χ^2 and Systematic Errors

The simplest definition

$$\chi_0^2 = \sum_{i=1}^{N} \frac{(D_i - T_i)^2}{\sigma_i^2} \qquad \left\{ \begin{array}{ll} D_i = & \text{data} \\ T_i = & \text{theory} \\ \sigma_i = & \text{"expt. error"} \end{array} \right.$$

is optimal for random Gaussian errors,

$$D_i = T_i + \sigma_i r_i \qquad \text{with} \qquad P(r) = \frac{e^{-r^2/2}}{\sqrt{2\pi}}.$$

With systematic errors,

$$D_i = T_i(a) + \alpha_i r_{\text{stat},i} + \sum_{k=1}^{K} r_k \beta_{ki} \, .$$

The fitting parameters will be $\{a_\lambda\}$ (theoretical model) _and_ $\{r_k\}$ (corrections for systematic errors).

Published experimental errors:

- α_i is the 'standard deviation' of the random uncorrelated error.

- β_{ki} is the 'standard deviation' of the kth (completely correlated!) systematic error on D_i.

To take into account the systematic errors, we define

$$\chi'^2(a_\lambda, r_k) = \sum_{i=1}^{N} \frac{\left(D_i - \sum_k r_k \beta_{ki} - T_i\right)^2}{\alpha_i^2} + \sum_k r_k^2,$$

and minimize with respect to $\{r_k\}$. The result is

$$\widehat{r}_k = \sum_{k'} \left(A^{-1}\right)_{kk'} B_{k'}, \qquad \text{(systematic shift)}$$

where

$$\begin{aligned} A_{kk'} &= \delta_{kk'} + \sum_{i=1}^{N} \frac{\beta_{ki}\beta_{k'i}}{\alpha_i^2} \\ B_k &= \sum_{i=1}^{N} \frac{\beta_{ki}\left(D_i - T_i\right)}{\alpha_i^2}. \end{aligned}$$

Note that the $\widehat{r}_k$'s depend on the PDF model parameters $\{a_\lambda\}$. Then

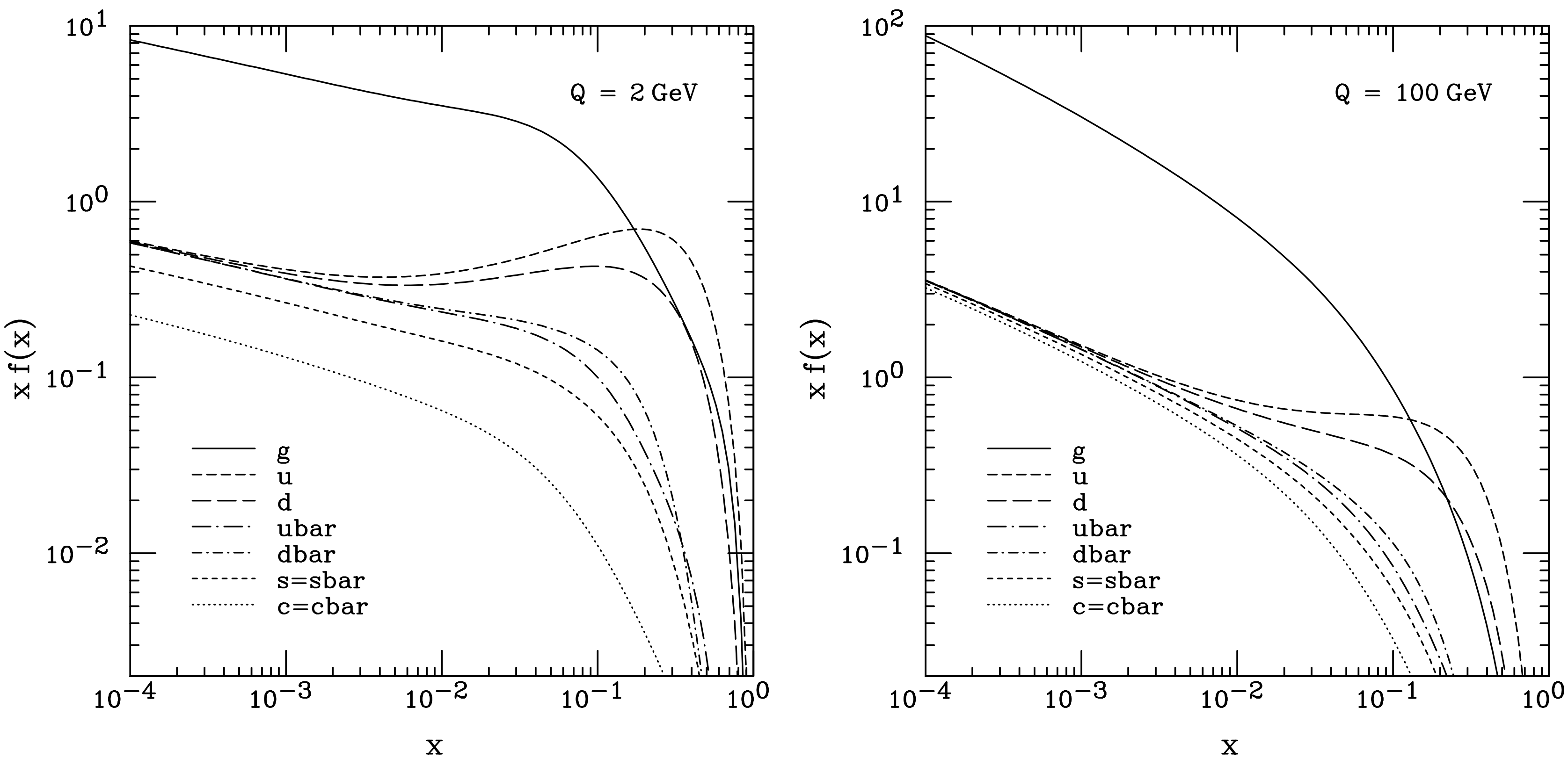
$$\boxed{\chi^2(a_\lambda) = \min_{\{r_k\}} \chi'^2(a_\lambda, r_k)}$$

Now minimize $\chi^2(a)$ with respect to the model parameters $\{a_\lambda\}$.

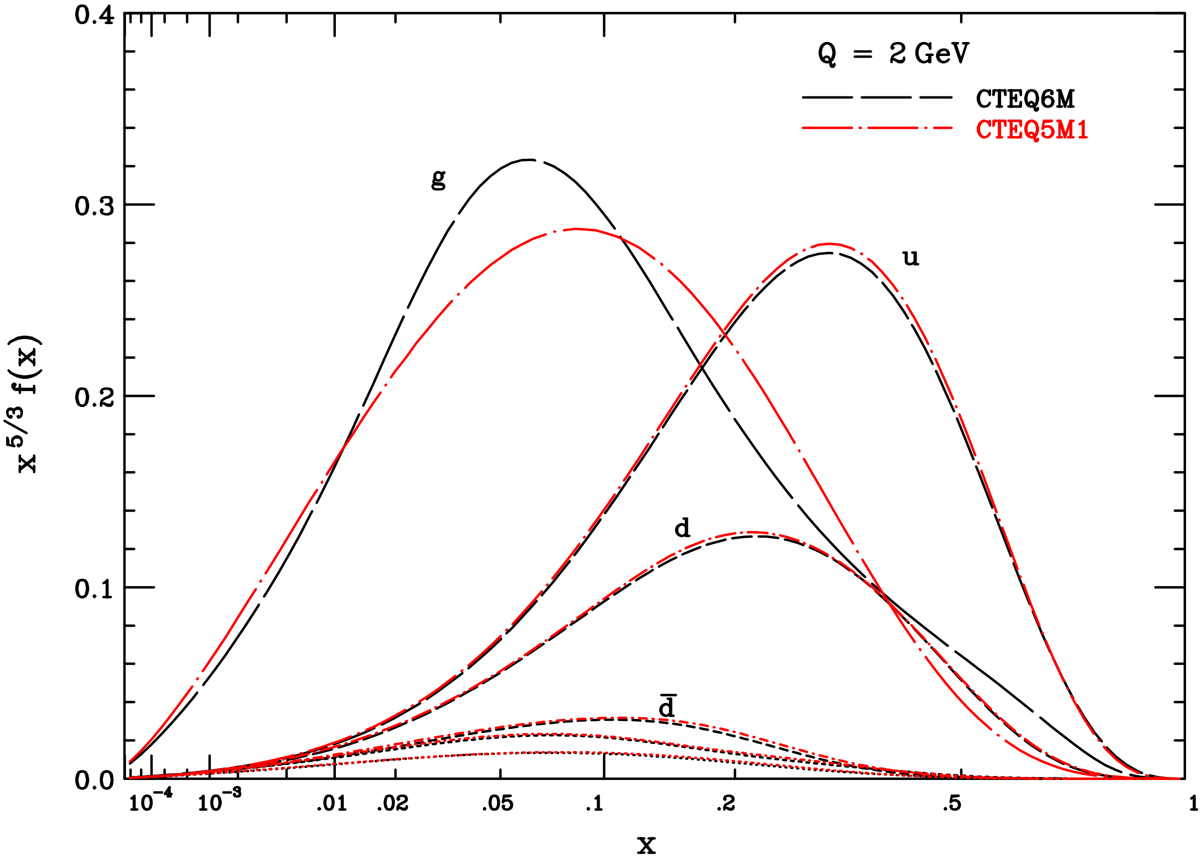
Output

- $\{a_\lambda\}$, which determine $f_i(x, Q_0^2)$.

- $\{\widehat{r}_k\}$, which are optimal "corrections" for systematic errors; i.e., systematic shifts to be applied to the data points to bring the data from different experiments into compatibility, within the framework of the theoretical model.



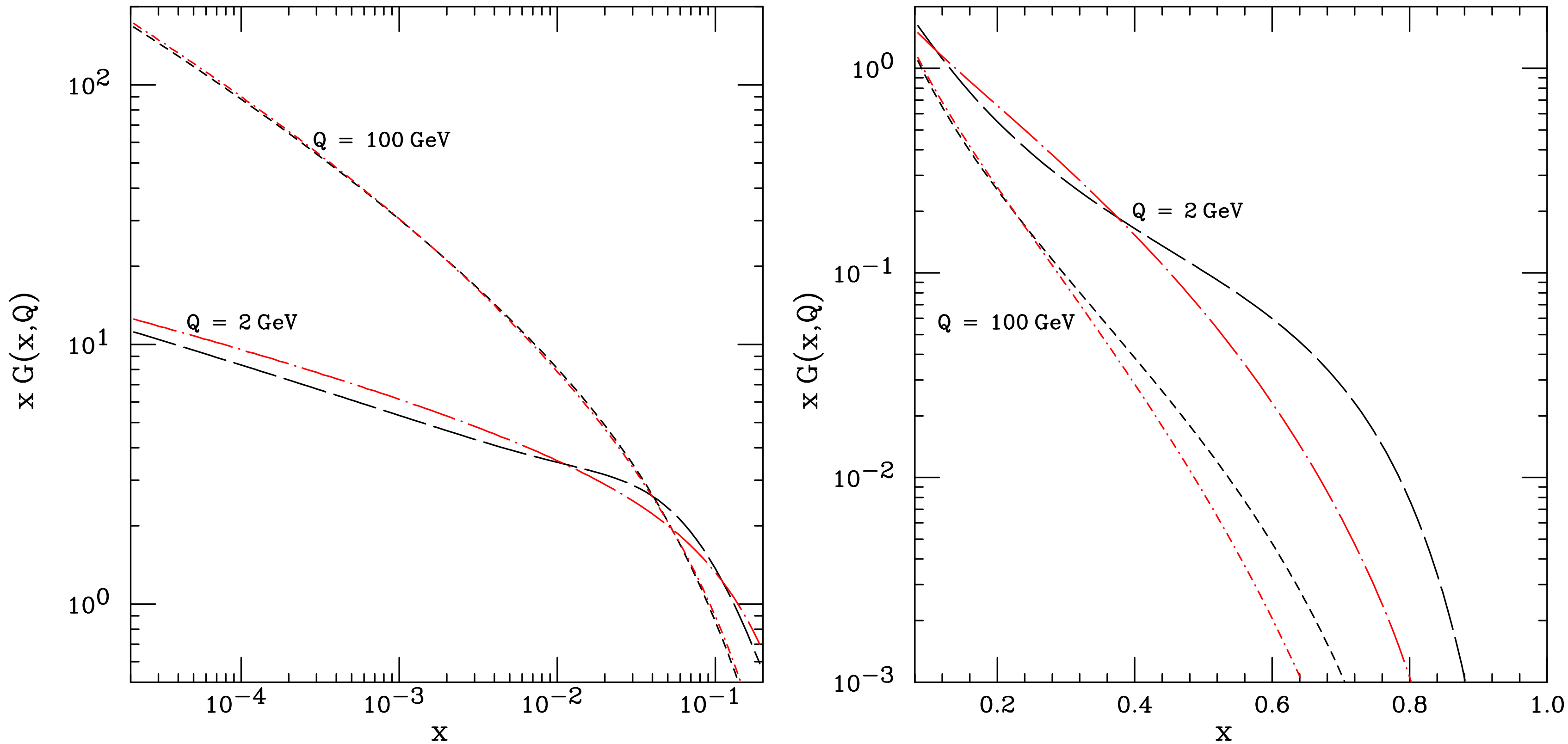
Overview of the CTEQ6M parton distribution functions at $Q = 2$ and 100 GeV.



Comparison of CTEQ6M (dashed) to CTEQ5M1 (dot-dashed) PDF's at $Q = 2$ GeV. (The unlabeled curves are $\bar{u}$ and $s = \bar{s}$.)

★ Quarks have not changed much.

★ Gluon is noticeably different.



Comparison of CTEQ6M (dashed) to CTEQ5M1 (dot-dashed) gluon distributions at $Q = 2$ and 100 GeV. (a) The small-x region; (b) the large-x region.

THE GLUON DISTRIBUTION