

## THE EXERCISE

A point charge  $q$  is located on the  $z$  axis at  $z = 2R$ . A grounded conducting sphere of radius  $R$  is centered at the origin. Then there is an image charge  $q' = -q/2$  on the  $z$  axis at  $z = R/2$ . What does the electric field look like?

The figure below shows the field lines (red and green curves) according to my calculation. Here are some interesting features:

- Field lines start at  $q$ . Some terminate on the sphere, normal to the surface. (In the figure I've shown them extending to the image charge inside the sphere, just so that we can see how the field of two point charges would look. But of course  $\mathbf{E} = 0$  inside a conducting sphere.) Other field lines extend to  $\infty$  because the net charge of the system is  $q/2$ .
- If the displayed field lines are uniformly distributed in angle at the charge  $q$  (the red curves) then one-half of the lines terminate on the sphere and the other half go to  $\infty$ . That's because the net charge is one-half of  $q$ .
- The interesting point marked P on the figure is the point where  $\mathbf{E} = 0$ . There is no field line through P, because the direction of the zero vector is undefined. I plotted the green curves to show how  $\mathbf{E}$  varies in the neighborhood of P.

It is a simple exercise to show that the position of P is  $z_0 = -(1 + 3/\sqrt{2})R$ . For negative  $z$  with  $z > z_0$  the electric field points in the direction of  $+\hat{\mathbf{k}}$ ; for  $z < z_0$  the direction of  $\mathbf{E}$  is  $-\hat{\mathbf{k}}$ .

- The picture on the cover of Pollack and Stump does not extend far enough in the negative  $z$  direction to show how the field lines separate at  $\infty$ . So in the cover picture there are two field lines that look like they might cross out of the range of the picture. In fact they will eventually separate as shown in the figure below.
- The field lines have nonzero curvature but that does not imply that  $\mathbf{E}$  has a nonzero curl. Imagine a closed loop anywhere on the figure; There is nothing about the figure that contradicts  $\oint \mathbf{E} \cdot d\mathbf{x} = 0$ .

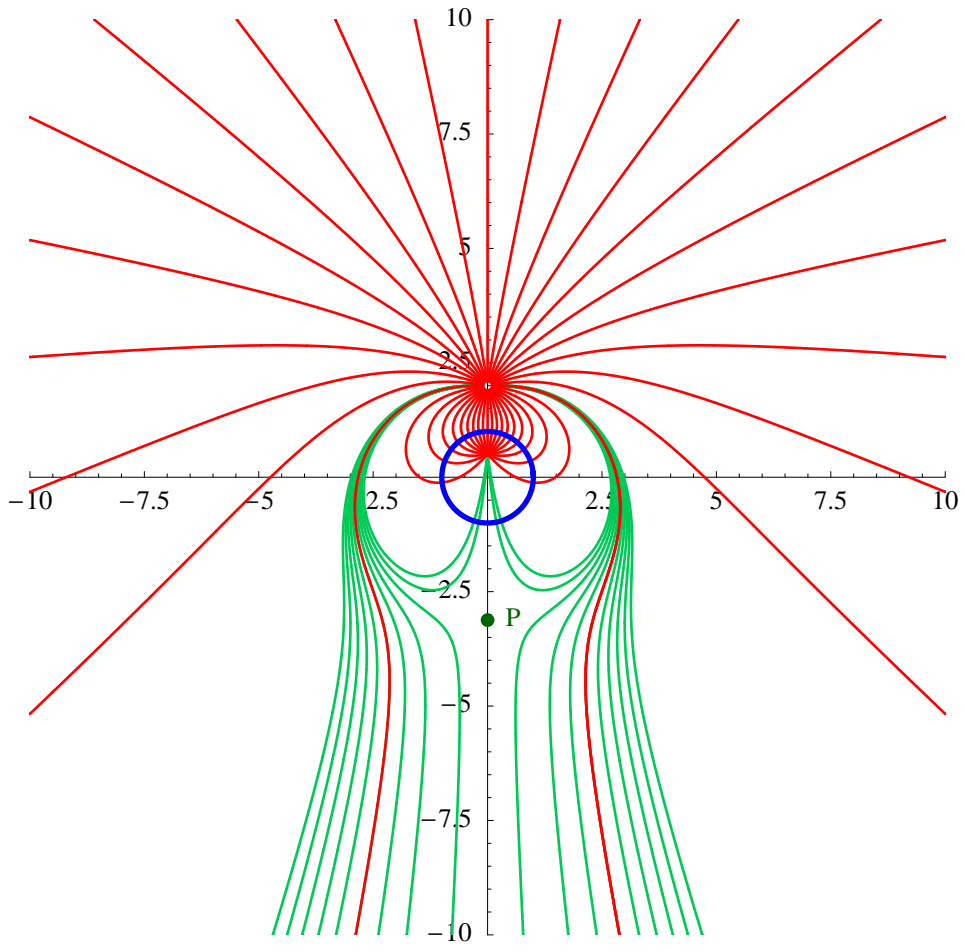


Figure 1: Electric field lines (red and green curves) of a point charge and grounded conducting sphere (in blue). There are no electric field lines inside the sphere (because  $\mathbf{E} = 0$ ) but the field lines to the fictitious image charge are shown to clarify the nature of the image charge solution.