3. Basic Principles of Electrostatics
Self-test questions

1. Find the electric field $\mathbf{E}(\mathbf{x})$ and potential $V(r)$ due to a uniformly charged sphere of radius a and total charge Q.

2. A semi-infinite uniformly charged wire is located on the negative z axis. Find the electric field on the positive z axis.

3. What is the charge distribution if the electric field is $\mathbf{E}(\mathbf{x}) = K\mathbf{x}/(r^2 + a^2)^{3/2}$?

4. A pointlike electric dipole is located at the origin and points in the $\widehat{\mathbf{k}}$ direction. The dipole moment is $\mathbf{p} = p_0\widehat{\mathbf{k}}$. Compare the electric field at the two points $(d, 0, 0)$ and $(0, 0, d)$.

5. Coulomb's law states that the electrostatic force between two point charges is inversely proportional to r^2. Give two examples in which this law is tested for atomic dimensions or smaller.

6. *A clever trick with superposition*

A hole of radius R whose center is at the origin is cut from the xy plane. The rest of the xy plane has constant surface charge density σ (measured in coulombs/meter^2).

(a) What is $\mathbf{E}$ on the z axis?

(b) Expand $E_z(0, 0, z)$ for $z \gg R$, and find the first two nonvanishing terms in the expansion.

For the next problem, computer graphics may help.

7. Four equal charges Q are located at the vertices of a square,

$$(a, a), (-a, a), (-a, -a), (a, -a).$$

Make a contour plot or a 3D surface plot of the potential in the xy plane. How much work is done in moving a test charge q from $(0, 0)$ to $(a, 0)$? Is there an energy barrier?