Parametrization uncertainty in PDFs

Jon Pumplin

CTEQ meeting at Argonne (11/18/2010)

1. Method for removing parametrization dependence: revised arXiv:0909.5176 to be published in PRD

2. Negative gluon distribution at small x?

PDF parametrizations

Typical recent gluon parametrization (CT10)

$$x\, g(x, \mu_0) = a_0\, x^{a_1}\, (1 - x)^{a_2}\, e^{p(x)}$$

where

$$p(x) = a_3\sqrt{x} + a_4 x + a_5 x^2$$

- Power-law dependence at $x \to 0$ (Regge) (However, NLO DGLAP doesn't do well by Regge theory.)
- Subleading terms down by $\sim x^{0.5}$ at $x \to 0$ (Regge)
- Spectator counting form at $x \to 1$
- $g(x)$ positive definite (However, possibly too strong)

New method

$$p(x) = \sum_{j=1}^{12} b_j\, x^{j/2}$$

Problems: How to obtain smooth, stable fits with so many parameters...

Chebyshev Polynomial method

Replace the fitting parameters $\{b_j\}$ by equivalent parameters $\{c_j\}$ where

$$p(x) = \sum_{j=1}^{12} b_j \, x^{j/2} = \sum_{j=1}^{12} c_j \, T_j(y)$$

where $y = 1 - 2\sqrt{x}$ maps the physical region $0 < x < 1$ to $-1 < y < 1$.

$$T_0(y) = 1, \quad T_1(y) = y \quad T_{n+1}(y) = 2yT_n(y) - T_{n-1}(y)$$

$$T_j(y) = \cos(j\theta) \quad \text{where} \quad y = \cos\theta \; .$$

$T_j(y)$ has extreme values of ± 1 at the endpoints and at $j - 1$ points in the interior of the physical region $0 < x < 1$. Chebyshev polynomials of increasingly large j thus model structure at an increasingly fine scale in x.

Smoothness penalties

The Chebyshev parametrizations can easily take on more fine structure in x than is plausible in the nonperturbative physics that is being described. To avoid this, we add a penalty to χ^2

Observe that the classic form

$$f(x) = a_0\, x^{a_1}\, (1-x)^{a_2}\ ,$$

surely embodies the appropriate smoothness.

This has

$$x\,(1-x)\, d(\ln f)/dx \;=\; a_1 - (a_1 + a_2)x$$

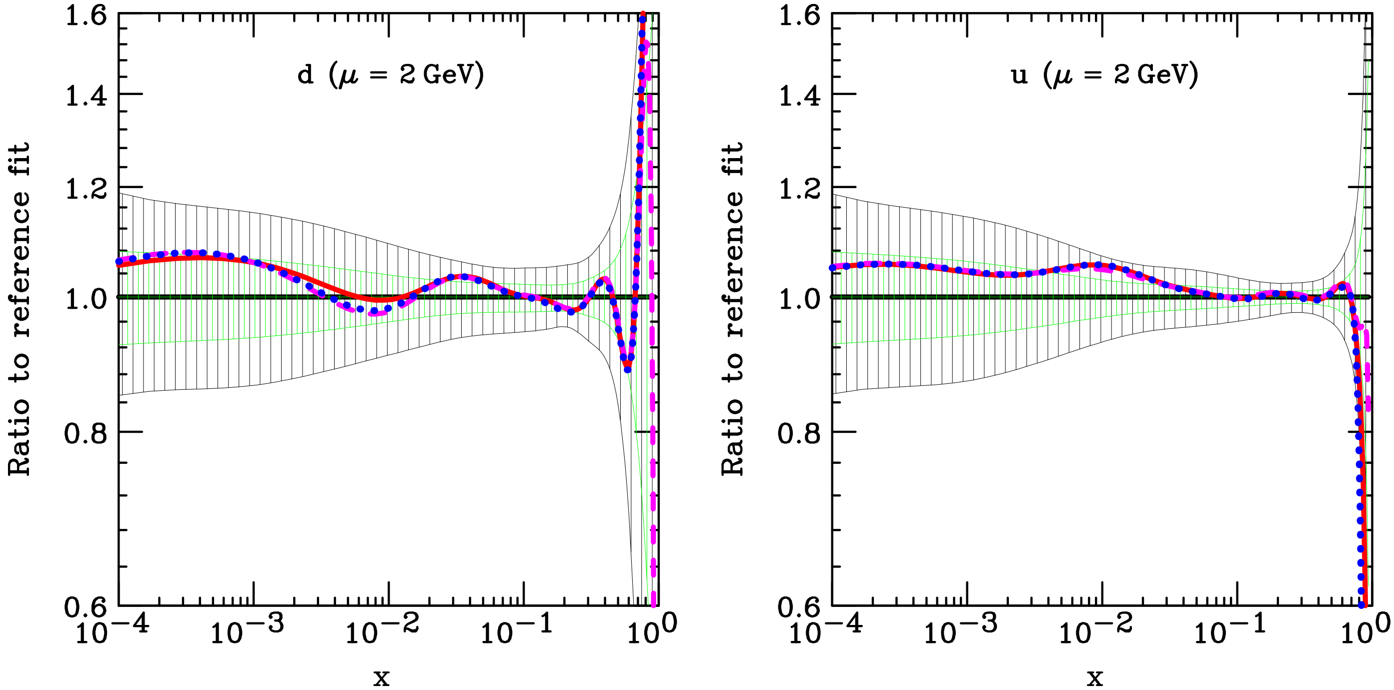
is linear in x. Hence it is natural to define

$$\Phi_a(x) \;=\; x\,(1-x)\, d(\ln f_a)/dx$$

$$S_a \;=\; \int_{x_1}^{x_2} \left(\frac{d^2\Phi_a}{dx^2}\right)^2 dx$$

Add $\sum_a C_a\, S_a$ to χ^2, with the weights C_a chosen to increase the overall χ^2 by ~ 5.

Results



Wide shaded region: fractional uncertainty from CT10 (26 fitting parameters)

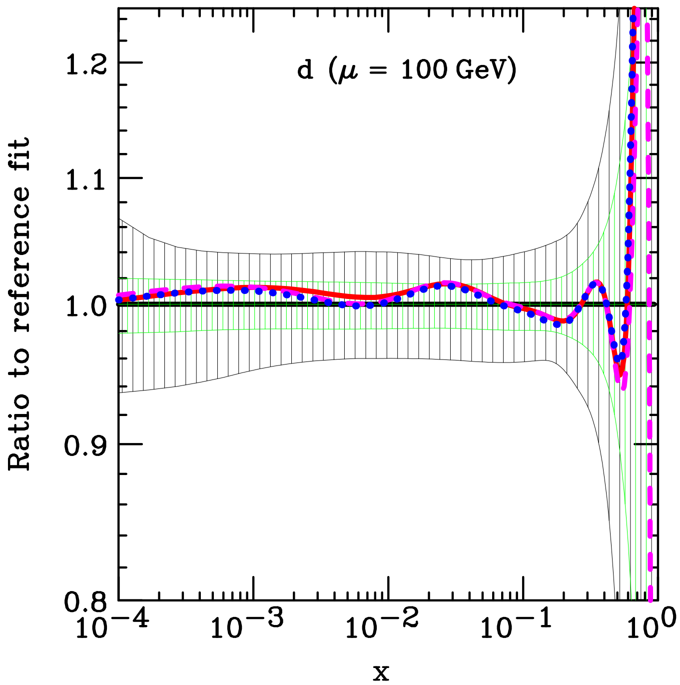
Narrow shaded regions: uncertainty for $\Delta\chi^2 = 10$.

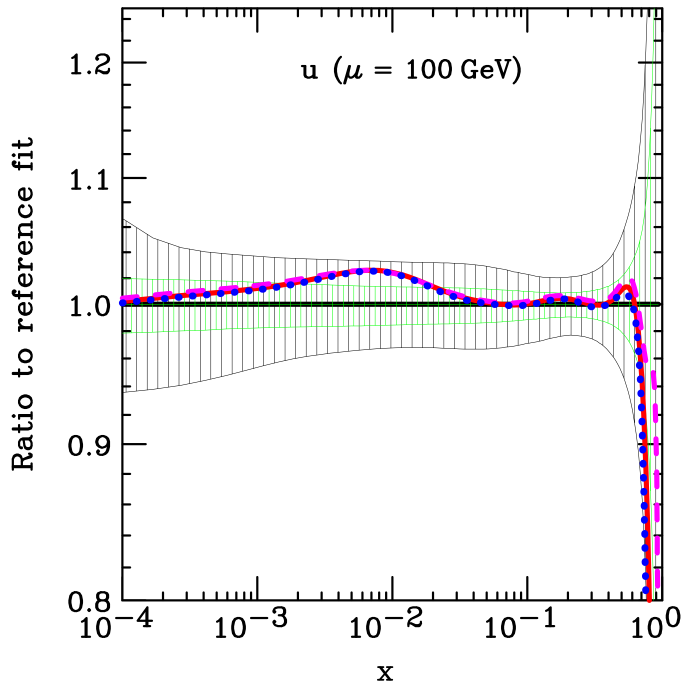
Solid curve: Chebyshev fit with 84 free parameters χ^2 lower than CT10 by 105.

> 21 better for BCDMS $\mu p \to \mu X$,
>
> 16 better for BCDMS $\mu d \to \mu X$,
>
> 17 better for HERA combined set

Dashed and Dotted curves: Chebyshev fits with different behaviors at large x, χ^2 within 5 of best fit.

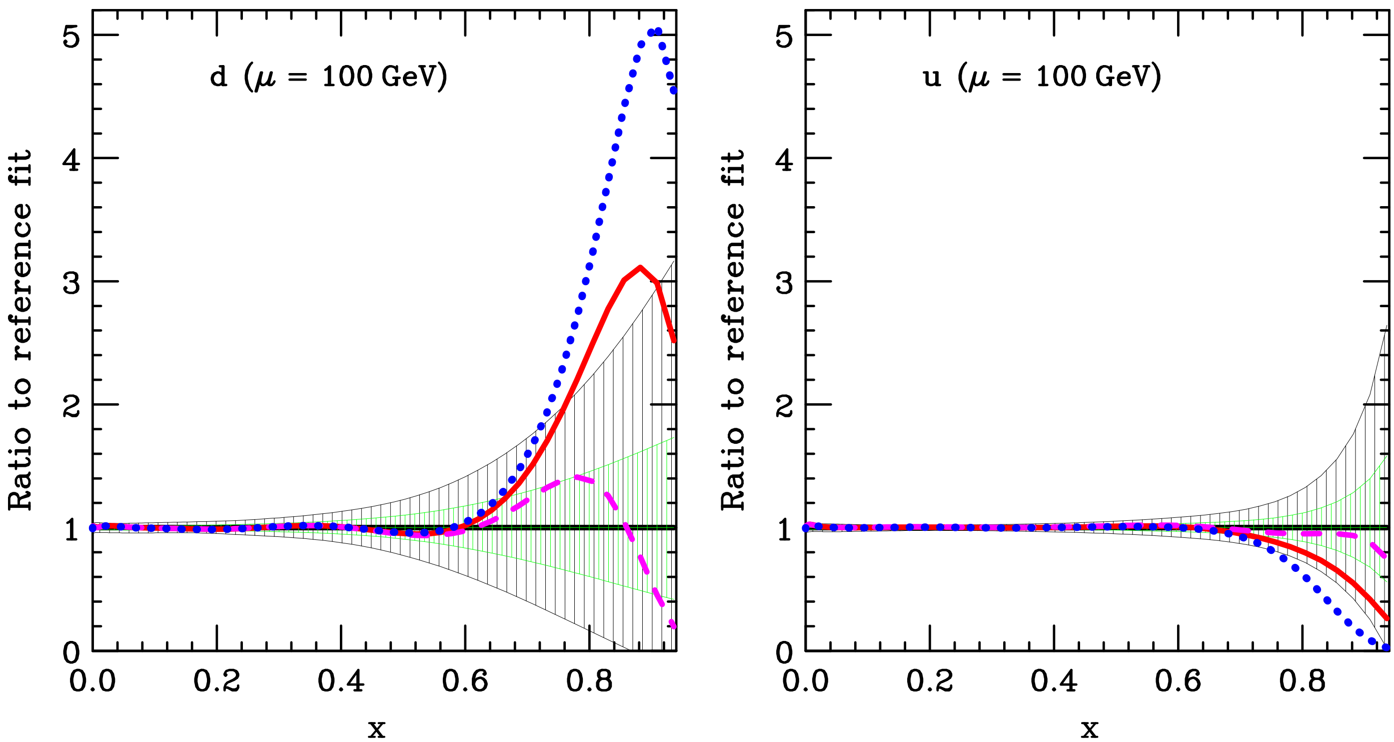
Results at $\mu = 100\,\mathrm{G}eV$





Parametrization effects are important at high scale, even for $u(x)$ which has nominally small uncertainty.

Results at large x

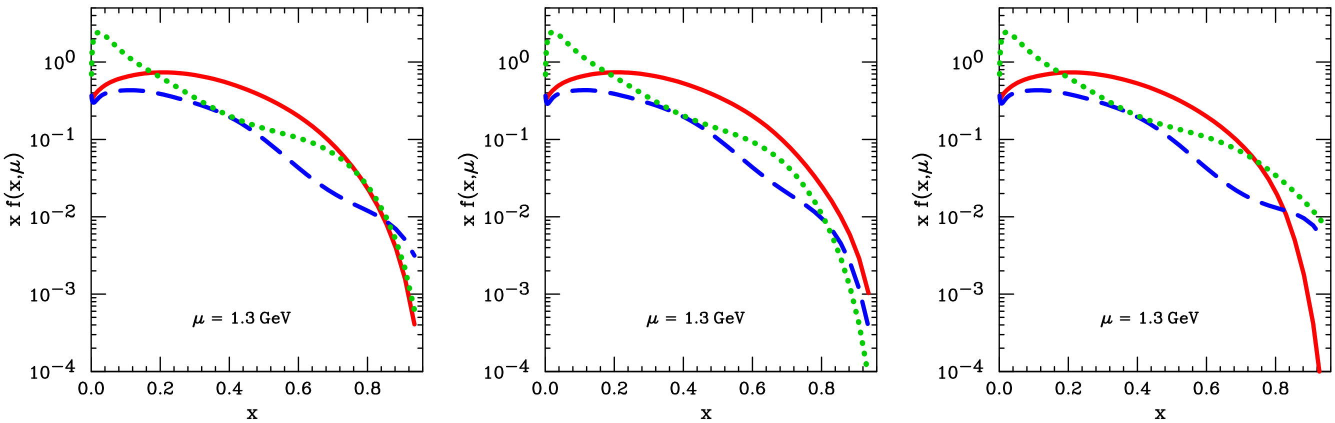


Wide shaded region: fractional uncertainty from CT10 (26 fitting parameters)

Solid curve: Chebyshev fit with 84 free parameters.

Dashed and Dotted curves: Chebyshev fits with different behaviors at large x, χ^2 within 5 of best fit.

Various possibilities at large x



Red = up quark

Blue = down quark

Green = gluon

As Jeff Owens remarked, the different versions agree quite well for $x < 0.6$ where there are direct constraints from data. In principal, the very large x region is constrained by data at higher scales, since it feeds down to lower x at large μ; but this constraint is weak because the absolute PDFs are so small at large x.