χ^2 minimization

The simplest fitting method is to define

$$\chi_0^2 = \sum_{i=1}^{N} \frac{(D_i - T_i)^2}{\sigma_i^2} \qquad \begin{cases} D_i = & \text{data point} \\ T_i = & \text{theory value} \\ \sigma_i = & \text{“expt. error”} \end{cases}$$

and minimize χ_0^2 with respect to the PDF model parameters $\{a_\lambda\}$. However, the systematic errors imply

$$D_i = T_i(a) + \alpha_i r_{\text{stat},i} + \sum_{k=1}^{K} r_k \beta_{ki}$$

where α_i = uncorrelated error on D_i

$\beta_{ki} = k^{\text{th}}$ systematic error on D_i

(numbers published by the expt.)

($r_{\text{stat},i}$ and r_k are random variables with standard deviation 1.)

To take into account the systematic errors[†] we define

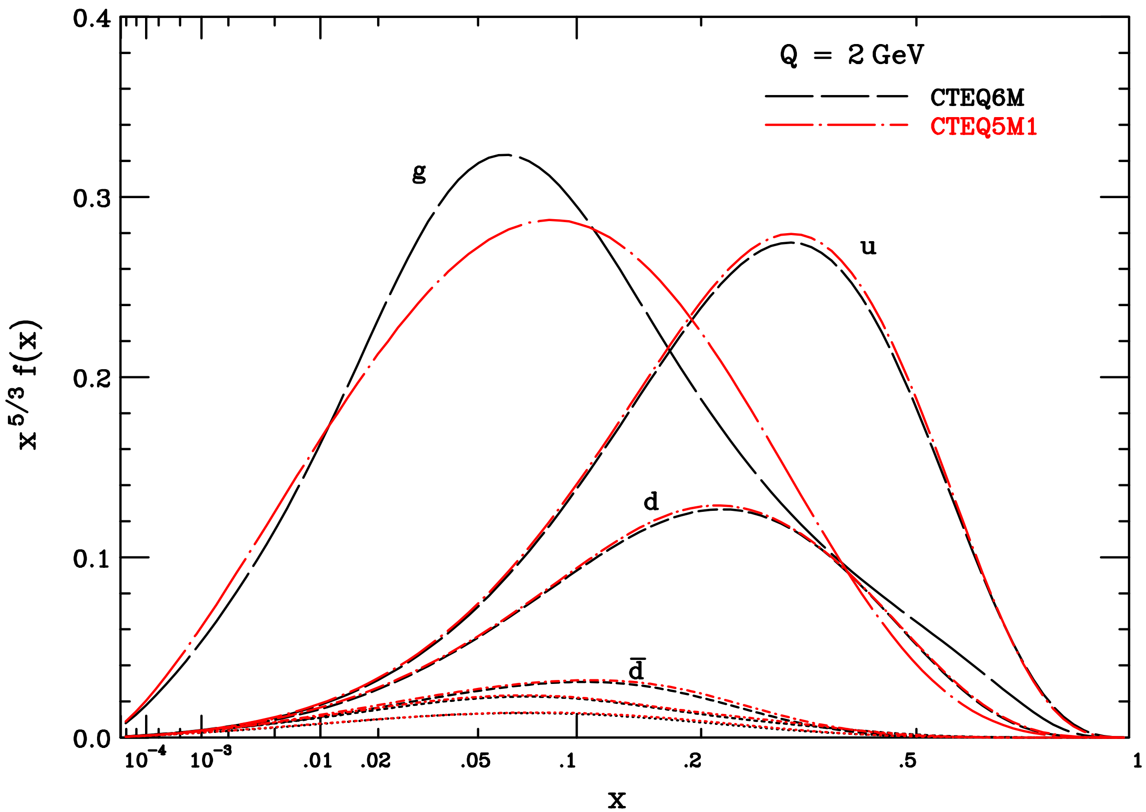
$$\chi'^2(a_\lambda, r_k) = \sum_{i=1}^{N} \frac{\left(D_i - \sum_k r_k \beta_{ki} - T_i\right)^2}{\alpha_i^2} + \sum_k r_k^2,$$

and minimize with respect to both $\{r_k\}$ ($\equiv$ the systematic shifts) and $\{a_\lambda\}$ ($\equiv$ the PDF model parameters).

Because we use a quadratic penalty term r_k^2, the minimization with respect to $\{r_k\}$ can be done analytically (for arbitrary $\{a_\lambda\}$). Then the minimization w. r. t. $\{a_\lambda\}$ is done numerically.

[†] In CTEQ6 we symmetrize the systematic errors.

Comparison of CTEQ6 and CTEQ5

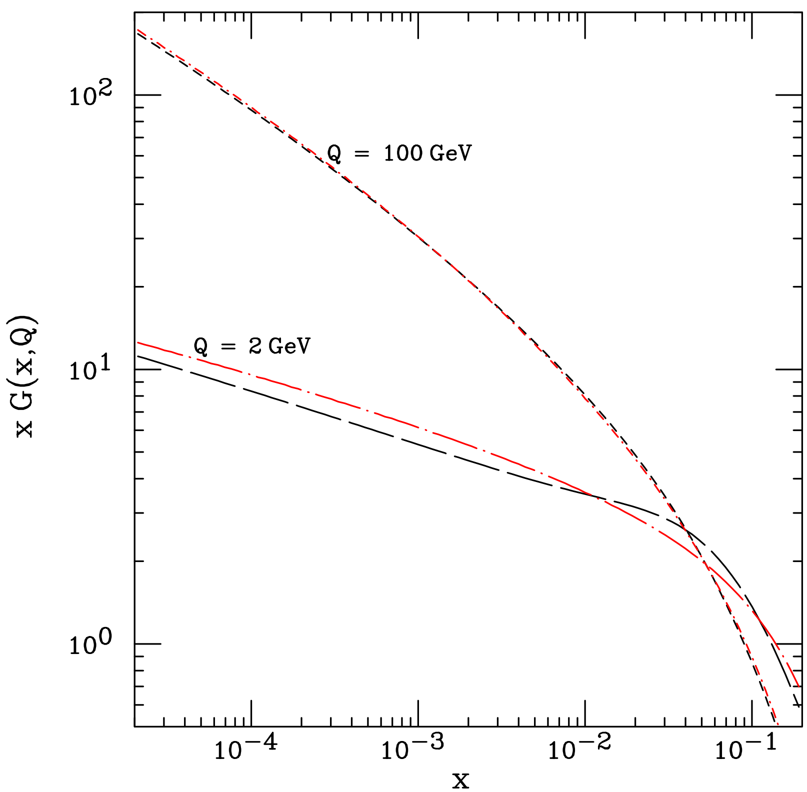


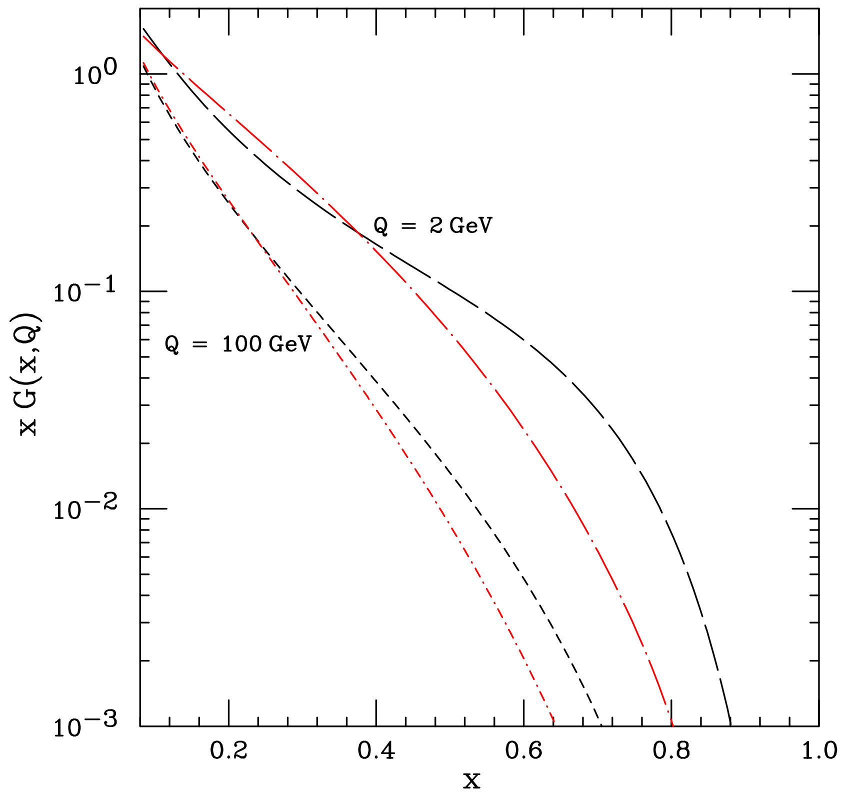
CTEQ6M and CTEQ5M1 PDF's at Q=2 GeV.

(unlabeled:
$\bar{u}$ and $s = \bar{s}$;
MS-bar scheme)

- ► The quark distributions have not changed much.
- ► The gluon is noticeably different.

The Gluon Distribution





CTEQ6M and CTEQ5M1 gluon distributions at Q = 2 and 100 GeV. (a) Small-x region; (b) large-x region.

► ***Note the hard gluon distribution in CTEQ6M.***