3. Basic Principles of Electrostatics
Chapter Summary

Electrostatics is concerned with three functions, $\rho(\mathbf{x})$, $\mathbf{E}(\mathbf{x})$, and $V(\mathbf{x})$. To proceed to the next few chapters, it is necessary to comprehend the six relations among these three functions. For each pair of functions there is an integral formula and a partial differential equation.

(i) Density and Field:

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\int \frac{\widehat{\mathbf{r}}}{r^2}\rho dV \quad \text{and} \quad \boldsymbol{\nabla}\cdot\mathbf{E} = \rho/\epsilon_0.$$

The electric field is determined by the density by this integral formula. The partial differential equation, which must hold at every point in space, is Gauss's Law. These are not independent principles. Either one may be derived from the other.

(ii) Field and Potential:

$$V(\mathbf{x}) = -\int_{\mathbf{x}_0}^{\mathbf{x}} \mathbf{E}\cdot d\boldsymbol{\ell} \quad \text{and} \quad \mathbf{E} = -\boldsymbol{\nabla}V.$$

The reason a static $\mathbf{E}$ may always be written as a gradient, and that the line integral is independent of the path from $\mathbf{x}_0$ to $\mathbf{x}$, is that $\boldsymbol{\nabla}\times\mathbf{E} = 0$.

(iii) Potential and Density:

$$V = \frac{1}{4\pi\epsilon_0}\int \frac{\rho dV'}{r} \quad \text{and} \quad -\boldsymbol{\nabla}^2 V = \rho/\epsilon_0.$$

The integral formula for $V(\mathbf{x})$ is written in a concise form here; r is the distance $|\mathbf{x}-\mathbf{x}'|$ from the source point $\mathbf{x}'$ (where ρ is evaluated) to the field point $\mathbf{x}$ (where V is evaluated), and dV' is the volume element d^3x' at $\mathbf{x}'$. The local equation is Poisson's equation. The integral is Green's solution to Poisson's equation, discussed in Section 3.5.