7. Electric Current
Self-test answers

1. The power is $P = V^2/R$, so the resistance is

$$R = \frac{V^2}{P} = 0.9\,\Omega.$$

The current is $I = V/R = 3.33\,\text{A}$. Also, the resistance is $\rho\ell/A$, so the cross section of the wire must be

$$A = \frac{\rho\ell}{R} = 2.22 \times 10^{-9}\,\text{m}^2.$$

Finally, the current density is

$$J = \frac{I}{A} = \frac{3.33\,\text{A}}{2.22 \times 10^{-9}\,\text{m}^2} = 1.5 \times 10^9\,\text{A}\,\text{m}^2.$$

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2. Let ℓ be the length of the wire. The potential across the first half of the wire is $V_1 = IR$, and across the second half is $V_2 = 2IR$, so the potential across the wire is $3IR$. The electric field in the two halves of the wire is

$$\begin{aligned} E_1 &= \frac{V_1}{\ell/2} = \frac{2IR}{\ell}, \\ E_2 &= \frac{V_2}{\ell/2} = \frac{4IR}{\ell}. \end{aligned}$$

The surface charge density on the boundary is

$$\sigma = \epsilon_0 (E_2 - E_1) = \frac{2\epsilon_0 IR}{\ell}.$$

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3. The internal resistance r is in series with the load R, so the current is $I = \mathcal{E}/(R + r)$. The power dissipated in R is

$$P = I^2 R = \frac{\mathcal{E}^2 R}{(R + r)^2}.$$

To find the maximum P, set dP/dR equal to 0. The result is $R = r$, i.e., load resistance equal to internal resistance. Then the powers in the load and internal resistance are equal, so 50% of the power is dissipated in R.

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4. In the steady state, the electron velocity is the terminal velocity v_0 given, for the specified model, by

$$eE = \gamma v_0^2.$$

Then the current is $I = nev_0$ where $n =$electron density; and $V = E\ell$ where $\ell =$wire length. That is,

$$I = ne\sqrt{\frac{eE}{\gamma}} = C\sqrt{V},$$

where $C = ne^{3/2}\left(\gamma\ell\right)^{-1/2}$. The proposed model is not reasonable, because real materials obey Ohm's law, $I \propto V$, whereas the model has $I \propto V^{1/2}$.