8. Magnetostatics
Self-test answers

1. (a) See Example 1 in Section 8.3.

(b) See Example 3 in Section 8.4.

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2. See Example 6 in Section 8.4.

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3. See Example 5 in Sec. 8.4, where the field is determined by Ampère's law. If the slab is parallel to the xy plane, and $\mathbf{J}(\mathbf{x}) = J_0\widehat{\mathbf{i}}$ in the slab, then

$$\mathbf{B}(\mathbf{x}) = \mp\frac{1}{2}\mu_0 J_0 \delta \widehat{\mathbf{j}} \quad \text{for points} \begin{cases} \text{above} \\ \text{below} \end{cases} .$$

For points inside the slab, B_y varies linearly with z, from $\frac{1}{2}\mu_0 J_0 \delta$ at the lower surface to $-\frac{1}{2}\mu_0 J_0 \delta$ at the upper surface.

It is interesting to verify the PDE for Ampère's law. Inside the slab,

$$\nabla \times \mathbf{B} = \begin{vmatrix} \widehat{\mathbf{i}} & \widehat{\mathbf{j}} & \widehat{\mathbf{k}} \\ \partial_x & \partial_y & \partial_z \\ 0 & B_y & 0 \end{vmatrix} = -\widehat{\mathbf{i}}\frac{\partial B_y}{\partial z} = \mu_0 J_0 \widehat{\mathbf{i}}.$$

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4. The magnetic moment is $\mathbf{m} = IA\widehat{\mathbf{k}}$, where A is the area of the loop, ℓ^2. Asymptotically the magnetic field may be approximated by the dipole field, (8.78) or (8.80). In spherical coordinates,

$$\mathbf{B}(\mathbf{x}) = \frac{\mu_0 m}{4\pi r^3}\left[2\widehat{\mathbf{r}}\cos\theta + \widehat{\boldsymbol{\theta}}\sin\theta\right].$$

(a) On the x axis, θ is $\pi/2$, $\widehat{\mathbf{r}}$ is $\widehat{\mathbf{i}}$, and $\widehat{\boldsymbol{\theta}}$ is $-\widehat{\mathbf{k}}$. There,

$$\mathbf{B}(x, 0, 0) = -\frac{\mu_0 m}{4\pi x^3}\widehat{\mathbf{k}}.$$

(b) On the positive z axis, θ is 0, $\widehat{\mathbf{r}}$ is $\widehat{\mathbf{k}}$ and $\widehat{\boldsymbol{\theta}}$ is $\perp \widehat{\mathbf{k}}$. There,

$$\mathbf{B}(0, 0, z) = +\frac{\mu_0 m}{2\pi z^3}\widehat{\mathbf{k}}.$$

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5. It is important to use relativistic equations, because the speed is nearly c. The equation of motion is

$$\frac{d\mathbf{p}}{dt} = e\mathbf{v} \times \mathbf{B}.$$

For circular motion, $\mathbf{p}$ is tangential and $d\mathbf{p}/dt$ is centripetal. Therefore

$$\mathbf{p}(t) = p_0 \left(-\sin \omega t \, \widehat{\mathbf{i}} + \cos \omega t \, \widehat{\mathbf{j}} \right)$$

where the equation if motion implies

$$\omega p_0 = evB.$$

Now substitute $v = R\omega$, and $p_0 \approx E/c$ where $E = 1\,\mathrm{TeV}$ is the energy (the mass energy may be neglected). The result is

$$B = \frac{E}{eRc} = 3.3\,\text{tesla}.$$