23}=3+3/2=9/2$ Ohm

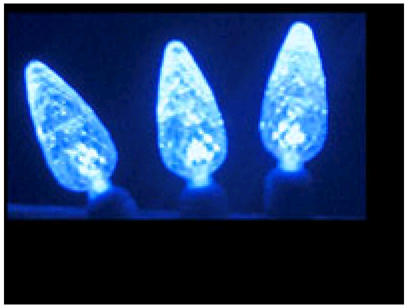
$I=V/R=12/(9/2)=24/9=8/3$ A



question: Christmas tree lights

- A tree is decorated with a string of many equal lights placed in parallel. If one burns out (no current flow through it), what happens to the others?
  - a) They all stop shining
  - b) the others get a bit dimmer
  - c) the others get a bit brighter
  - d) the brightness of the others remains the same





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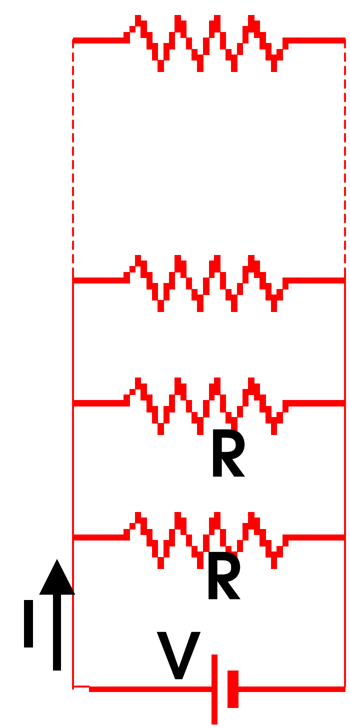


Before the one light fails:
$1/R_{eq}=1/R_1+1/R_2+\ldots+1/R_n$
if there are 3 lights of 1 Ohm: $R_{eq}=1/3$
$I=V/R_{eq}$ $I_j=V/R_j$ (if 3 lights: $I=3V$ $I_j=V/1$
After one fails:
$1/R_{eq}=1/R_1+1/R_2+\ldots.+1/R_{n-1}$
if there are 2 lights left: $R_{eq}=1/2$
$I=V/R_{eq}$ $I_j=V/R_j$ (if 2 lights: $I=2V$ $I_j=V/1$) The total resistance increases, so the current drops. The two effects cancel each other

A different Christmas tree

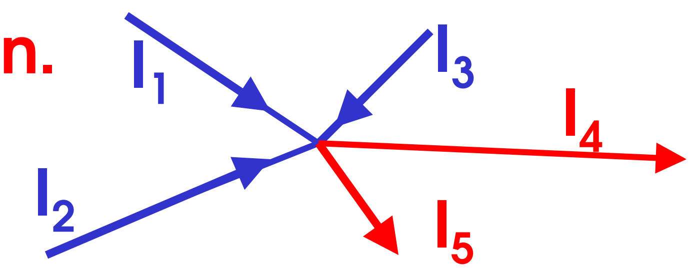
- a person designs a new string of lights which are placed in series. One fails, what happens to the others?
  - a) They all stop shining
  - b) the others get a bit dimmer
  - c) the others get a bit brighter
  - d) the brightness of the others remains the same

Assume: If one fails, the wire inside it is broken and Current cannot flow through it any more.

Kirchhoff's rules

- To solve complex circuits, we can use the following rules:

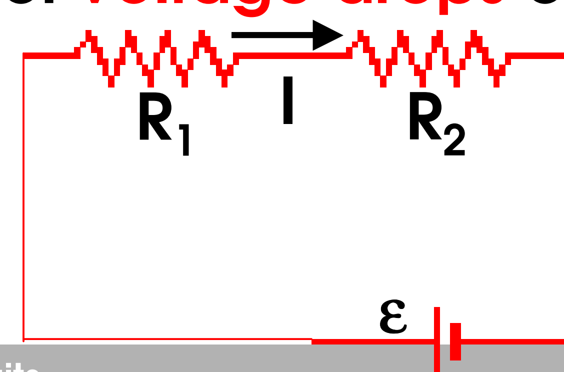
- **Kirchhof 1:** The sum of the currents flowing into a junction must be the same the the sum of the current flowing out of the junction.



$I_1+I_2+I_3=I_4+I_5$

- **Kirchhof 2:** The sum of voltage gains over a loop (I.e. due to emfs) must be equal to the sum of voltage drops over the loop.

$\varepsilon_I=IR_1+IR_2$

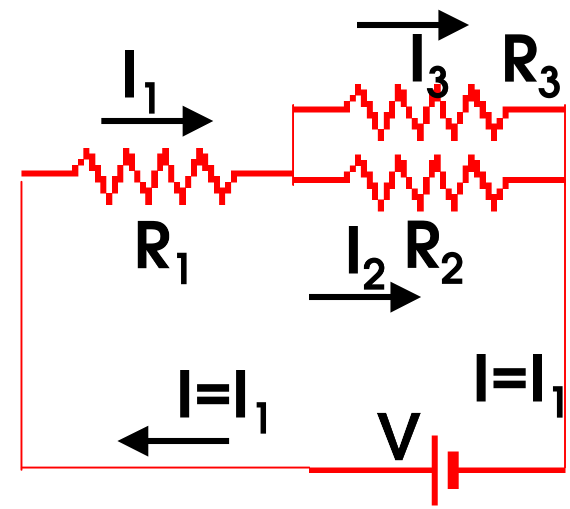


question

- Given V=12V, what is the current through and voltage over each resistor

1) Slide 12: $I_1=8/3$ A
Kirchhof 2
2) $V-I_1R_1-I_2R_2=0$ $12-3I_1-3I_2=0$
Kirchhof 2
3) $V-I_1R_1-I_3R_3=0$ $12-3I_1-3I_3=0$
Kirchhof 2
4) $0-I_3R_3+I_2R_2=0$ $-3I_3+3I_2=0$
Kirchhoff I
5) $I_1-I_2-I_3=0$ $I_1=I_2+I_3$
1) & 2) $12-8-3I_2=0$ so $4=3I_2$ and $I_2=4/3$ A
1) & 3) $12-8-3I_3=0$ so $4=3I_3$ and $I_3=4/3$ A
Use V=IR for R_1 $V_1=8/3*3=8$ V
for R_2 $V_2=4/3*3=4$ V
for R_3 $V_3=4/3*3=4$ V

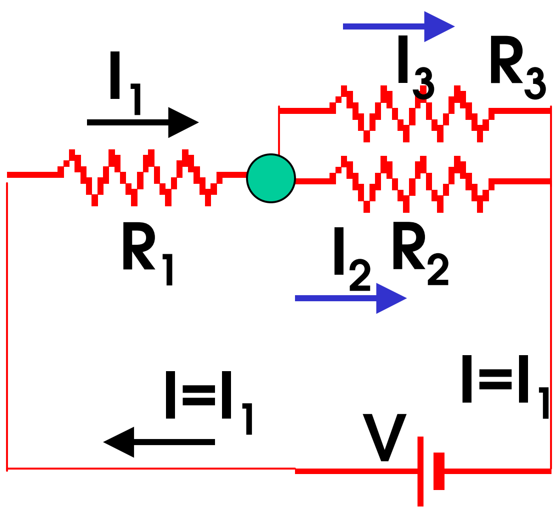
$R_1=3$ Ohm
$R_2=3$ Ohm
$R_3=3$ Ohm
V=12V



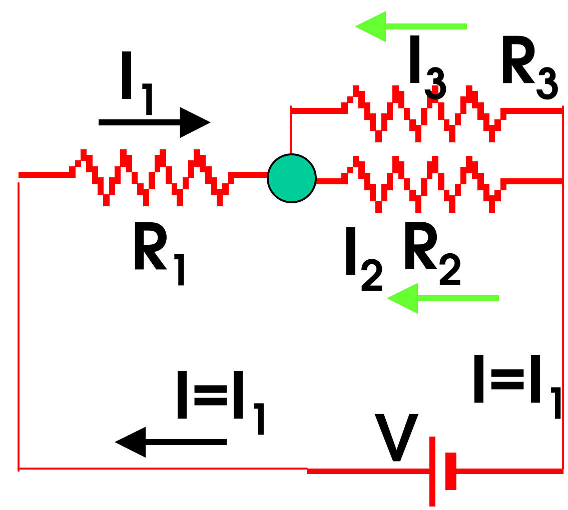
demo
lightbulb
circuit

IMPORTANT

- **When starting a problem we have to assume something about the direction of the currents through each line. It doesn't matter what you choose, as long as you are consistent throughout the problem example:**

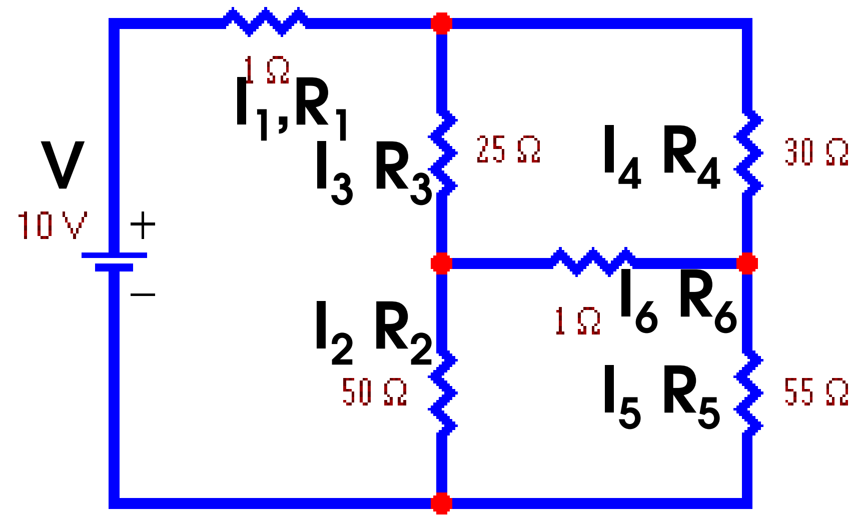


both are okay



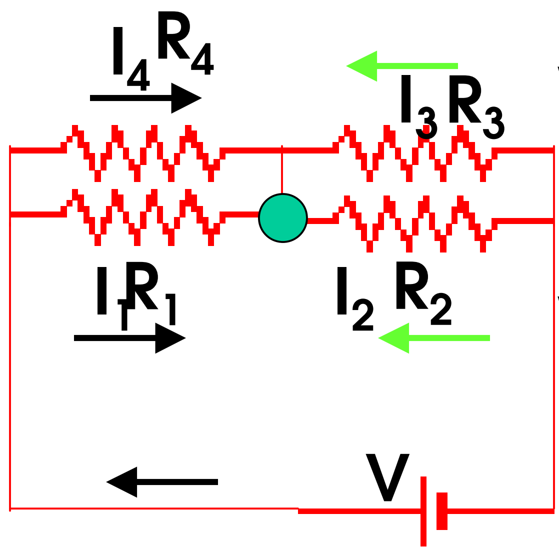
Kirchhoff I: $I_1-I_2-I_3=0$	Kirchhoff I: $I_1+I_2+I_3=0$
Kirchhoff 2: $V-I_1R_1-I_2R_2=0$	Kirchhoff 2: $V-I_1R_1+I_2R_2=0$

question



- which of the following cannot be correct?
- a) $V-I_1R_1-I_3R_3-I_2R_2=0$
- b) $I_1-I_3-I_4=0$
- c) $I_3R_3-I_6R_6-I_4R_4=0$
- d) $I_1R_1-I_3R_3-I_6R_6-I_4R_4=0$ NOT A LOOP
- e) $I_3+I_6+I_2=0$

question



What is Kirchhoff I for ● ?

a) $I_1+I_2-I_3-I_4=0$ b) $I_1+I_2+I_3+I_4=0$ c) $I_1-I_2-I_3-I_4=0$

What is Kirchhoff II for the left small loop (with R_4 and R_1?)

a) $I_4R_4+I_1R_1=0$ b) $I_4R_4-I_1R_1=0$ c) $I_4R_4+I_1R_1-V=0$

What is Kirchhoff II for the right small loop (with R_2 and R_3)?

a) $I_3R_3+I_2R_2=0$ b) $I_3R_3-I_2R_2=0$ c) $I_3R_3-I_2R_2+V=0$

What is Kirchhoff II for the loop (with V,R_4 and R_3)?

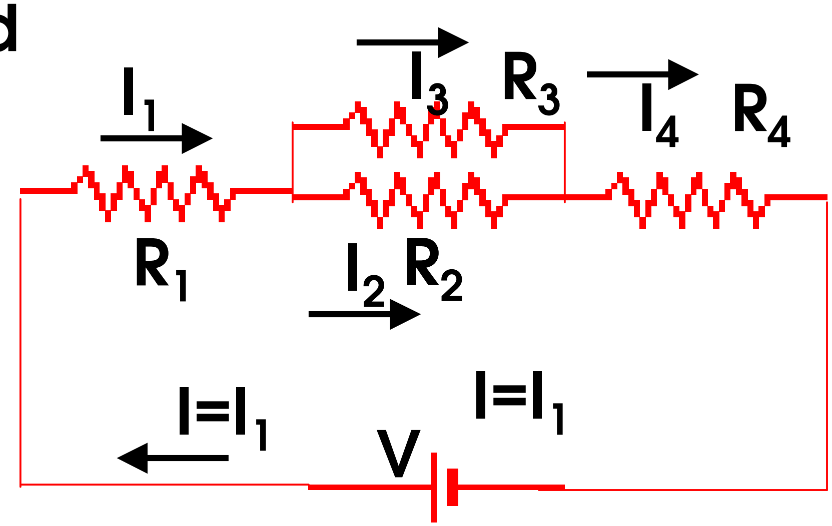
a) $V-I_4R_4+I_3R_3=0$ b) $V+I_4R_4-I_3R_3=0$ c) $V-I_4R_4-I_3R_3=0$

question

- what is the power dissipated by R_3?

$P=VI=V^2/R=I^2R$

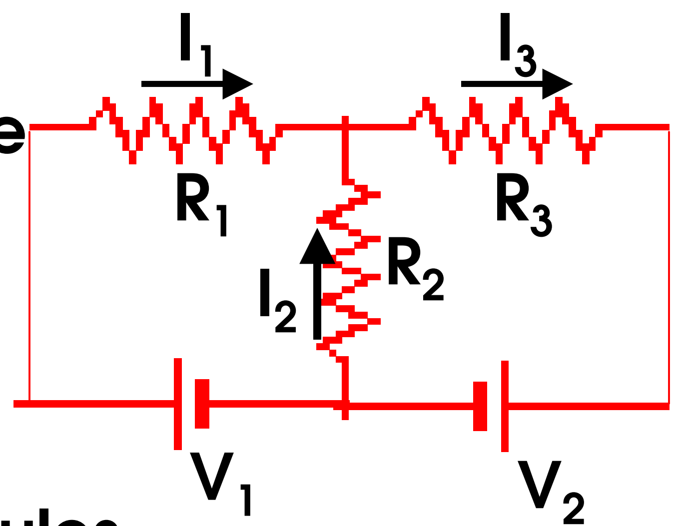
$R_1= 1$ Ohm
$R_2=2$ Ohm
$R_3=3$ Ohm
$R_4=4$ Ohm
$V=5V$



We need to know V_3 and/or I_3.
Find equivalent R of whole circuit.
$1/R_{23}=1/R_2+1/R_3=1/2+1/3=5/6$ $R_{23}=6/5$ Ohm
$R_{1234}=R_1+R_{23}+R_4=1+6/5+4=31/5$ Ohm $I=I_1=I_4=V/R_{1234}=5/(31/5)$
$I=25/31$ A
Kirchhoff 1: $I_1=I_2+I_3=25/31$ Kirchhoff 2: $I_3R_3-I_2R_2=0$ so $3I_3-2I_2=0$ $I_2=3/2\ I_3$
Combine: $3/2\ I_3+I_3=25/31$ so $5/2\ I_3=25/31$ $I_3=10/31$ A
$P=I^2R$ so $P=(10/31)^2*3=(100/961)*3=0.31$ J/s

more than one emf

what is the current through and voltage over each R?



R_1=R_2=R_3=3 Ohm
V_1=V_2=12 V

Ø apply kirchhoff's rules

1) $I_1+I_2-I_3=0$ (kirchhoff I)

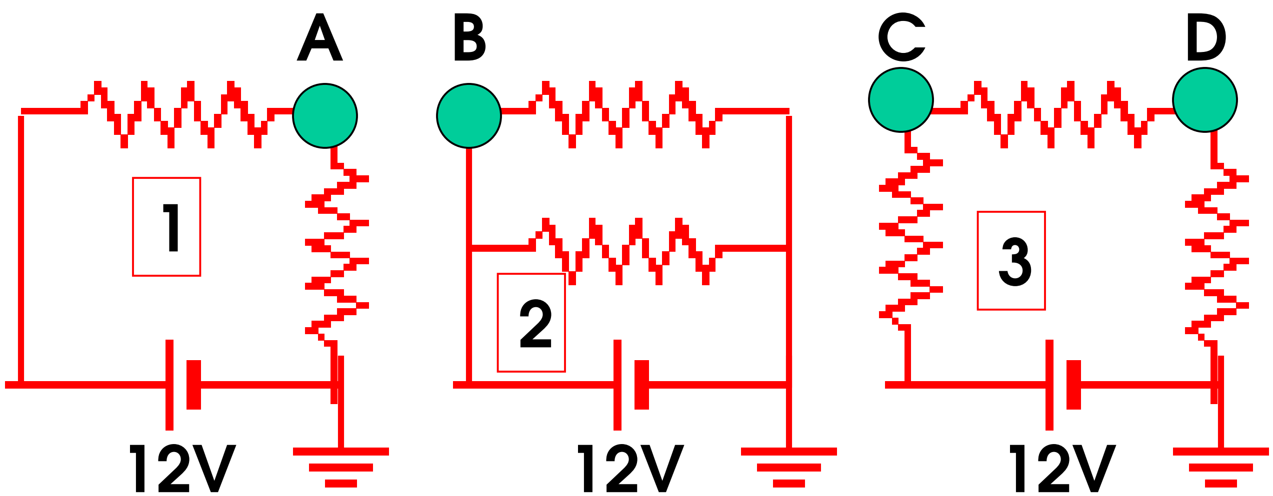
2) left loop: $V_1-I_1R_1+I_2R_2=0$ so $12-3I_1+3I_2=0$

3) right loop: $V_2+I_3R_3+I_2R_2=0$ so $12+3I_3+3I_2=0$

4) outside loop: $V_1-I_1R_1-I_3R_3-V_2=0$ so $-3I_1-3I_3=0$ so $I_1=-I_3$

combine 1) and 4) $I_2=2I_3$ and put into 3) $12+9I_3=0$ so $I_3=4/3$ A and $I_1=-4/3$ A and $I_2=8/3$ A

question



At which point (A,B,C,D) is the potential highest and at which point lowest? All resistors are equal.

a) highest B, lowest A
b) highest C, lowest D
c) highest B, lowest D
d) highest C, lowest A
e) highest A, lowest B

circuit 1: $I=12/(2R)=6/R$

$V_A=12-6/R*R=6V$

circuit 2: $V_B=12V$

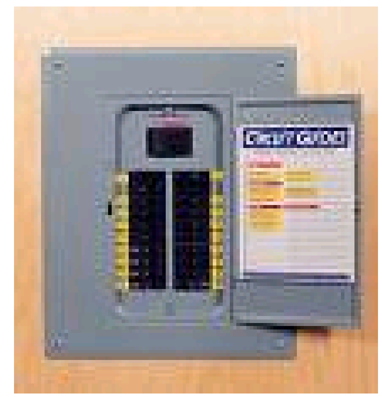
circuit 3: $I=12/(3R)=4/R$

$V_C=12-4/R*R=8V$

$V_D=12-4/R*R-4/R*R=4V$

circuit breakers

Circuit breakers are designed to cut off power if the current becomes too high. In a house a circuit breaker is rates at 15A and is connected to a line that holds a coffee maker (1200 W) and a toaster (1800 W). If the voltage is 120 V, will the breaker cut off power?



$P=VI$ $\quad 1800+1200=120 \times I$

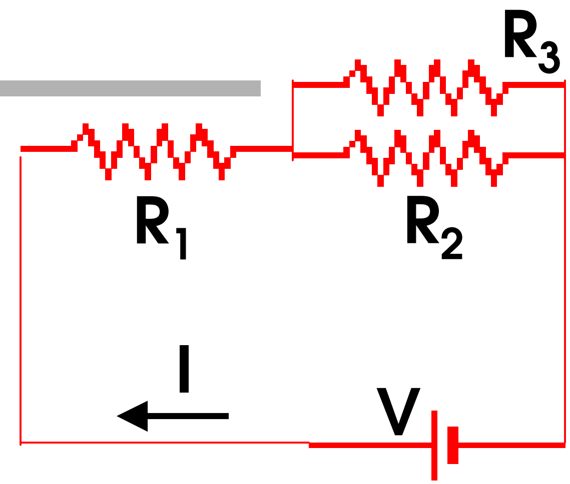
$I=3000/120=25$ A

25A>15 A the breaker will cut off power

Question:

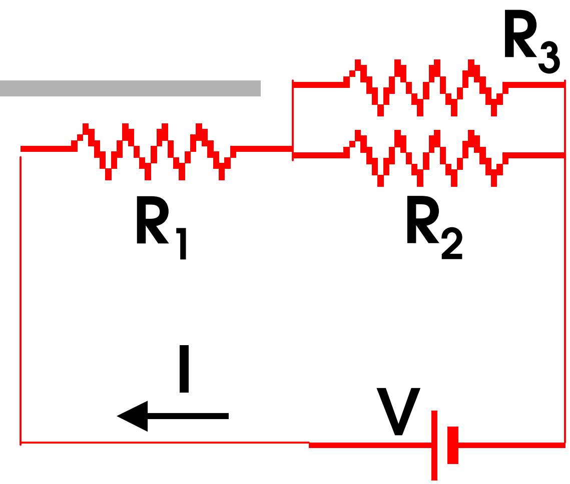
Ø consider the circuit. Which of the following is/are not true?

1. If $R_2=R_3=2R_1$ the potential drops over R_1 and R_2 are the same
2. for any value of R_1,R_2 and R_3 the potential drop over R_1 must be equal to the potential drop over R_2
3. The current through R_1 is equal to the current through R_2 plus the current through R_3 ($I_1=I_2+I_3$)



a) 1 is not true
b) 2 is not true
c) 3 is not true
d) 1&2 are not true
e) 1&3 are not true

answer



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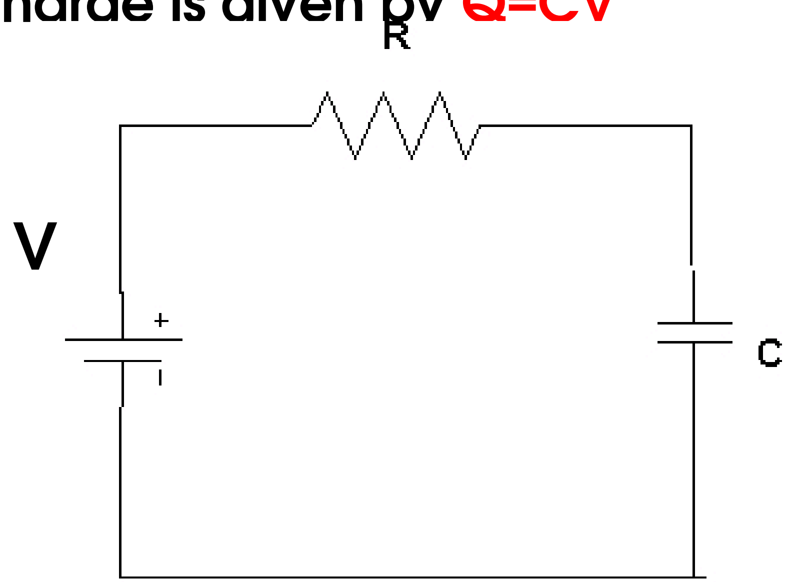
1) if $R_2=R_3=2R_1$ then $1/R_{23}=1/R_2+1/R_3=1/R_1$ so $R_{23}=R_1$ and $I_1=I_{23}$ and potential of R_1 equals the potential over R_{23} and thus R_2 and R_3. THIS IS TRUE
2) no, this is only TRUE in the case of 1)
3) true: conservation of current.

RC circuits

Consider the below circuit.

- When the battery is connected, a current passes through the resistor, and the capacitor begins to charge up.
- As the capacitor gets more charge, and hence more voltage, the voltage across the resistor decreases, so the current decreases.
- Eventually, the capacitor becomes essentially fully charged, so the current becomes essentially zero.
- The maximum charge is given by $Q=CV$



RC circuit II

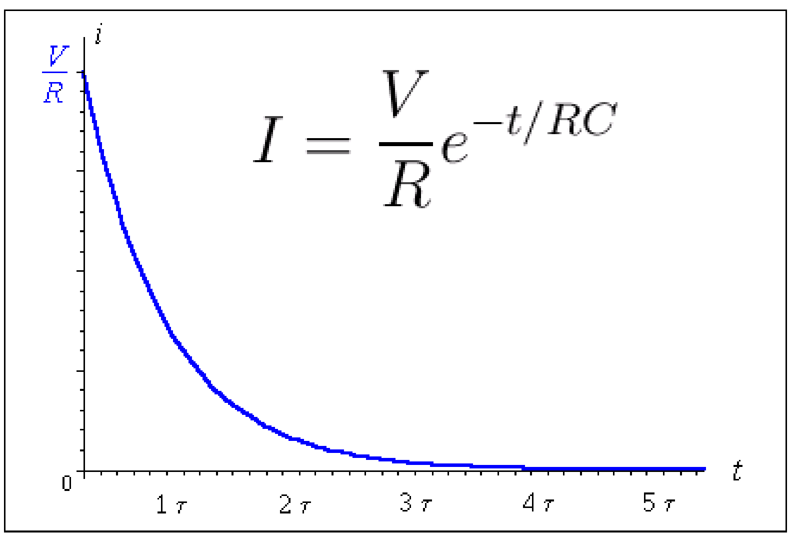
for the charge on the capacitor $q = Q(1 - e^{-t/RC})$

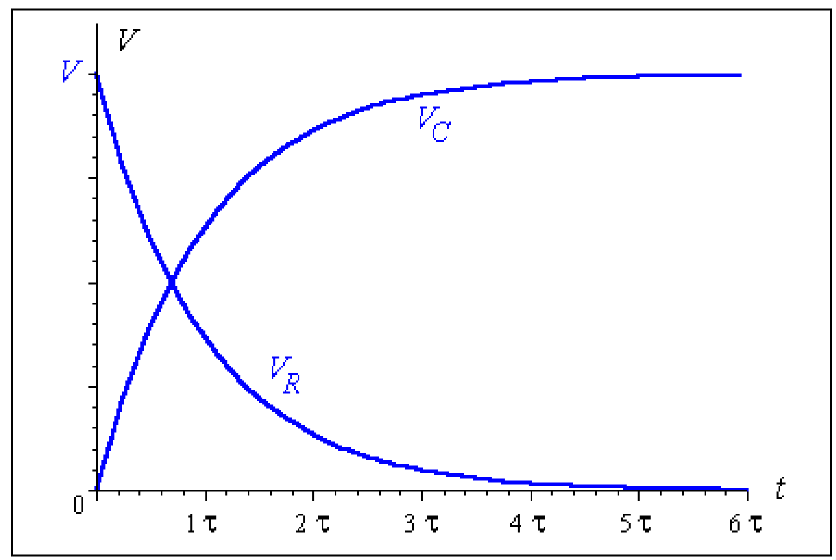
for the voltage over the capacitor $V_C = \frac{Q}{C}(1 - e^{-t/RC}) = V(1 - e^{-t/RC})$

for the voltage over the resistor $V_R = V - V_C$

for the current $I = \frac{V}{R} e^{-t/RC}$

e = 2.718…

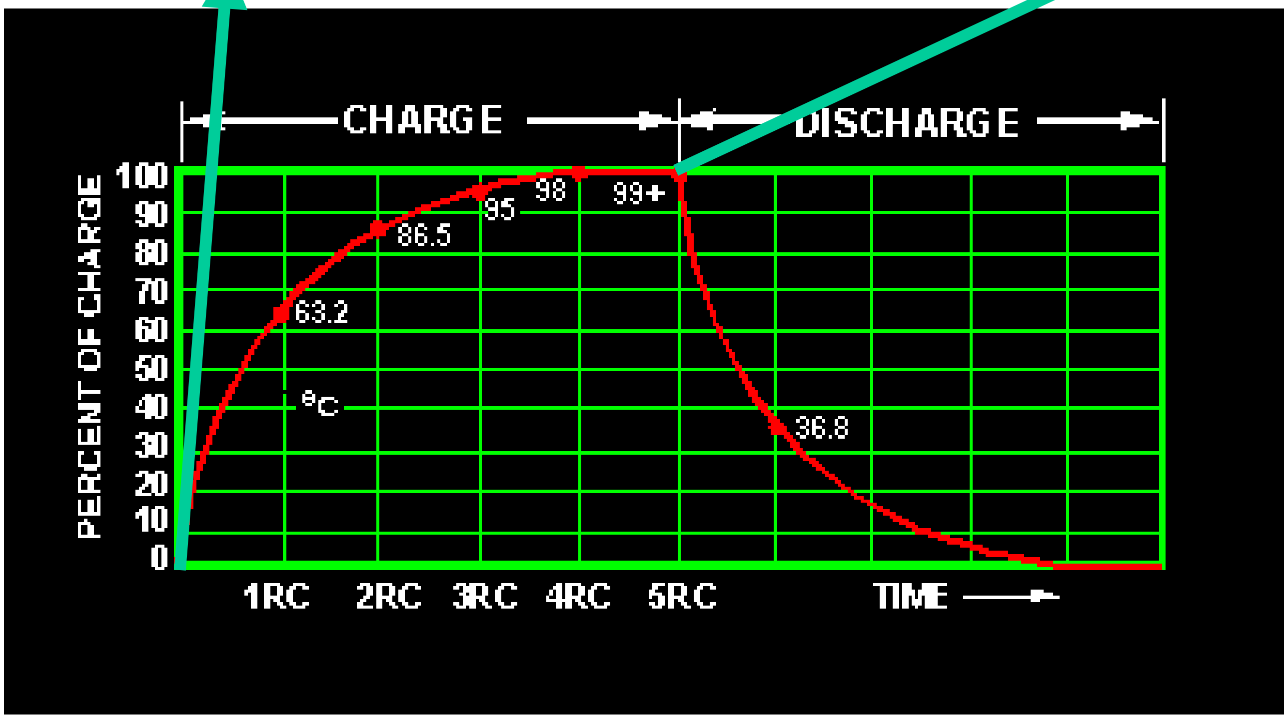




RC time

Voltage switched on

$$q = Q(1 - e^{-t/RC})$$



voltage switched off

$$q = Qe^{-t/RC}$$

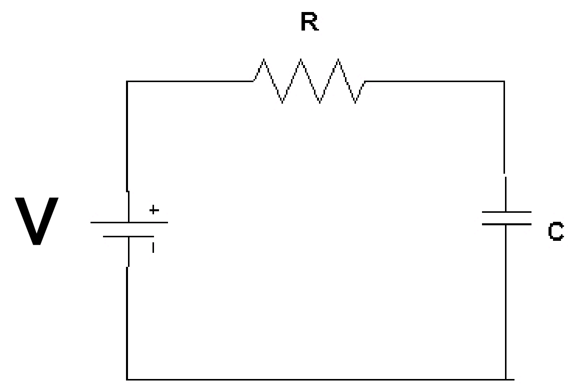
Ø The value $\tau = RC$ is the time constant. It is the time it takes to increase the stored charge on the capacitor to ~63% of its maximum value (1/e=0.63)

question

- given: $V=10$ V

  $R=100$ Ohm

  $C=10\times10^{-6}$ F



The emf source is switched on at t=0. a) After how much time is the capacitor C charged to 75% of its full capacity?

b) what is the maximum current through the system?

a) if $(1-e^{-t/RC})=0.75$ then

charged for 75%, so $e^{-t/RC}=0.25$ $t/RC=-\ln(0.25)$

ln: natural logarithm $t=-RC \times \ln(0.25)=-100\times10^{-5}\times(-1.39)=$ 1.39×10^{-3} seconds.

b) maximum current: at t=0 it is as if the capacitor C is not present so $I=V/R=0.1$ A

$$q = Q(1-e^{-t/RC}) = CV(1-e^{-t/RC}) \qquad I = \frac{V}{R}e^{-t/RC}$$

warning

- there is a question in lon-capa that looks like an RC question, but the current is constant… be careful.