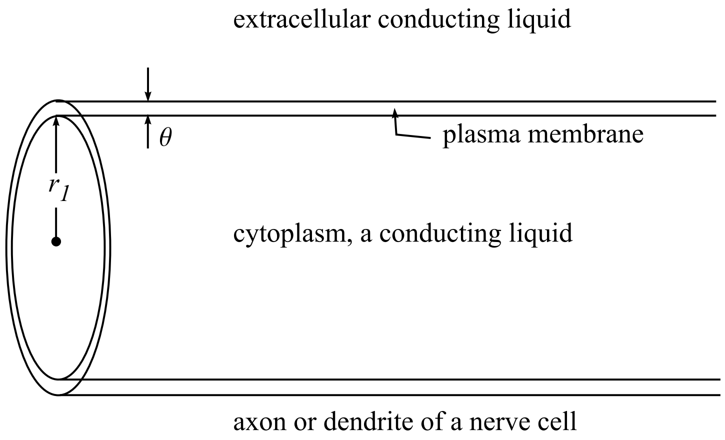
Capacitance in the Nervous System

Electrical signals propagate in nerve cells along axons and dendrites, which may be modeled as long cylinders of conducting fluid (cytoplasm), enclosed by a thin annular layer of dielectric (the plasma membrane), with another conducting fluid outside. The figure below shows a model of such a nerve process; we call the radius r_1, and the membrane thickness θ. In all real cells, $\theta \ll r_1$; for clarity in the figure the membrane thickness has been exaggerated. Propagation of signals along axons and dendrites is determined in part by the capacitance of cell membranes, so it is interesting to examine it in more detail. Neuroscientists have determined from experiment that the capacitance per unit area of cell membranes is typically $C''_m \approx 7\,\mathrm{mF/m^2}$. We consider the question: What is the effective dielectric constant of the material which makes up the plasma membrane?



The capacitance per unit length of a cylindrical capacitor whose inner and outer conductors have radii r_1 and r_2, respectively, is (see, for example, Exercise 6.30)

$$C' = \frac{2\pi k\epsilon_0}{\ln(r_2/r_1)} \tag{1}$$

where k is the dielectric constant of the material between the conductors. Referring now to the figure, we find for the capacitance per unit area of a membrane

$$C''_m = \frac{2\pi k\epsilon_0}{2\pi r_1 \ln\left[(r_1+\theta)/r_1\right]}. \tag{2}$$

The range of radii in the nervous system is very large; e.g., for a typical thin dendrite $r_1 \approx 0.5\,\mu\mathrm{m}$, the thinnest processes have $r_1 \approx 0.01\,\mu\mathrm{m}$, and for squid axons $r_1 \approx 0.5\,\mathrm{mm}$. Membrane thicknesses are rather uniform so that to a good approximation we can take $\theta \approx 3\,\mathrm{nm}$. If we then assume in Eq. (2) that $\theta/r_1 \ll 1$, and expand the logarithm as $\ln\left(1+\dfrac{\theta}{r_1}\right) \approx \dfrac{\theta}{r_1}$, we have the result

$$C''_m = \frac{k\epsilon_0}{\theta}\ \text{farads/meter}^2, \tag{3}$$

which is, surprisingly, independent of the radius. Putting in the appropriate numerical values for C''_m, ϵ_0, and θ, we obtain $k \approx 2$ for the dielectric constant of the membrane material.

The plasma membrane is actually a bi-lipid matrix with an as yet only partially understood, but wonderful, variety of properties and structures whose study is an active and important part of molecular and cell biology. What we have found is that for this simple model the effective dielectric constant of the membrane in living systems is essentially the same as that of other nonpolar insulators such as octane, decane, or Teflon.

Reference

M. J. Zigmond, *et al.*, *Fundamental Neuroscience* (Academic Press, San Diego, CA, 1999).