Electromagnetism
G. L. Pollack and D. R. Stump

2. Vector Calculus
Chapter Summary

• The grad, div, and curl in Cartesian coordinates are

$$\begin{aligned}
\boldsymbol{\nabla} f &= \widehat{\mathbf{i}}\frac{\partial f}{\partial x} + \widehat{\mathbf{j}}\frac{\partial f}{\partial y} + \widehat{\mathbf{k}}\frac{\partial f}{\partial z} \\
\boldsymbol{\nabla} \cdot \mathbf{F} &= \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} \\
\boldsymbol{\nabla} \times \mathbf{F} &= \begin{vmatrix} \widehat{\mathbf{i}} & \widehat{\mathbf{j}} & \widehat{\mathbf{k}} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ F_x & F_y & F_z \end{vmatrix}
\end{aligned}$$

• The coordinate-independent definitions of grad, div, and curl are

$$\begin{aligned}
df &= \boldsymbol{\nabla} f \cdot d\mathbf{x} \\
\boldsymbol{\nabla} \cdot \mathbf{F} &= \lim_{dV \to 0} \frac{1}{dV} \oint \mathbf{F} \cdot d\mathbf{A} \\
\boldsymbol{\nabla} \times \mathbf{F} &= \lim_{dA \to 0} \frac{1}{dA} \oint \mathbf{F} \cdot d\boldsymbol{\ell}
\end{aligned}$$

• The grad, div, and curl in cylindrical or spherical coordinates are not simple generalizations of the Cartesian equations, because of scale factors. The formulas for these operators are in Tables 2.3 and 2.4.

• Two important integral identities are

$$\begin{aligned}
&\text{Gauss's theorem} \qquad \int_V \boldsymbol{\nabla} \cdot \mathbf{F} d^3x = \oint_S \mathbf{F} \cdot d\mathbf{A}, \\
&\text{Stokes's theorem} \qquad \int_S \boldsymbol{\nabla} \times \mathbf{F} \cdot d\mathbf{A} = \oint_C \mathbf{F} \cdot d\boldsymbol{\ell}.
\end{aligned}$$