PDF Parametrization Issues

Jon Pumplin *

Department of Physics and Astronomy, Michigan State University, USA

The role of parametrization dependence in the measurement of parton distributions is discussed, using the strangeness component in the CTEQ6.6 fit as an example. A method to impose theoretical constraints in PDF fitting is proposed. The Regge behavior of PDFs is examined.

1 Parametrization dependence

In the standard PDF fitting paradigm, one parametrizes each flavor $f_a(x, Q_0)$ at a small Q_0 using currently a total of about 20 parameters. One then computes the PDFs $f_a(x, Q)$ for $Q > Q_0$ by DGLAP evolution; followed by the cross section predictions for DIS(e,μ,ν), Drell-Yan, inclusive jets, ... by perturbation theory; followed by a "χ^2" measure of agreement between the predictions and measurements. One minimizes χ^2 with respect to the parameters at Q_0 to find Best Fit PDFs, and explores the neighborhood of the minimum via eigenvector sets to estimate an uncertainty range in which all of the data sets are described tolerably well. (Weight factors are included in the definition of χ^2 to keep experiments with a small number of data points from being unduly neglected.)

Parametrization dependence is the systematic error caused by making specific choices of the functional forms in $f_a(x, Q_0)$. It was interesting to observe at this meeting that the major fitting groups all use significantly different parametrizations, so at least the community is not being mislead by a single choice. Meanwhile, Neural Net methods may be able to avoid this problem entirely.

Previous CTEQ PDF analyses generally assumed $s(x) = \bar{s}(x) \propto \bar{d}(x) + \bar{u}(x)$ at Q_0. We dropped that ansatz in CTEQ6.6. A preliminary version used the innocent-looking form $s(x) = \bar{s}(x) = a_0\, x^{a_1}\, (1-x)^{a_2}$, with a_1 the same as for $\bar{d}$ and $\bar{u}$, as predicted by Regge theory. The resulting strangeness looked OK by itself; but gave $\bar{s}(x) > \bar{u}(x)$, $\bar{d}(x)$ at small x, which violates theory prejudice and perhaps Hermes data. A more elaborate parametrization was chosen in a somewhat ad hoc manner for the final CTEQ6.6 to avoid this.

2 Applying theory constraints to PDF fitting

A more general method to handle this sort of problem is as follows: One can choose a very flexible parametrization such as $\bar{s}(x) = a_0\, x^{a_1}\, (1-x)^{a_2}\, e^{a_3 \sqrt{x} + a_4 x}$ that has more parameters than can be determined from the data. One then adds a "penalty" to χ^2 to force parameters such as a_2 and $\bar{s}(x)/(\bar{d}(x) + \bar{u}(x))$ at $x \to 0$ to fall within the range allowed by our theoretical prejudices. For the central "Best Fit," this may be not much different from the previous method of just freezing some parameters at plausible values if they cannot be determined unambiguously by the fitting. But for the uncertainty analysis, it has the advantage of capturing the actual wider range of uncertainties, since a new eigenvector direction (two new extreme eigenvector sets) is generated for each parameter that is treated in this way.

*This work was supported by the U.S. National Science Foundation under award PHY-0354838.

In the standard analysis, there are several parameters, such as the $(1-x)^{a_2}$ behavior of $g(x, Q_0)$ or $d_v(x, Q_0)$ which could benefit from this treatment in the future. The goal in all this is to avoid repeating the "HJ" scenario, in which new data (Run I inclusive jet data at the Tevatron) appeared to lie outside standard model predictions; but were later found consistent using an unanticipated HJ form for $g(x, Q_0)$.

This method could also be useful for the HERA-only type fits which currently use only 11 fit parameters. Those authors could add one more parameter for each flavor in this way, traded by constraint penalties on a_2, to obtain a more realistic measure of uncertainty at large x in their analysis.

3 Regge behavior

For a **valence quark**, the Regge behavior $f(x, Q) \propto x^{a_1}$ that we assume for $x \to 0$ at Q_0 is quite well preserved to higher Q by DGLAP evolution. This can be seen by the nearly straight-line behavior on a log-log plot, with slope nearly independent of Q in Fig. 1. The numerical value of the slope a_1 agrees well with expectations from Regge, which supports the use of the ansatz $f(x, Q) \propto x^{a_1}$. However, the uncertainty in a_1 from fitting is small compared to the uncertainty of estimates based on Regge, so the Regge theory does not provide a useful numerical constraint. It would be interesting to follow up this study by seeing if the small variation in slope is consistent with other estimates of Regge "shrinkage."

For a **sea quark**, the Regge behavior $f(x, Q) \propto x^{a_1}$ that we assume for $x \to 0$ at Q_0 is also well preserved by DGLAP evolution as seen in Fig. 2. Again the observed slope value a_1 is consistent with expectations from Regge theory, which supports the choice of functional form. And again the uncertainty in a_1 from PDF fitting is small compared to the uncertainty of its estimate based on Regge theory, so traditional Regge phenomenology doesn't provide a useful constraint on a_1 to improve PDFs.

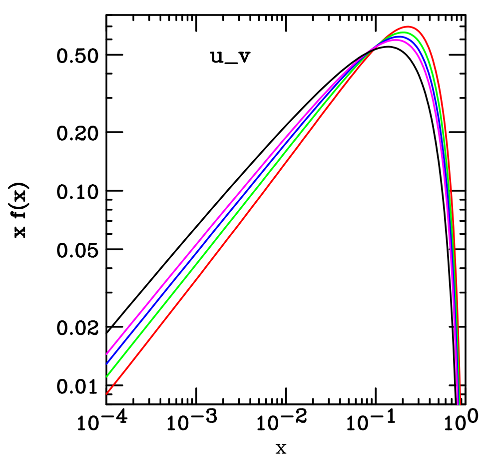

Figure 1: $u_v(x, Q) = u(x, Q) - \bar{u}(x, Q)$: Curves from top to bottom on left are $Q = 20$, 5, 3.16, 2, 1.3 GeV.

For the **gluon**, in contrast to valence and sea quark distributions, the NLO evolution at small x shown in Fig. 3 is so rapid that no simple comparison can be made with expectations from Regge theory. This rapid change in slope is related to the rapid variation of the effective power $F_2 \sim x^{\lambda(Q^2)}$. An interesting speculation is that perhaps it is a bit more extreme than the behavior of F_2; and that small-x resummation corrections to DGLAP might restore Regge behavior for $g(x, Q)$. This remains to be studied.

A link to the original slides for this talk is given below [1]. A related consideration of the implications of Regge theory for PDF fitting is given in [2].

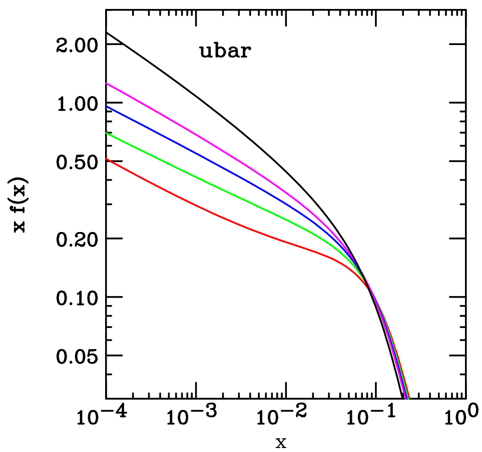

Figure 2: $\bar{u}(x, Q)$: Curves from top to bottom on left are $Q = 20$, 5, 3.16, 2, 1.3 GeV.

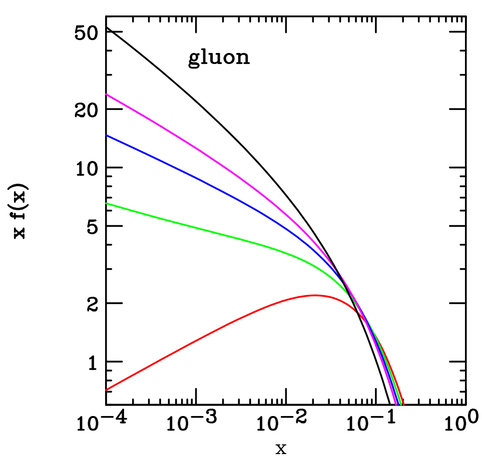

Figure 3: $g(x, Q)$: Curves from top to bottom on left are $Q = 20$, 5, 3.16, 2, 1.3 GeV.

References

[1] Slides:
http://indico.cern.ch/contributionDisplay.py?contribId=92&sessionId=17&confId=24657

[2] G. Soyez, AIP Conf. Proc. **775** (2005) 88 [arXiv:hep-ph/0502158]; Phys. Rev. D **71** (2005) 076001 [arXiv:hep-ph/0407098].