2. Vector Calculus
Self-test questions

1. Calculate the divergence of the vector function $\mathbf{C}(\mathbf{x}) = \mathbf{x}/(r^2+a^2)^{3/2}$, where $r = |\mathbf{x}|$. Describe the tangent curves of $\mathbf{C}(\mathbf{x})$.

2. Let $\mathbf{F}(\mathbf{x}) = \widehat{\boldsymbol{\phi}} f(r)$, where (r, ϕ, z) are cylindrical coordinates and $f(r)$ is an arbitrary function of r. Calculate the line integral of $\mathbf{F}$ around a circle in the xy plane centered at the origin. Verify that Stokes's theorem is satisfied.

3. For the vector function $\mathbf{G}(\mathbf{x}) = \mathbf{x}$, verify that the flux of $\mathbf{G}$ through the unit sphere (centered at the origin) is equal to the volume integral of $\boldsymbol{\nabla} \cdot \mathbf{G}$.

4. Consider the vector function

$$\mathbf{F}(\mathbf{x}) = \mathbf{x} \times (\mathbf{c} \times \mathbf{x})$$

where $\mathbf{c}$ is a constant vector. Calculate the curl of $\mathbf{F}(\mathbf{x})$.

5. The electric field of a point dipole is described by the function

$$\mathbf{E}(\mathbf{x}) = \frac{3\mathbf{x}\,(\mathbf{p} \cdot \mathbf{x}) - \mathbf{p}r^2}{4\pi\epsilon_0 r^5}.$$

Let $\mathbf{p} = p_0\widehat{\mathbf{k}}$. Make a sketch of the field in the xz plane. For points on the x axis, indicate the direction of the vector $\mathbf{E}$. Do the same for points on the z axis, and on the lines $z = +x$ and $z = -x$. Finally, sketch the tangent curves, i.e., the field lines.