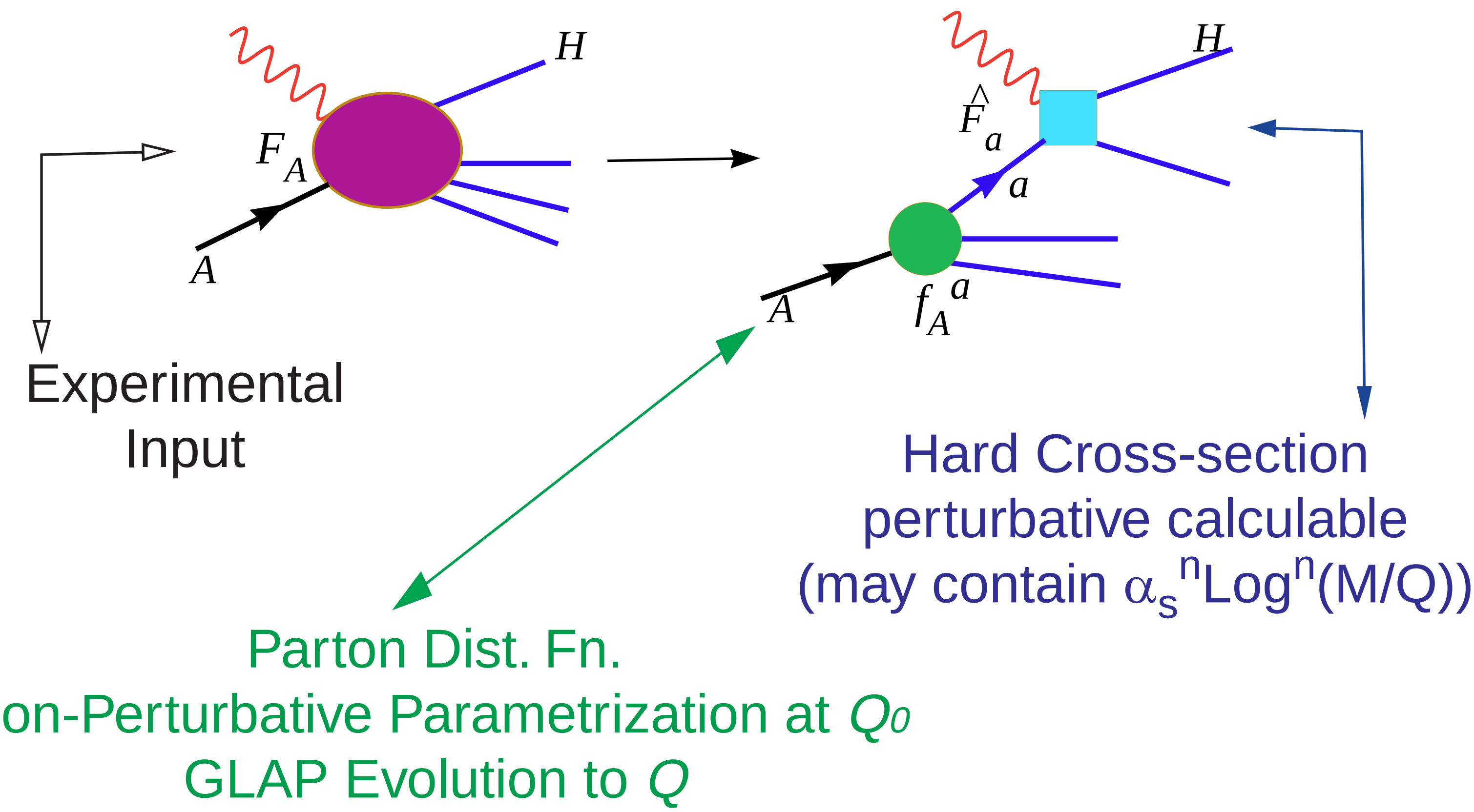
Global QCD Analysis in a Nutshell

Master Equation for QCD Parton Model
– the Factorization Theorem

$$F_A^\lambda(x, \frac{m}{Q}, \frac{M}{Q}) = \sum_a f_A^a(x, \frac{m}{\mu}) \otimes \hat{F}_a^\lambda(x, \frac{Q}{\mu}, \frac{M}{Q}) + \mathcal{O}((\frac{\Lambda}{Q})^2)$$

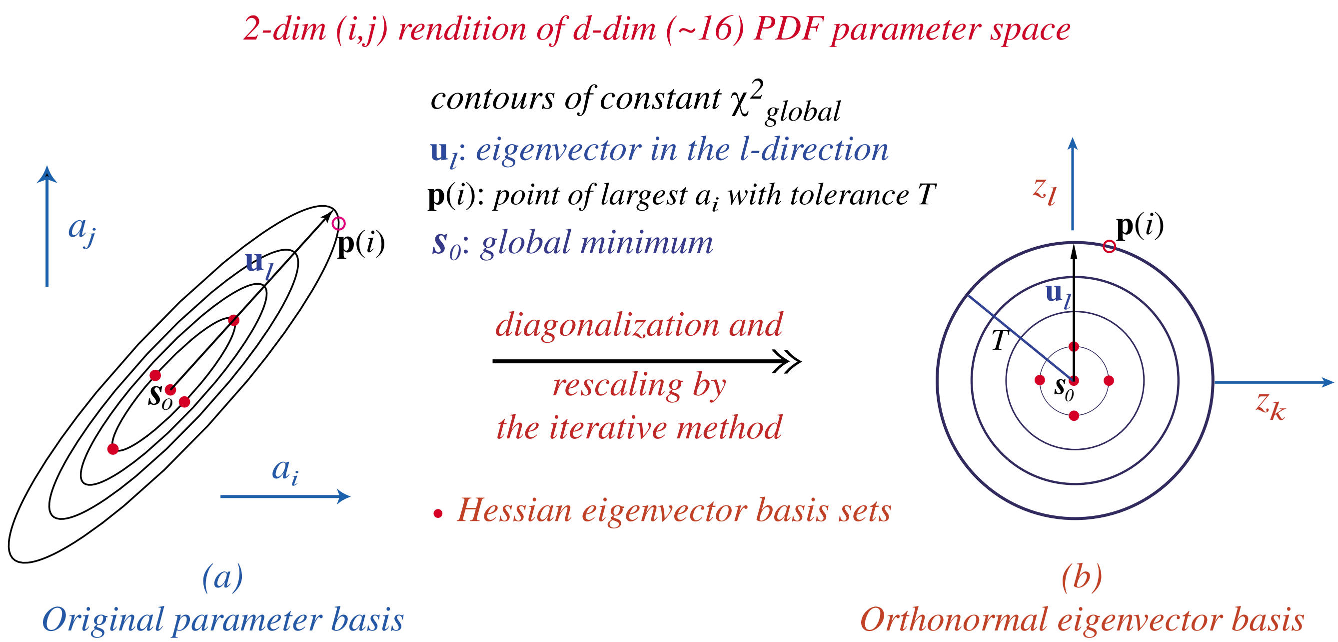


Sources of Uncertainties and Challenges:

- Experimental errors; (and uncertainties on errors!)
- Parametrization dependence;
- Higher-order corrections; Large Logarithms;
- Power-law (higher twist) corrections.

Optimized Sampling of PDF Parameter Space – "Alternate Hypotheses" in uncertainty study

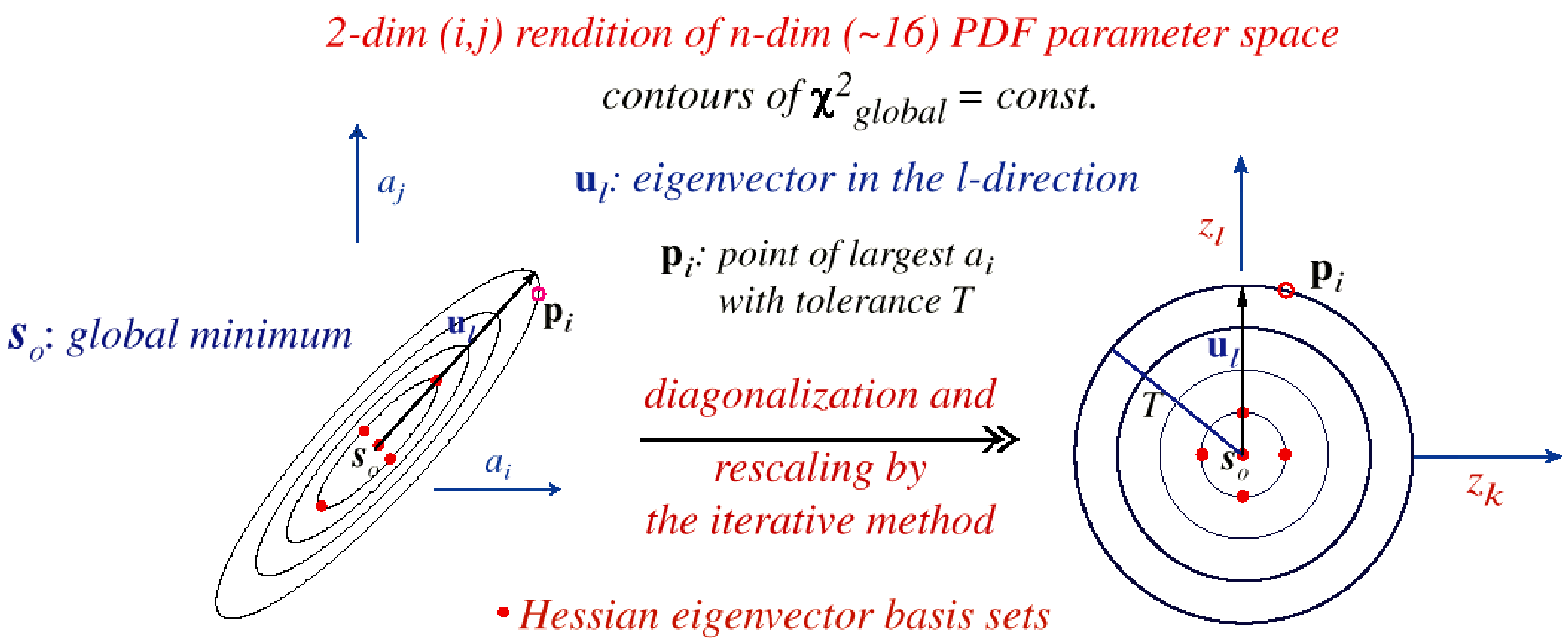
Hessian Matrix: (general but approx.)
Diagonalize the Hessian matrix calculated from χ_g^2, then move along each of the eigenvectors, i, to get up/down PDF sets $\{S_i^\pm\}$.



- $\Rightarrow$ PDF sets $\{S_i^\pm\}$, $i = 1, \ldots, n$, spanning the *full* PDF parameter space in the neighborhood of the global mimimum;
- The uncertainty of *any physical variable* X can be calculated as: (the master eq.)

$$\Delta X = \frac{T}{2t}\sqrt{\sum_i (X(S_i^+) - X(S_i^-))^2}$$

The Hessian Method of quantifying uncertainties by a complete set of orthonormal eigenvector PDFs



(a)
Original (physical) parameter basis

(b)
Orthonormal eigenvector basis

Iterative Method to generate Eigenvectors:
(and dramatically improve numerical reliability)

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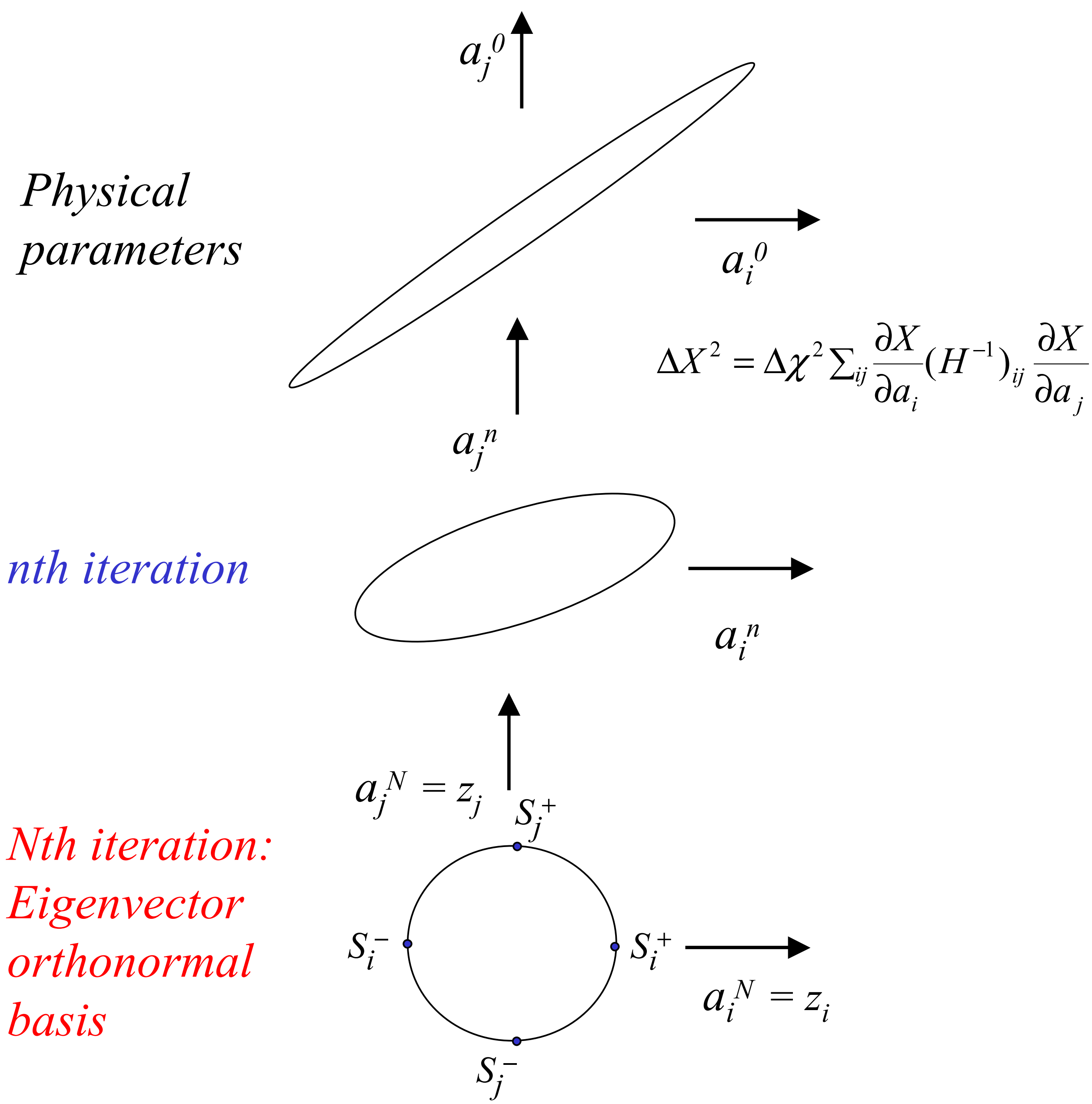
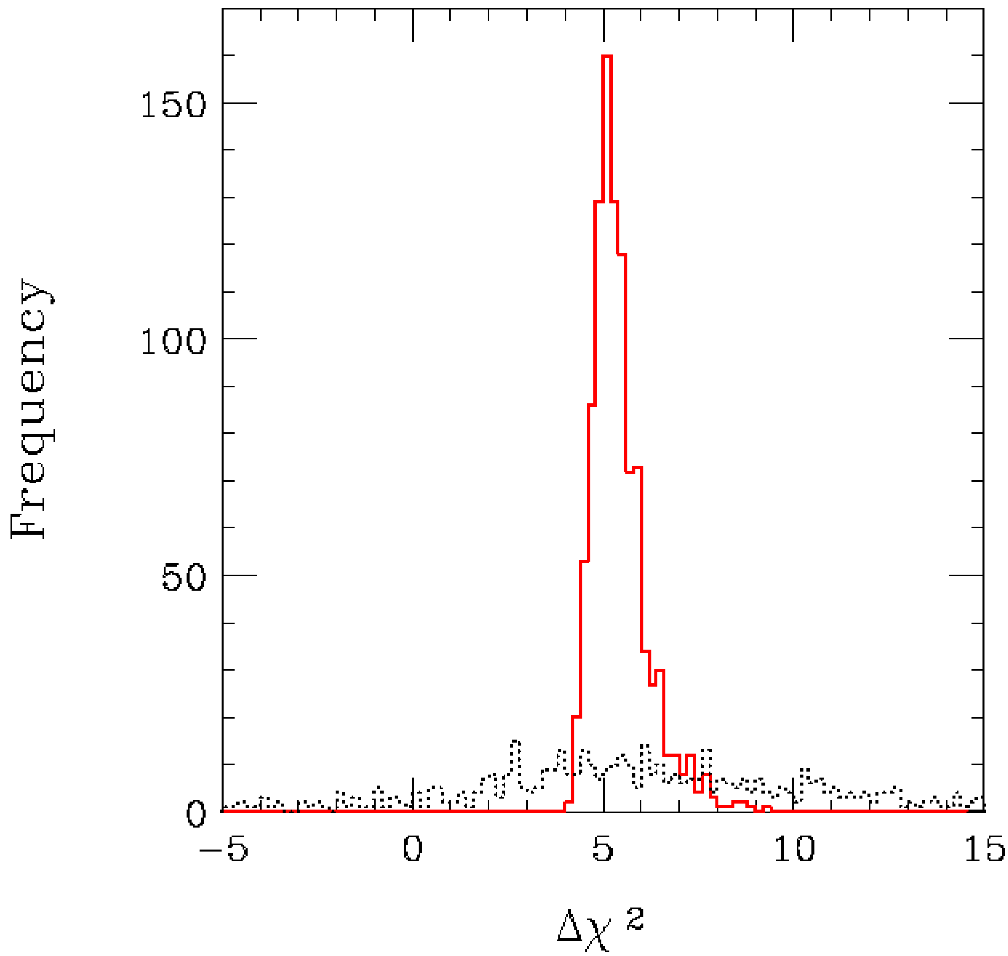
the $\chi^2 = const.$ ellipsoid



$$\Delta X^2 = \Delta\chi^2 \sum_i \left(\frac{\partial X}{\partial z_i}\right)^2 = \sum_i [X(S_i^+) - X(S_i^-)]^2$$



Frequency distribution of $\Delta\chi^2$ *according to the Hessian approximation for displacements in random directions for which the true value is* $\Delta\chi^2=5$*:*
Solid histogram: results from our iterative method;
Dotted histogram: results obtained from MINUIT.

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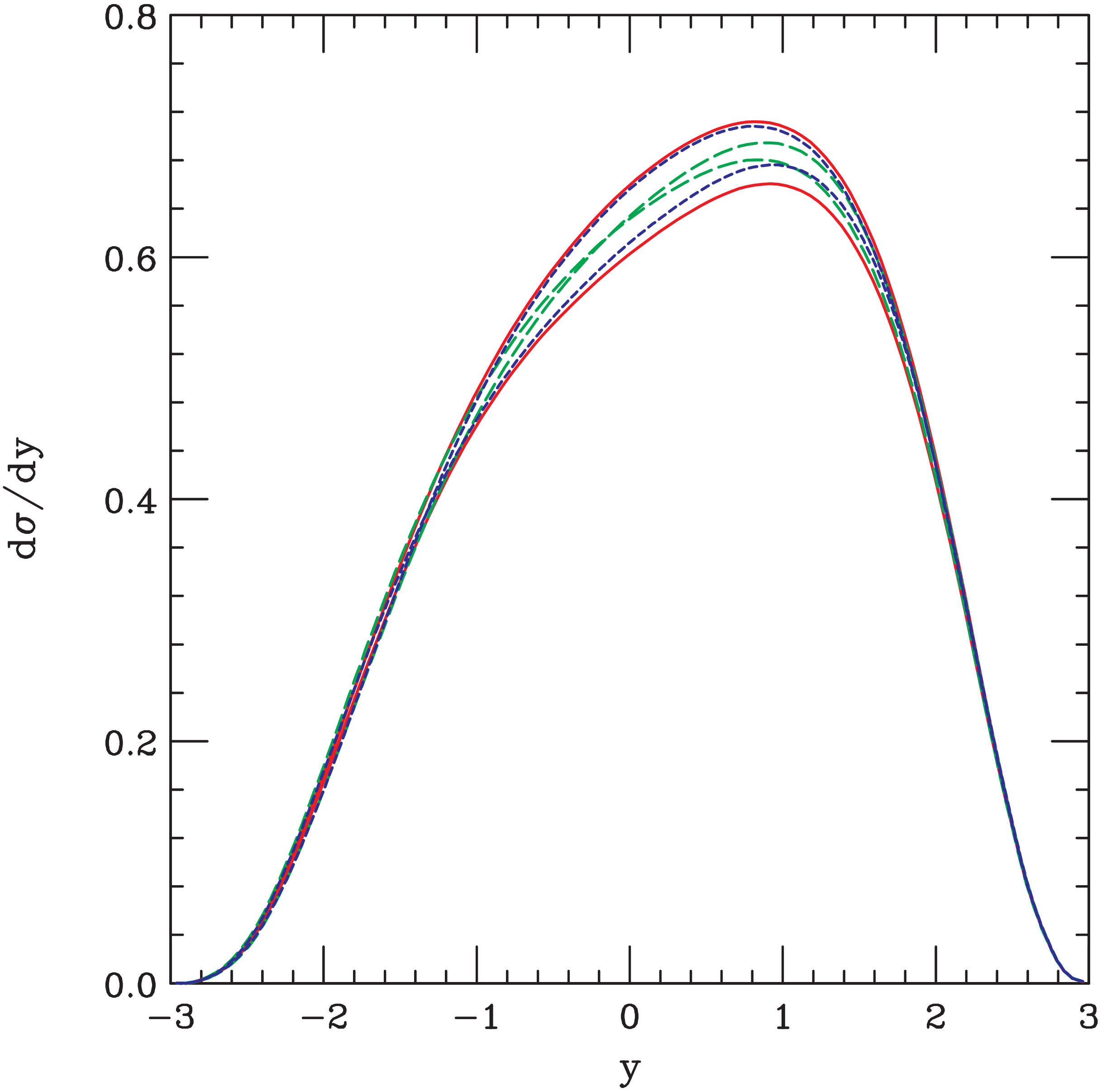
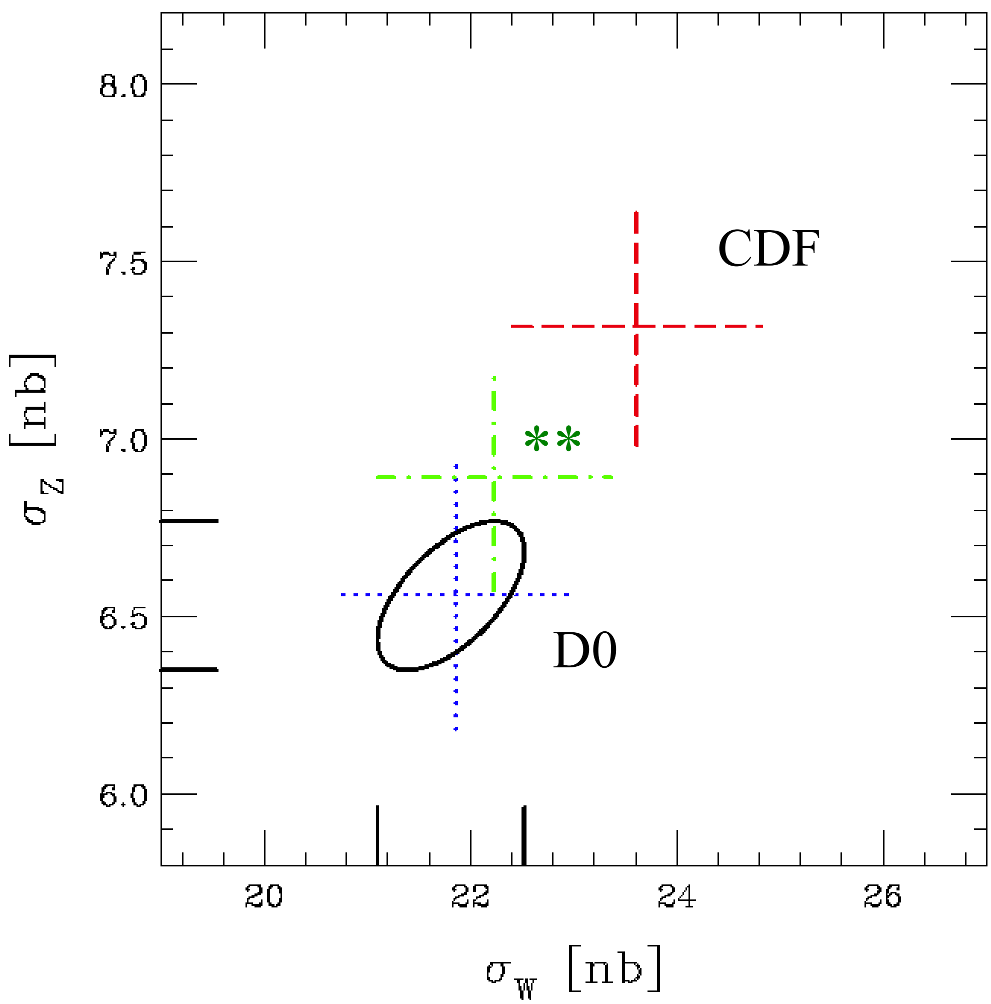
* This iterative method has been shown by Botje and Zomer to greatly improve their own global analyses, and it is now being considered for adoption in MINUIT (as an option) by its author F. James.

Predicted rapidity distribution for W production at the Tevatron



Six curves represent the alternate hypotheses of one $\pm\Delta\chi^2$ for the physical variables σ_t, $<y>$, and $<y^2>$.

Correlated uncertainties in W/Z production
Cross-section at the Tevatron



Ellipse : Allowed region with Tolerance = 10.

** CDF with the same luminosity input as D0