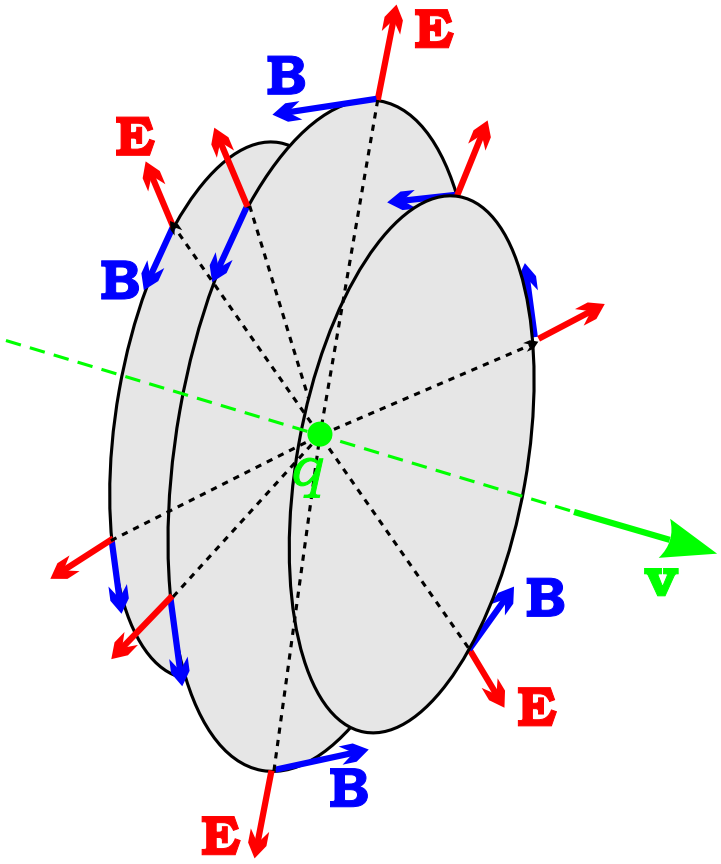
Electromagnetic Field of a Moving Charge

Suppose a charge q moves with constant velocity $\mathbf{v}$ in an inertial frame $\mathcal{F}$. What fields would be observed by an observer at rest in $\mathcal{F}$? In other words, what are the fields due to a moving charge?

If the velocity $\mathbf{v}$ is constant, then the fields can be derived by the Lorentz transformation from the rest frame of q. (In the rest frame, the only field is the Coulomb electric field.) The fields are derived in Sec. 12.5. The figure shows $\mathbf{E}$ and $\mathbf{B}$ schematically. $\mathbf{E}$ is directed away from the instantaneous position of q. $\mathbf{B}$ is directed around the trajectory of q.



Please do not overlook Exercise 12.26, in which the covariant field tensor is determined. The result is

$$F^{\mu\nu} = \frac{q}{4\pi\epsilon_0 c}\frac{r^{\mu\nu}}{(r^2)^{3/2}},$$

where

$$r^{\mu\nu} = \frac{1}{c}\left(\eta^\mu x^\nu - \eta^\nu x^\mu\right).$$

Here η^μ is the 4-velocity of q, and $r^2 = -r^{\mu\nu}r_{\mu\nu}/2$. Written covariantly, this very basic result in electrodynamics has a beautifully simple form.