Three Ways to Calculate $\mathbf{E}(\mathbf{x})$

1. By direct integration The first method is to apply the basic equation

$$\mathbf{E}(\mathbf{x}) = \int d\mathbf{E} = \int \frac{(\mathbf{x} - \mathbf{x}')dq}{4\pi\epsilon_0 \left|\mathbf{x} - \mathbf{x}'\right|^3},$$

which expresses the superposition principle. The electric field at $\mathbf{x}$ is the integral of the contributions of elemental charge dq at $\mathbf{x}'$. This method is the simplest conceptually, but the most difficult mathematically. It is rarely the best method. See Examples 2 – 4 in Chapter 3.

2. By Gauss's law The integral form of Gauss's law is

$$\oint_S \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0}.$$

If the system has spherical, cylindrical, or planar symmetry, then the form of $\mathbf{E}(\mathbf{x})$ can be identified *a priori*, and the field can be determined by applying Gauss's law to an appropriate Gaussian surface. For systems with symmetry, this is the best method. See Examples 5 – 7 in Chapter 3.

3. From the potential The electric field is

$$\mathbf{E}(\mathbf{x}) = -\boldsymbol{\nabla} V$$

where $V(\mathbf{x})$ is the electrostatic potential. $V(\mathbf{x})$ may be calculated by integration over elemental charges (see Examples 8 – 10 in Chapter 3), or by solving a boundary-value problem. Then the electric field is calculated by differentiation. This method is very powerful, and is employed in many examples in Chapters 4 and 5.