

6. Electrostatics and Dielectrics

Chapter Summary

- A single atom or molecule responds to an electric field by becoming polarized, with dipole moment $\mathbf{p} = \alpha\mathbf{E}$. α is called the polarizability.

Polarization occurs by two mechanisms: (1) displacement of charge within the atom or molecule; (2) orientation of a molecule that has a permanent dipole moment.

- The polarization field $\mathbf{P}(\mathbf{x})$ in a dielectric is the dipole moment density,

$$\mathbf{P}(\mathbf{x}) = \frac{1}{\delta V} \sum_{i=1}^{\delta N} \mathbf{p}_i.$$

- For a polarized dielectric, the volume density of bound charge is $\rho_b(\mathbf{x}) = -\nabla \cdot \mathbf{P}$. The surface density is $\sigma_b = \hat{\mathbf{n}} \cdot \mathbf{P}$.
- The displacement field $\mathbf{D}(\mathbf{x})$ is defined by $\mathbf{D} = \epsilon_0\mathbf{E} + \mathbf{P}$. Its field equation is Gauss's Law,

$$\nabla \cdot \mathbf{D} = \rho_{\text{free}}.$$

- The constitutive equation for an isotropic linear dielectric may be written as $\mathbf{D} = \epsilon\mathbf{E}$, or $\mathbf{P} = \chi_e\epsilon_0\mathbf{E}$. The dielectric constant κ is ϵ/ϵ_0 .
- The Clausius-Mossotti formula (6.39) relates the dielectric constant κ to the atomic polarizability α .
- A capacitor filled with a dielectric has capacitance $C = \kappa C_{\text{vacuum}}$ where C_{vacuum} is the capacitance without the dielectric.
- **Boundary conditions.** The tangential components of \mathbf{E} are continuous at any surface, e.g., the boundary surface between two dielectrics. The normal component of \mathbf{D} is continuous at a surface with no free surface charge; if there is a free surface charge density σ_{free} then $D_{2n} - D_{1n} = \sigma_{\text{free}}$.