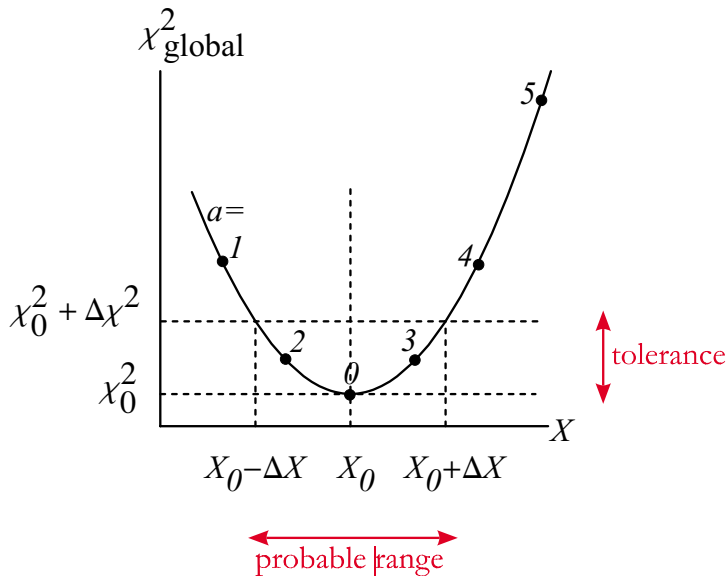


Quantitative Uncertainties

A SIMPLE IDEA



Tolerance

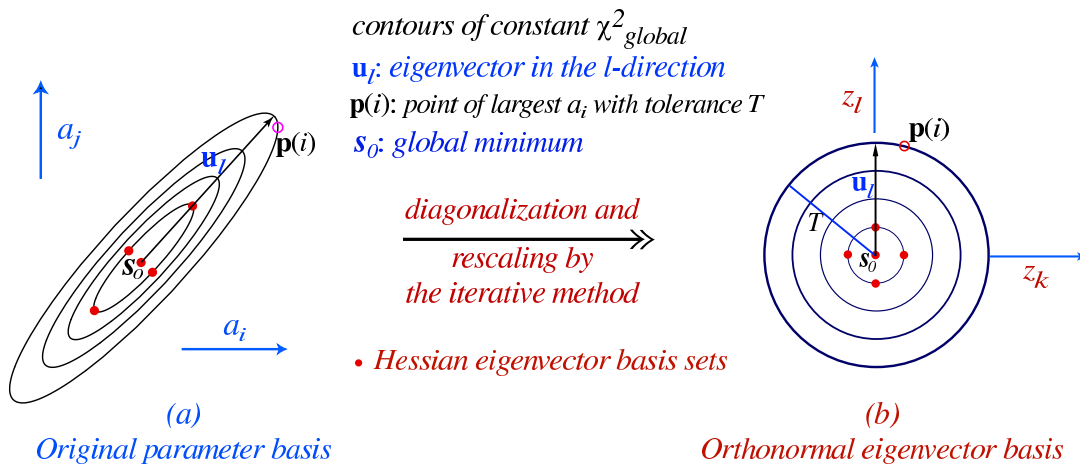
$$T^2 = \text{max. allowed } |\Delta\chi^2|$$

$$T \approx 10$$

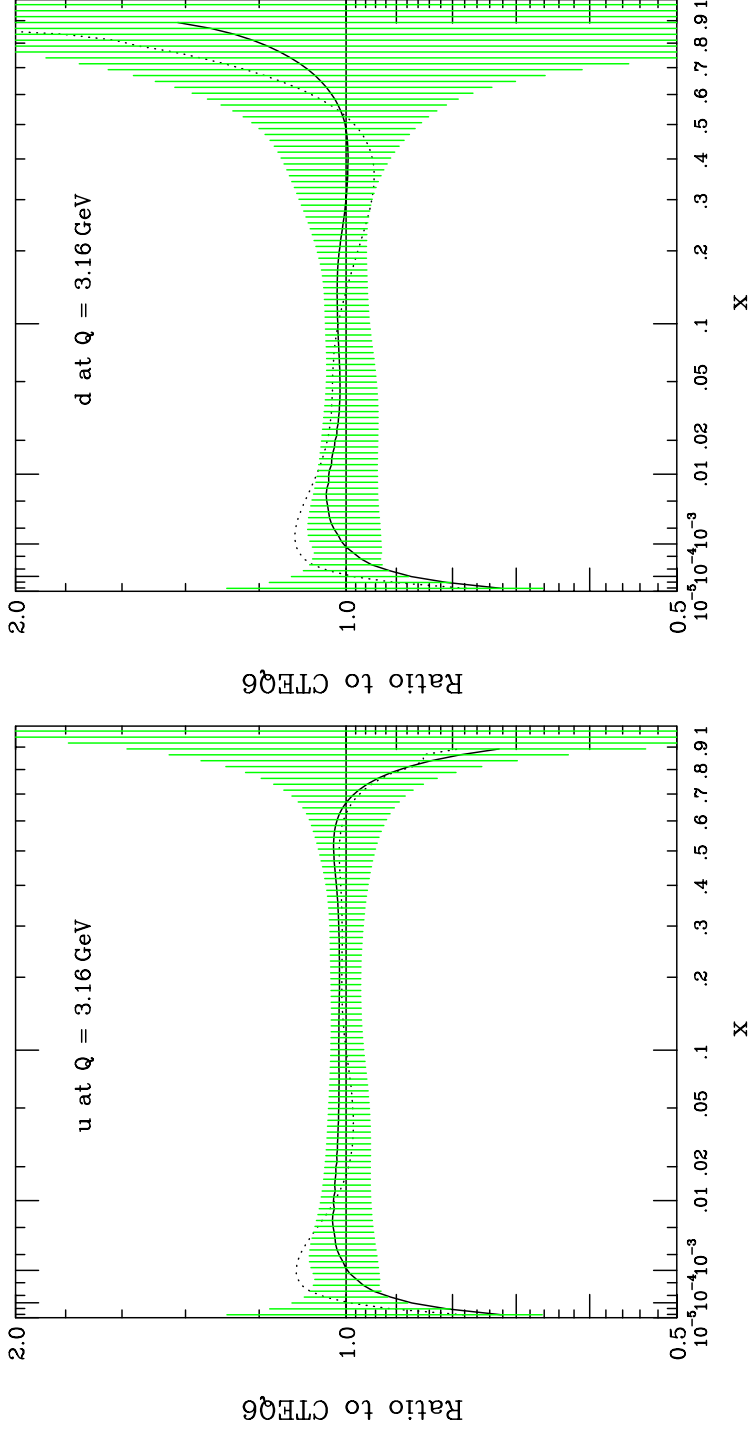
COMPUTATIONAL METHODS

- Lagrange Multiplier Method – constrained minimization to obtain the best fit as a function of X .
- Hessian Matrix Method – explore the variation of $\chi^2(a)$ in the neighborhood of the global minimum in the n dimensional parameter space.

2-dim (i,j) rendition of d-dim (~20) PDF parameter space

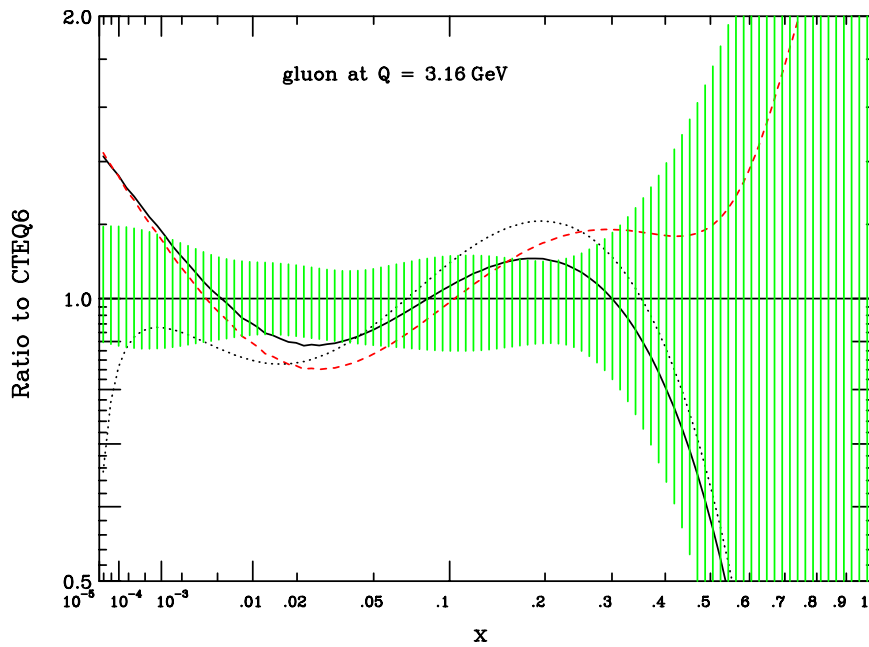


- Hessian eigenvector basis sets



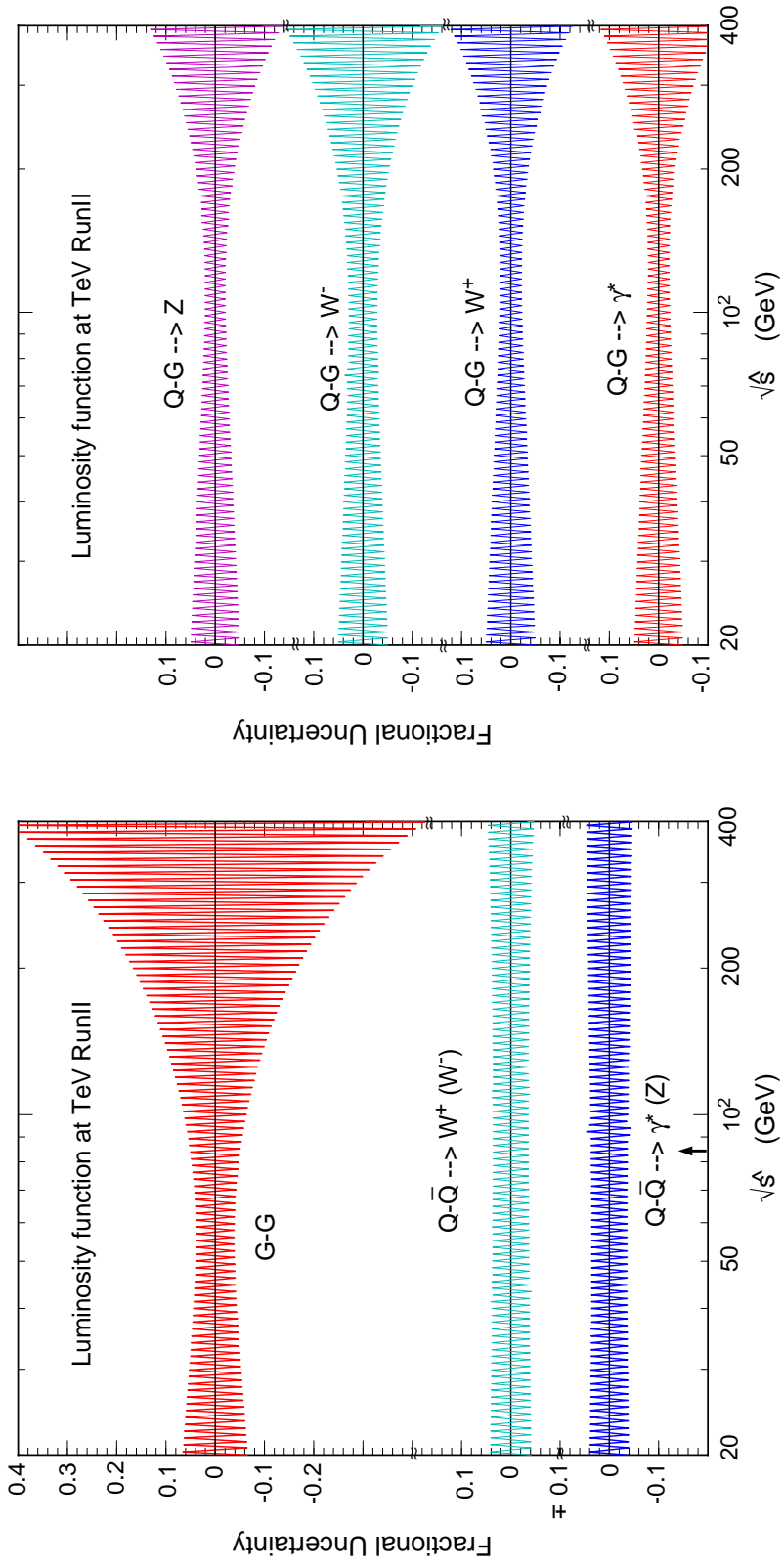
Uncertainty bands for the u - and d -quark distribution functions at $Q^2 = 10 \text{ GeV}^2$. The solid line is CTEQ5M1 and the dotted line is MRST2001.

(Shaded: envelopes of extreme pdf's)



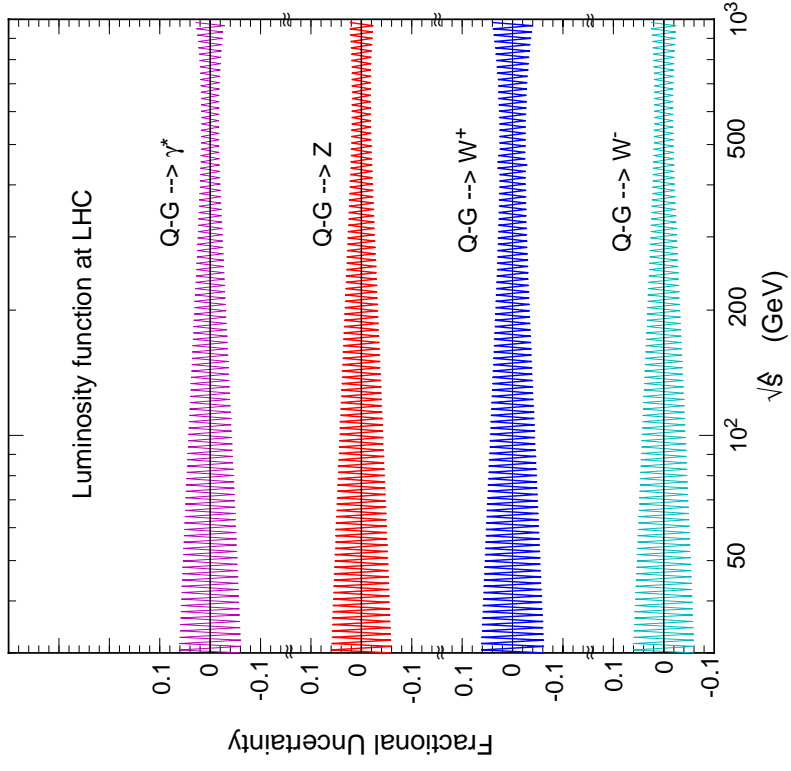
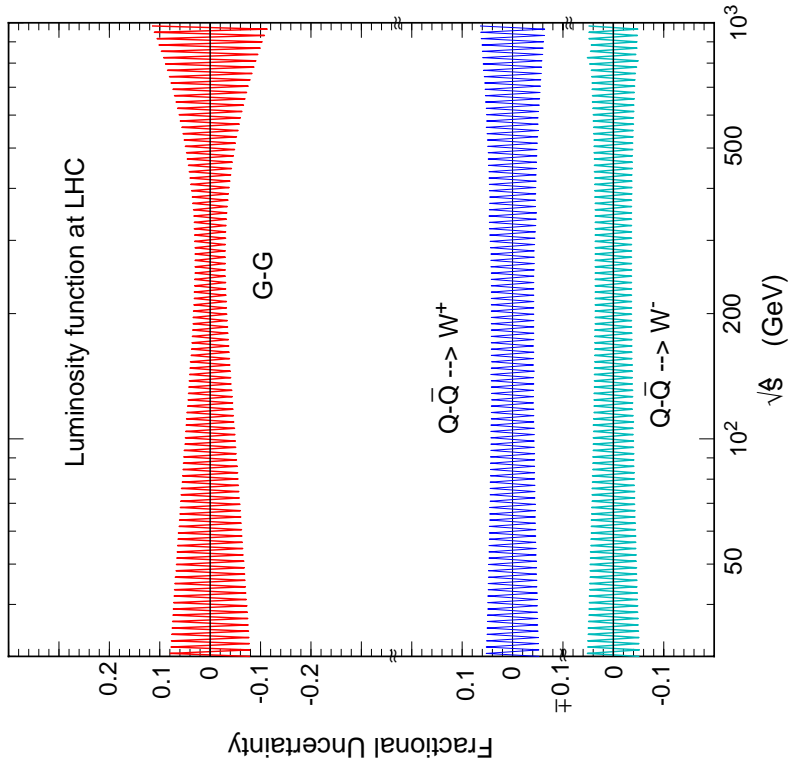
Uncertainty band for the gluon distribution function at $Q^2 = 10 \text{ GeV}^2$. The curves correspond to CTEQ5M1 (solid), CTEQ5HJ (dashed), and MRST2001 (dotted).

The gluon is very uncertain for $x \gtrsim 0.4$.
 [$g(x) \rightarrow 0$ as $x \rightarrow 1$.]



Uncertainties of the luminosity functions at the Tevatron.

$$\mathcal{L}(\hat{s}) = \sum_{i,j} C_{ij} \int f_i(x_1) f_j(x_2) \delta(\hat{s} - x_1 x_2 s) dx_1 dx_2$$

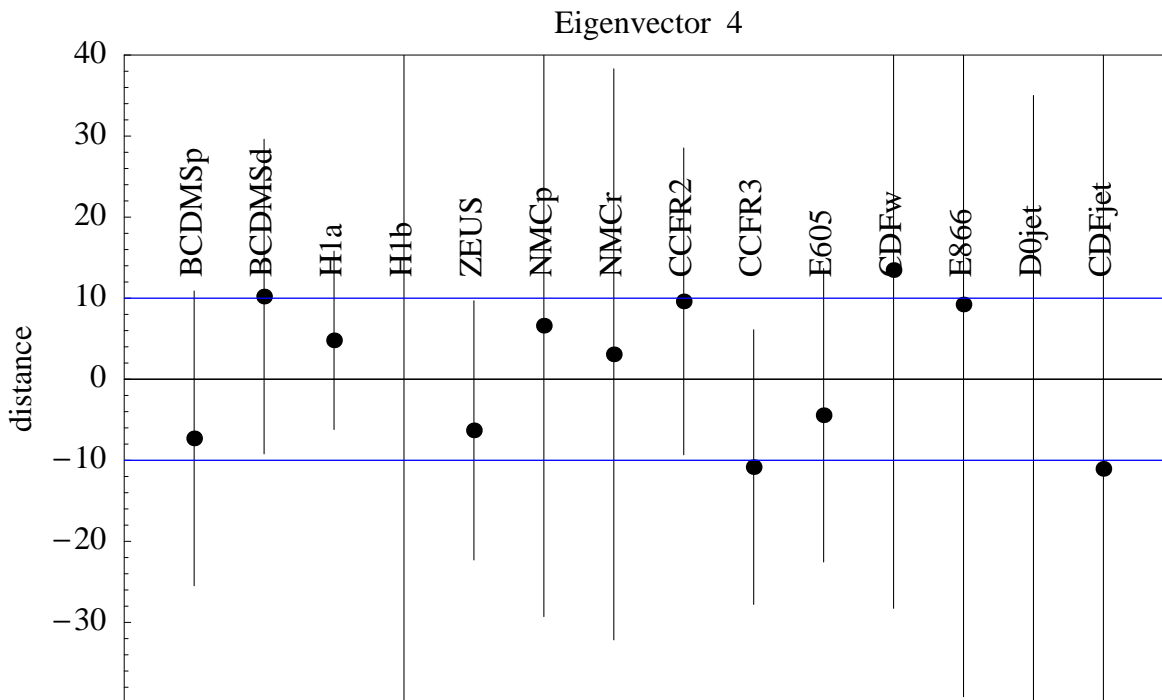


Uncertainties of the luminosity functions at the LHC.

A knotty problem

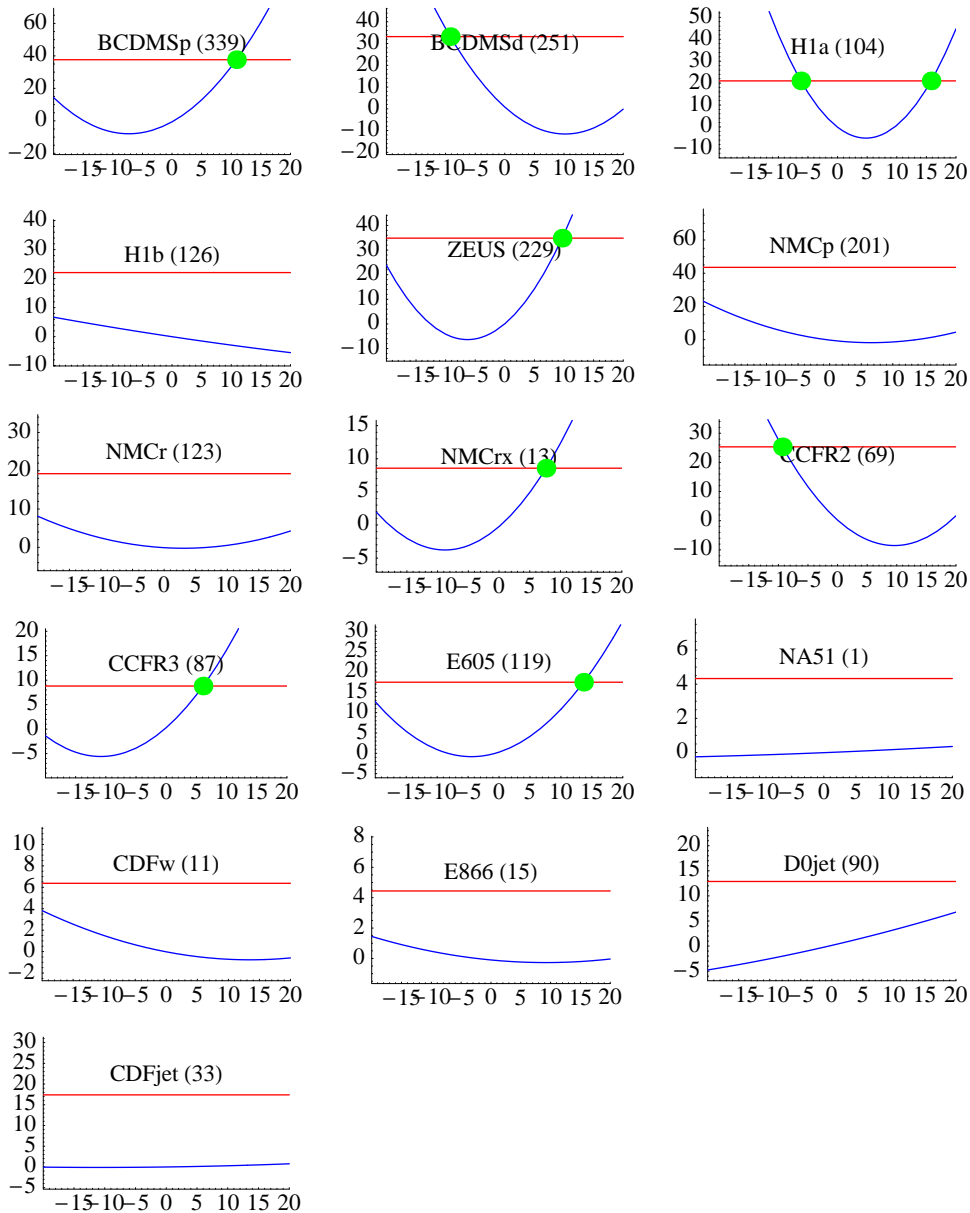
What's the tolerance?

Our analysis: Consider the χ^2 's of individual experiments, as any variable changes from its value at the global minimum.



Uncertainty ranges (vertical lines) for the input experiments along the Eigenvector 4 direction. The horizontal lines indicate the *tolerance* T . (Points: positions of minimum χ^2 for individual experiments)

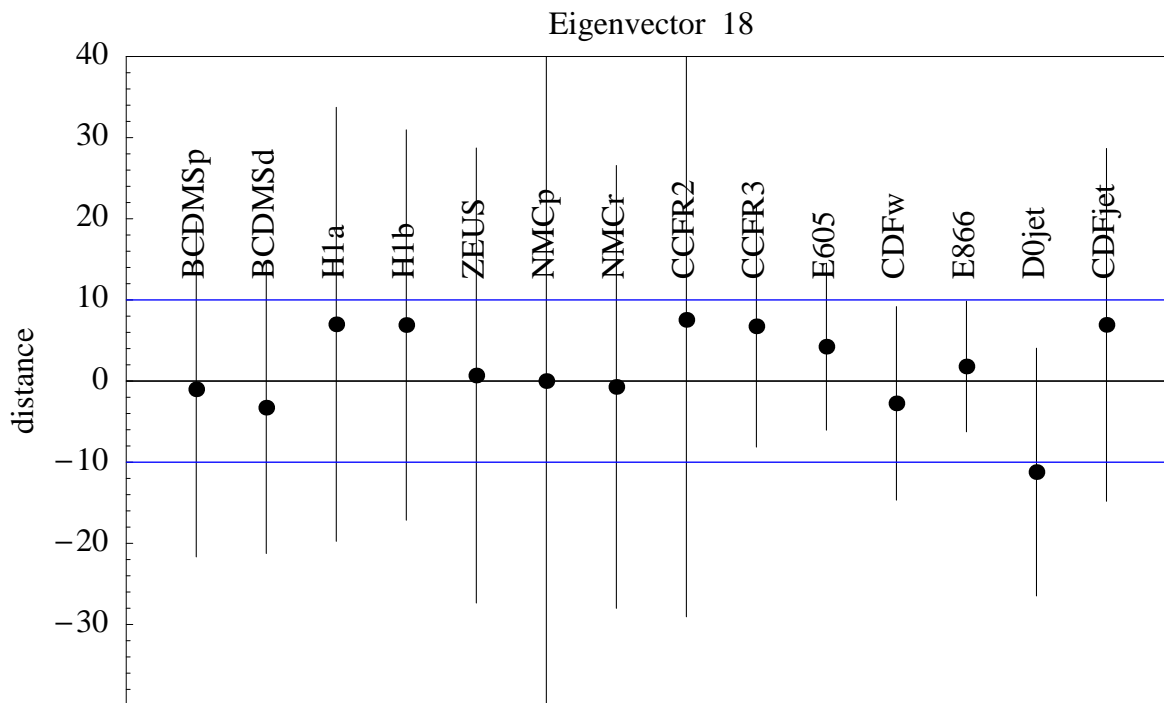
$$(\text{distance} = \sqrt{\Delta\chi_{\text{global}}^2})$$



$\Delta\chi^2$ of individual data sets versus distance along Eigenvector 4.

The red line is the 90% confidence range for χ^2_{scaled} .

$$(\chi^2_{\text{scaled}} \equiv \frac{\chi^2}{\chi_0^2})$$



Uncertainty ranges for the input experiments along the Eigenvector 18 direction.

A *reasonable* (as opposed to strict) tolerance criterion leading to $T \approx 10$:

take the *intersection* of the individual 90% confidence ranges ...

... but a single anomalous experiment cannot set the bound.

The uncertainties on predictions are $\propto T$.