

# Electromagnetism

Daniel R. Stump

Michigan State University

**Electromagnetism** is the science of electric and magnetic fields and of the interactions of these fields with electric charges and currents.

## I Introduction

Electromagnetism is one of the fundamental interactions of nature. Its physical origin lies in a property possessed by elementary particles of matter—electrons and protons—called electric charge. The electromagnetic interaction also governs light and other forms of radiation. Electricity and magnetism appear in many natural phenomena, and are applied in many inventions of technology and everyday life. The purpose of this article is to describe the basic science and applications of electromagnetism.

The earliest discoveries of electric and magnetic forces were made by philosophers of ancient Greece. They observed that when amber is rubbed with animal fur, it acquires the ability to attract small bits of reed or feathers. This small effect was the first observation of static electricity. (The Greek word for amber, *ελεκτρον*, is the origin of our word “electric.”) They also observed that a lodestone exerts a force on iron—the first example of magnetism. (The Greek province of Magnesia, where the iron ore magnetite occurs naturally, is the origin of our word “magnetic.”) These discoveries of weak and mysterious forces were the first steps toward our scientific understanding of electromagnetism. Today science has dispelled much of the mystery of electricity and magnetism, and created technological power beyond the dreams of the ancient philosophers.

## II Electric charge and current

### II.1 Electrostatics and electric charge

Electric charge is a property of matter. At the most basic level, the constituents of atoms are charged particles—electrons with negative charge ( $-e$ ) and protons with positive charge ( $+e$ ). An atom has equal numbers of electrons and protons so it is electrically neutral. However, a sample of matter becomes electrically charged if the balance between electrons and protons is broken. For example, when amber is rubbed with fur, electrons are transferred from the fur to the amber; the amber then has net negative charge.

Like charges repel and unlike charges attract. That is, if two samples of matter have net positive charge (or both have net negative charge) they exert equal but opposite repulsive forces on one another. If the samples have unlike charges, one positive and the other negative, then each exerts an attractive force on the other. The strength  $F$  of the electric force  $\mathbf{F}$  was measured accurately by Charles Augustin de Coulomb.<sup>1</sup> The force is proportional to the product of the charges,  $q_1$  and  $q_2$ , and inversely proportional to the square of the distance of separation  $r$ ,

$$F = \frac{kq_1q_2}{r^2} \quad \text{where} \quad k = 8.99 \times 10^9 \text{ Nm}^2/\text{C}^2.$$

This simple mathematical formula has been tested to extreme precision.<sup>2</sup> It forms part of the foundation for the theory of electromagnetism.

### II.2 Magnetostatics and electric current

Everyone has observed magnets and their forces, which can be quite strong even in small magnets. Every magnet has polarity—north and south poles. Two magnets repel if their north poles approach one another, and repel if the south poles approach, but attract if north approaches south. Magnets have a special attraction to iron; if either pole is placed near an iron object, a magnetic force pulls

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<sup>1</sup>Vectors are indicated in bold face, scalars in plain face.

<sup>2</sup>The dimensional units for the constant  $k$  are  $\text{Nm}^2/\text{C}^2$ , where N = newton = unit of force, m = meter = unit of length, and C = coulomb = unit of charge. The proton charge is  $e = 1.602 \times 10^{-19}$  C.

the magnet and the object toward one another. Science has identified the origins of magnetic forces. Technologies based on magnetic forces are used every day throughout the world.

Magnetism is very closely connected to the electric charge of subatomic particles. The most familiar example of a magnet is a ferromagnet—a piece of magnetized iron.<sup>3</sup> However, a ferromagnet is not the only source of magnetism nor even the most basic. Electric currents also produce magnetic forces, and in some ways the magnetic field associated with electric current is the most basic form of magnetism.<sup>4</sup>

### II.2.1 Electric current as a source of magnetic field

An electric current is a stream of electrically charged particles (of one sign) moving in the same direction. The current may be constant in time (DC, or direct current), oscillating in time with a constant frequency (AC, or alternating current), or varying in time. Currents can exist in metals and in several other forms of conducting matter. In a metal, some electrons occupy states that extend over large distances, i.e., not bound to a single atomic core. If an electric force is applied then these conduction electrons will move in response, creating an electric current. (Ohm's law,  $V = IR$  where  $V$  = potential difference in volts,  $I$  = current in amps, and  $R$  = resistance in ohms, expresses quantitatively the relation between the electric force and the current.) In metals the positively charged atomic nuclei are fixed in a crystalline lattice, so the electric current is due to motion of electrons.

The first observation of the magnetic field associated with an electric current was an accidental discovery by Hans Christian Oersted during a public lecture in 1819. The current existed in a metal wire connected across a battery. Oersted noticed that a nearby compass needle deflected while the current was flowing. The strength of the magnetic field at a distance of 1 centimeter from a 1 ampere current is  $2 \times 10^{-5}$  tesla, comparable to the Earth's magnetic field of approximately  $5 \times 10^{-5}$  tesla.

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<sup>3</sup>Cobalt and nickel are also ferromagnetic elements but iron is the most common example.

<sup>4</sup>The term "magnetic field" used in this section refers to any magnetic effect. The more technical meaning of the term is explained in Sec. III.

Oersted's discovery was studied in detail by Jean Marie Ampère. The magnetic field can be determined quantitatively by measuring the force on a pole of a magnet or the torque on a compass needle. A compass needle in equilibrium points in the direction of the field vector at the position of the compass. Therefore a compass can be used to map the field directions. Ampère found that the field direction “curls around” the current. Figure 1 shows a segment of current-carrying wire and the associated magnetic field encircling the wire. **Fig. 1** The field directions follow the right-hand rule: With the thumb of your right hand pointing along the wire in the direction of the current, the fingers naturally curl in the direction of the magnetic field around the wire. The theory of the magnetic field created by an electric current is called Ampère's law.

Electric currents can also exist in materials other than metals, such as plasmas and ionic solutions. The associated magnetic fields may be measured for scientific purposes. For example, the current in a neuron in the brain creates a magnetic field that is measured in magnetoencephalography. Or, a lightning strike (current in an ionized path through the atmosphere) creates a magnetic field that may be measured to study the properties of lightning. The magnetic field of the Earth is another natural example of a field produced by an electric current. The core of the Earth is highly metallic and at high temperature and pressure. Electric currents in this metal core create the Earth's magnetism, which we observe at the surface of the Earth.

**Electromagnets.** Ampère's law is applied in electromagnets. The magnetic field due to a straight length of current-carrying wire is weak. However, if the wire is wound around a cylinder, making a coil with many turns as illustrated in Fig. 2, then the field inside and near the ends of the cylinder can be strong for a practical current. **Fig. 2** The cylindrical coil is called a solenoid. The field strength can be increased by putting an iron core in the cylinder. Unlike a permanent ferromagnet, the field of an electromagnet can be turned on and off by an electric switch. Electromagnets are commonly used in electric motors and relay switches.

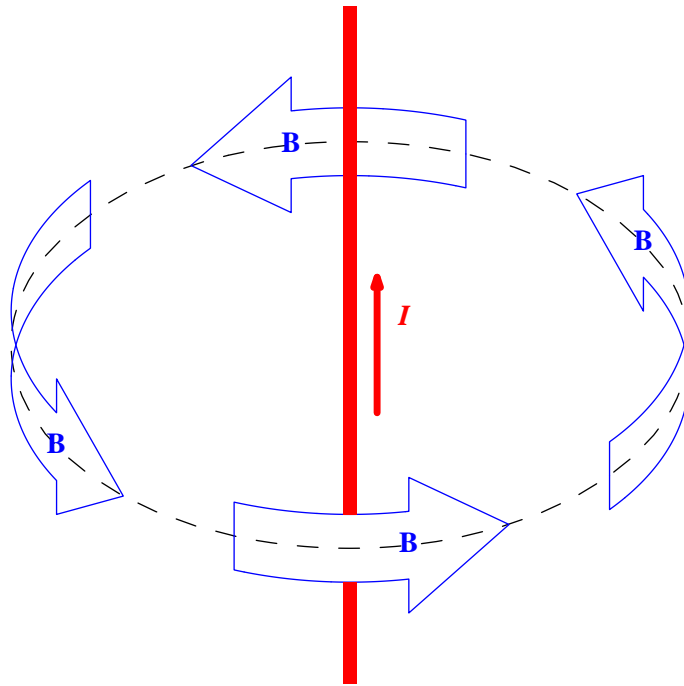


Figure 1: The magnetic field  $\mathbf{B}$  curls around the current  $I$ . The dashed curve indicates an imaginary circle around the wire segment. A compass needle placed near the wire will point in the direction of the field.

## II.2.2 The magnetic force on an electric current

A magnetic field, whether produced by a ferromagnet or by an electric current, exerts a force on any electric current placed in the field. Indeed there exists a magnetic force on any moving charged particle that moves across the magnetic field vectors. So the interaction between electric charges in motion and a magnetic field has two aspects: (i) an electric current creates a magnetic field (Ampère's law); (ii) a magnetic field exerts a force on an electric current.

**The electric motor.** The magnetic force on an electric current is the basis for all electric motors. The magnetic field may be produced by a permanent magnet (in small DC motors) or by an electromagnet (in large DC and AC motors). The current flows in a rotating coil of wire, and may be produced by a battery or some other source of electromotive force. Many practical designs have been invented, with the common feature that a magnetic force acts on the current-carrying coil, in opposite directions on opposite sides of the coil, creating a torque that drives the rotation of the coil. A co-rotating shaft attached to the coil is then connected to a mechanical system to do useful work.

## III The field concept

In field theory, electric and magnetic forces are described as the effects of electric and magnetic fields. The theory of electromagnetic phenomena is entirely based on the field concept.

Any electrically charged particle  $q$  creates an associated electric field  $\mathbf{E}$ . The field extends throughout the space around  $q$ , and varies with the position  $\mathbf{x}$  in space. If the particle is at rest (in some frame of reference) then its field is independent of time. In a system with many charges, the full electric field is the sum of the electric fields of the individual charges. Thus the electric field in an electrostatic system, denoted mathematically by  $\mathbf{E}(\mathbf{x})$ , is a function of position  $\mathbf{x}$  and depends on the locations and charge strengths of all the charged particles.

The electric field extends throughout a volume of space. It exerts a force on any charge in

the space. To consider the field concept, suppose two charged particles,  $q_1$  and  $q_2$ , are located at positions  $\mathbf{x}_1$  and  $\mathbf{x}_2$ , respectively. (See Fig. 3.) The electric field at an arbitrary point  $\mathbf{x}$  is

$$\mathbf{E}(\mathbf{x}) = \frac{kq_1}{r_1^2}\mathbf{e}_1 + \frac{kq_2}{r_2^2}\mathbf{e}_2$$

where  $r_1$  is the distance from  $q_1$  to  $\mathbf{x}$ ,  $\mathbf{e}_1$  is the unit vector in the direction from  $q_1$  to  $\mathbf{x}$ , and  $q_2$  and  $\mathbf{e}_2$  are the analogous quantities for  $q_2$ . A small test charge  $q$  placed at  $\mathbf{x}$  will experience a force  $\mathbf{F} = q\mathbf{E}(\mathbf{x})$ . Since  $\mathbf{E}(\mathbf{x})$  is the sum of the fields due to  $q_1$  and  $q_2$ , the force  $\mathbf{F}$  on the test charge is the sum of the two forces exerted on  $q$ . The charges  $q_1$  and  $q_2$  also experience forces due to the presence of each other. For example, the force on  $q_2$  due to  $q_1$  is  $q_2\mathbf{E}_1(\mathbf{x}_2)$  where  $\mathbf{E}_1$  is the field due to  $q_1$  alone. The field due to a charged particle is inversely proportional to the square of the distance from the particle, so the force between charges obeys the inverse-square law observed by Coulomb.

In field theory, the force on a charged particle  $q$  is attributed to the field  $\mathbf{E}(\mathbf{x})$  created by the other charges. Thus the field concept is significantly different from “action at a distance.” The force on the particle, equal to  $q\mathbf{E}(\mathbf{x})$ , is exerted by the field at the position of  $q$ , rather than by direct actions of the distant charges. In other words, an electrostatic system consists of two physical entities: a set of charged particles and an electric field  $\mathbf{E}(\mathbf{x})$ . The field is just as real as the particles.

Figure 3 illustrates the electric field for a system of two charged particles with equal but opposite charges. **Fig. 3** The curves in Fig. 3, called the electric field lines, represent the field. The electric field is a vector at each point throughout the space around the charges. A positive test charge  $q$  in the field would experience a force in the direction of the field vector at its position. We visualize the field by drawing the field lines, which are curves that are everywhere tangent to the field vector directions.

Electric charges at rest create an electric field  $\mathbf{E}(\mathbf{x})$ . Ferromagnets and electric currents create another field—the magnetic field  $\mathbf{B}(\mathbf{x})$ . Figure 4 illustrates the magnetic fields for two elementary current sources: (a) a small current loop and (b) a long current-carrying wire. **Fig. 4** In a ferromagnet the atoms behave as small current loops with a uniform orientation. The combined atomic fields make up the field of the magnet.

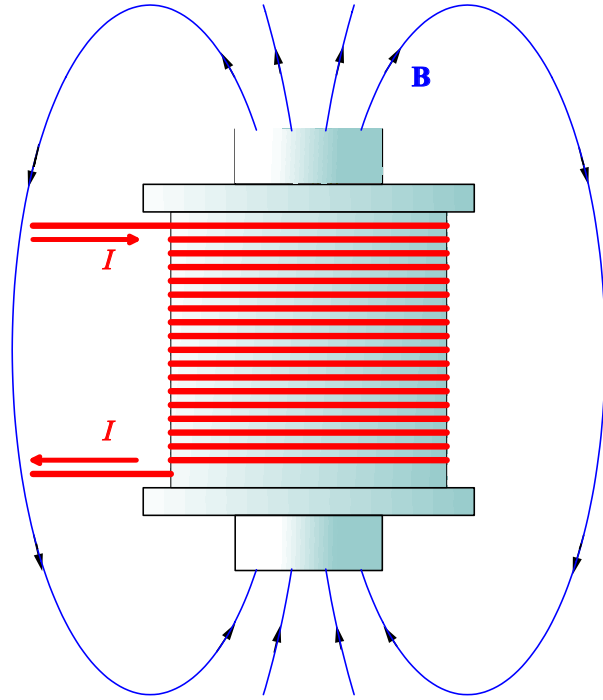


Figure 2: An electromagnet.

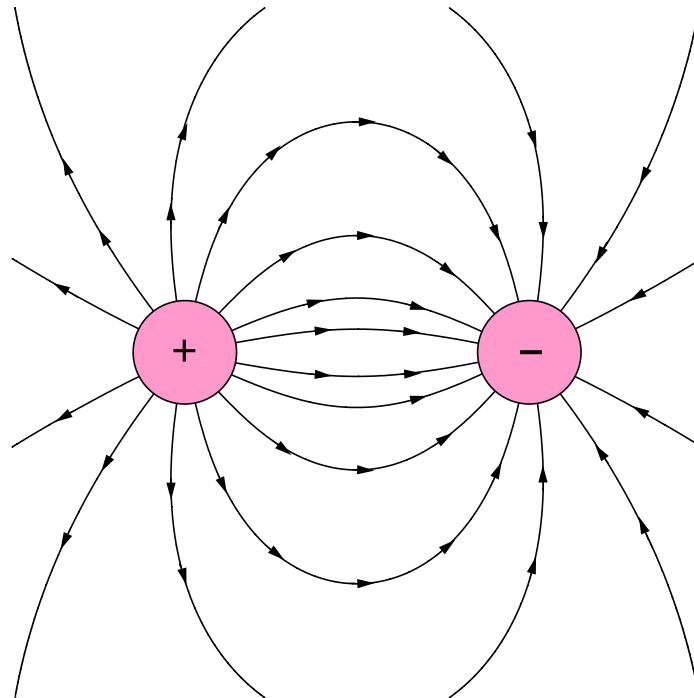


Figure 3: The electric field lines of a system of two charges with equal but opposite charge.

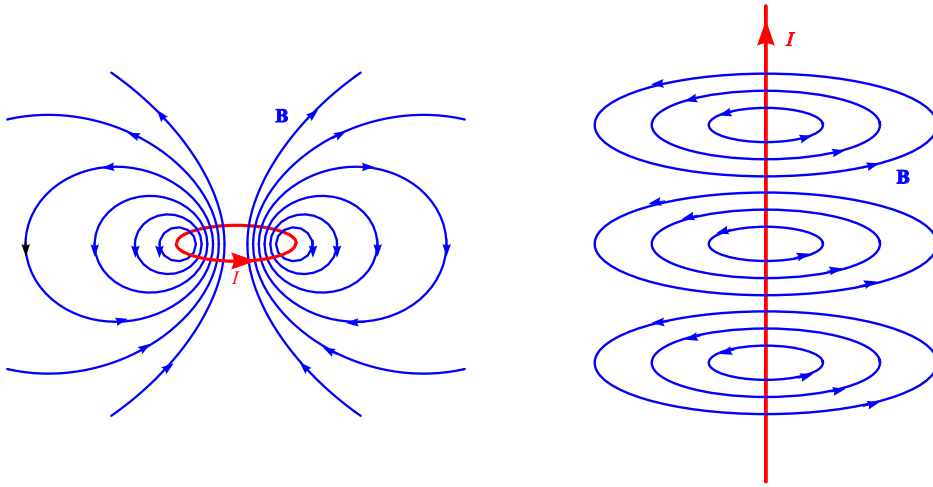


Figure 4: Magnetic field of (a) a small current loop and (b) a long straight wire segment.

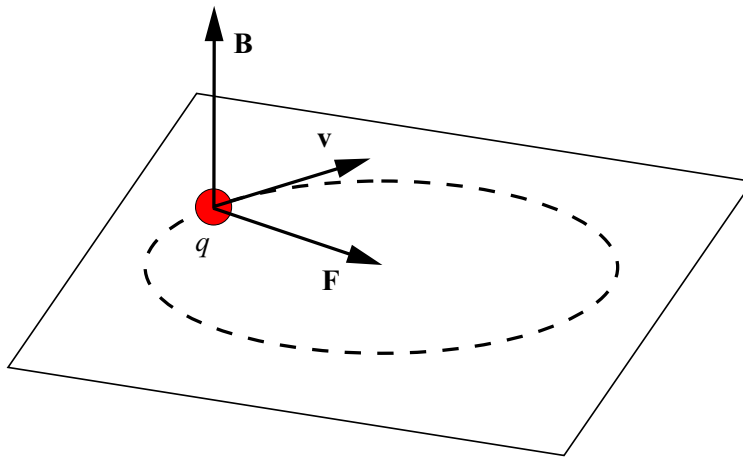


Figure 5: The magnetic force on a moving charged particle (or current segment). The direction of the force is perpendicular to both the particle velocity  $\mathbf{v}$  (or current) and the magnetic field:  $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$ . A charged particle moving on a plane perpendicular to a uniform magnetic field moves along a circle.

Both  $\mathbf{E}$  and  $\mathbf{B}$  are force fields, which extend throughout a volume of space. But they exert distinct and different forces. The electric field  $\mathbf{E}$  exerts a force on a charge  $q$  in the direction of the field vector. The magnetic field  $\mathbf{B}$  exerts a force on moving charges or current-carrying wires. The direction of the magnetic force is perpendicular to both the current and the field, as illustrated in Fig. 5. **Fig. 5** The magnetic field also exerts forces on the poles of a ferromagnet. The direction of the force is parallel (for a north pole) or antiparallel (for a south pole) to the vector  $\mathbf{B}$ . A compass needle aligns with the magnetic field because the equal but opposite forces on the two poles of the needle compose a torque that twists the needle toward alignment with the field. The magnetic forces on the current elements around a small current loop also compose a torque; in equilibrium the plane of the loop is oriented perpendicular to the  $\mathbf{B}$  vector. The combined torques on the atoms in a ferromagnet make up the net torque on the magnet.

The field concept was first stated by Michael Faraday. From years of experimental studies on electricity and magnetism, Faraday had formed the idea that a physical entity extends throughout the space outside charges or magnets, and exerts forces on other charges or magnets in the space. He referred to this extended entity as the “lines of force.” The term *Electromagnetic Field* was coined by James Clerk Maxwell, the renowned theoretical physicist who developed a mathematical theory of electromagnetism based on the field concept.

## IV Electromagnetic Induction

The previous sections were concerned with static systems of electric charge or current, i.e., in which the fields  $\mathbf{E}(\mathbf{x})$  and  $\mathbf{B}(\mathbf{x})$  do not change in time. Some relationships between electricity and magnetism were discussed: a steady electric current produces a magnetic field (Ampère’s law); a magnetic field exerts a force on any electric charge moving across the field lines. However, for static fields electric and magnetic phenomena appear to be rather distinct.

Time-dependent fields will be described next: The electric field  $\mathbf{E}(\mathbf{x}, t)$  and magnetic field  $\mathbf{B}(\mathbf{x}, t)$  are functions of time  $t$  as well as position  $\mathbf{x}$ . In dynamic systems the two fields affect each other

significantly. Therefore electric and magnetic phenomena are connected, and must be described by a unified theory. Electricity and magnetism are then combined into electromagnetism.

The phenomenon of electromagnetic induction was discovered in 1831 by Michael Faraday in England and independently by Joseph Henry in the United States. The effect is that when a magnetic field changes in time, an electric field is induced in directions that curl around the change of the magnetic field. This phenomenon has important technological applications.

Electromagnetic induction may be observed directly in simple physical demonstrations. Figure 6 shows schematically a coil of conducting wire C connected to a galvanometer G. **Fig. 6** The galvanometer acts as a current indicator: When no current flows around C the galvanometer needle points toward the central (zero current) position; when current flows the needle is deflected. No current source (such as a battery) is connected to the wire coil. In Fig. 6, M is a magnet that can be moved toward or away from the coil C. When the magnet is at rest no current flows in C and the galvanometer needle points in the central direction.

If the magnet in Fig. 6 moves toward the coil, the galvanometer needle will be deflected in one direction, indicating that a current is flowing in C. The current exists while M is moving. When the motion of M ceases, the needle will return to the central position indicating that the current has stopped. If the demonstration is repeated with the magnet moving away from the coil, the galvanometer needle will be deflected in the opposite direction while M moves. These demonstrations show directly that a change in the magnetic field through the coil induces a current around the coil.

The magnetic field in the demonstration might be varied in a different way. Suppose the bar magnet M in Fig. 6 is replaced by an electromagnet that does not move. Electric current  $I(t)$  in the electromagnet solenoid produces a magnetic field  $\mathbf{B}(\mathbf{x}, t)$  according to Ampère's law. If the solenoid current is constant then  $\mathbf{B}$  is constant in time and there is no current in the coil C. But if the solenoid current changes, the magnetic field changes. A deflection of the galvanometer needle will be observed while the magnetic field is varying in time. A current around the sensing coil C is again induced when the magnetic field through the coil is changing.

These demonstrations show that when a magnetic field  $\mathbf{B}(\mathbf{x}, t)$  changes in time, a current is induced in a conductor that is present in the field. However, the induced current is actually a secondary effect. The current in C is created by an induced electric field  $\mathbf{E}(\mathbf{x}, t)$ , and  $\mathbf{E}$  is the primary effect. Electromagnetic induction is fundamentally a phenomenon of the fields,  $\mathbf{B}(\mathbf{x}, t)$  and  $\mathbf{E}(\mathbf{x}, t)$ . A magnetic field that changes in time induces an electric field in directions curling around the change of the magnetic field. If there happens to be a conducting coil present, as for example C in Fig. 6, then the induced electric field drives an electric current around C. But the primary effect is induction of the electric field. The induced electric field  $\mathbf{E}(\mathbf{x}, t)$  exists while the magnetic field  $\mathbf{B}(\mathbf{x}, t)$  is varying in time.

The apparatus shown in Fig. 6 is only schematic. The induced current in C would be very small for an ordinary bar magnet M moving at reasonable velocities. A practical demonstration would require a sensing coil with many turns of wire and a sensitive galvanometer. The effect might be increased by putting an iron core inside the coil to enhance the magnetic field.

Faraday and Henry performed laboratory experiments similar to the demonstrations illustrated in Fig. 6 in their discoveries of electromagnetic induction. Faraday described the results of his detailed studies in terms of the lines of force—his concept of a magnetic field filling space around a magnet. In modern language, a statement summarizing his observations is

**Faraday’s Law:** When the flux of magnetic field through a loop changes in time, an electromotive force (EMF) is induced around the loop.

This statement is Faraday’s law of electromagnetic induction. In equation form,  $d\Phi/dt = -\mathcal{E}$  where  $\Phi$  is the magnetic flux through the loop,  $d\Phi/dt$  is the rate of change of the flux, and  $\mathcal{E}$  is the electromotive force around the loop. The “flux” is a quantitative measure of the field lines passing through the loop, defined by the product of the component of the magnetic field vector perpendicular to a surface bounded by the loop and the area of the surface.

The “loop” in Faraday’s law is a closed curve, e.g., a circle. The loop may be a conducting loop, in which case the induced EMF drives a current; or it may just be an imaginary curve in space. In

either case the EMF is an induced electric field curling around the change of the magnetic field.

Another, related demonstration may be carried out with the simple apparatus of Fig. 6. Instead of moving the magnet M and holding the coil C fixed, suppose the coil is moved while the magnet is held fixed. Again a current will be observed around C. The phenomenon in this case is called “motional EMF;” an electromotive force is induced in a conductor that moves relative to a nonuniform magnetic field. In the language of Faraday, when a conducting wire moves through magnetic lines of force an induced current flows in the wire. Evidently any change of the magnetic flux through a conducting loop will induce a current in the loop.

Yet another way to change the magnetic flux through the coil C in Fig.6 is to change the orientation of the coil. In Fig.6 the plane of the coil is shown perpendicular to the bar magnet. In this orientation the magnetic field lines pass straight through C; the flux is the product of the magnetic field strength  $B$  and the area  $A$  of the loop,  $\Phi = BA$ . Now suppose the coil rotates about a vertical axis, with M fixed. Then the flux of magnetic field through the coil changes as the plane of the loop is at a varying angle to the field vector. When the plane of the loop is parallel to the field lines the flux is zero because no field lines pass through the coil. While the coil is rotating, a deflection of the galvanometer needle will be observed, consistent with Faraday’s law, because of the changing flux. This is another example of motional EMF: The conducting wire moves through the magnetic field lines and there is an induced current in the wire.

## V Applications of Electromagnetic Induction

A number of important inventions are based on the phenomenon of electromagnetic induction. Two that have great technological significance will be described here.

### V.1 The electric generator

An electric generator (or dynamo) is a machine that generates an electromotive force (EMF) by electromagnetic induction. One or more coils of wire rotate in a magnetic field. The magnetic

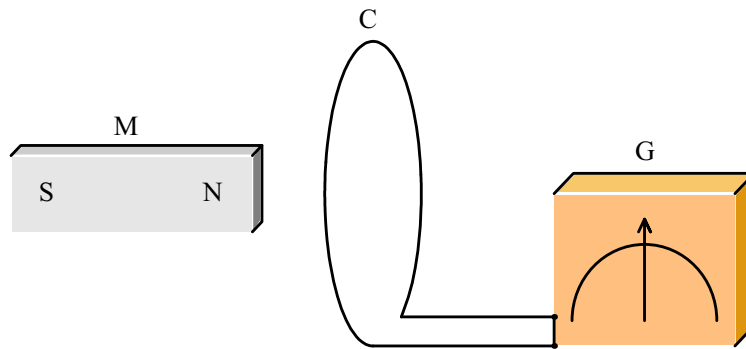


Figure 6: Schematic apparatus for demonstrations of electromagnetic induction.

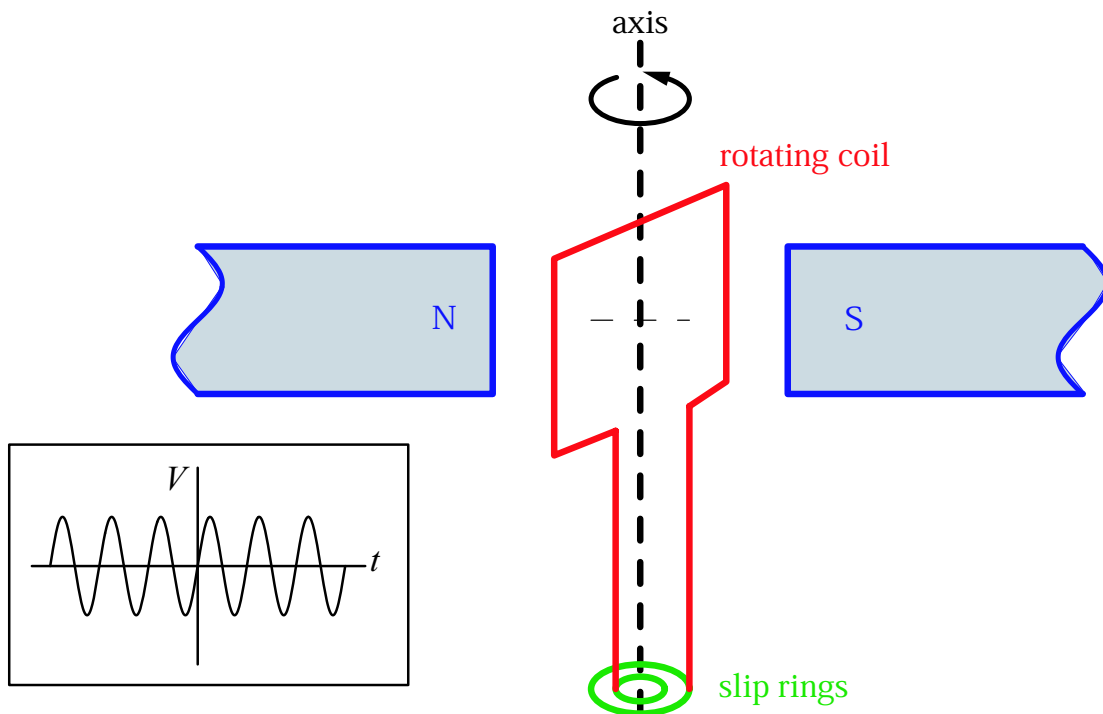


Figure 7: The design principle of an electric generator. The magnetic flux through the coil varies as the coil rotates and an EMF is induced in accord with Faraday's law. The voltage difference  $V$  between the slip rings varies sinusoidally in time (inset graph).

flux through a coil changes as the coil rotates, inducing an EMF around the coil by Faraday's law,  $\mathcal{E} = -d\Phi/dt$ .

Figure 7 shows schematically the design principle of an electric generator. **Fig. 7** In the model, the square coil rotates about the vertical axis that bisects the coil. The model field  $\mathbf{B}$  is the constant field between north and south poles of ferromagnets. As the coil rotates, the flux through it varies sinusoidally as a function of time. Then there is an induced alternating EMF around the coil.

The model generator in Fig. 7 has the ends of the coil in contact with two slip rings. The voltage difference between the rings is equal to the EMF  $\mathcal{E}$  around the coil. Thus the device generates an alternating voltage  $V$  (shown in the inset graph) with constant frequency equal to the rotational frequency of the coil. If the rings are connected by a wire with resistance  $R$ , an alternating current (AC)  $I$  will occur in the wire with  $V = IR$  by Ohm's law.

An external torque must be applied to the coil of the generator to maintain the constant rotation. If the slip rings are connected to an electric appliance, then energy is supplied by the generator because the induced EMF drives an alternating current in the appliance. But energy cannot be created—only converted from one form to another. If no external torque is applied, the generator coil will slow down and stop as its rotational kinetic energy is converted to the work done by the appliance. The slowing of the rotation is an example of magnetic braking: A conductor moving in a magnetic field experiences a force opposite to the motion. To maintain a constant rotation of the generator coil (and hence AC power to the appliance) the coil must be driven by a torque exerted by an external device. In a portable generator the external torque is produced by a gasoline-powered engine. In an electric power plant the external torque is produced by a turbine driven by a flow of hot steam; the steam is produced in a boiler, e.g., from burning coal in a coal-fired power plant or from the heat of nuclear reactions in a nuclear reactor.

The electric generators producing electric power around the world are large and complex machines; but the fundamental physics in these devices is simple: Faraday's law and the design principle in Fig. 7.

## V.2 The transformer

A transformer takes an input alternating EMF  $V_1$ , e.g., from an electric generator, and makes an output EMF  $V_2$  that might be supplied to an electric appliance. (See Fig. 8.) The frequencies of  $V_1$  and  $V_2$  are equal but the amplitudes are different. A step-down transformer has  $V_2 < V_1$ ; for example, a transformer might take in 110 volts (alternating) from a wall socket and put out 10 volts to a small electric motor in an appliance. A step-up transformer has  $V_2 > V_1$ ; high-voltage applications require such a transformer.

The physical phenomenon acting in a transformer is electromagnetic induction. Figure 8 shows a simple transformer: Two coils of conducting wire are wrapped around a toroidal ferromagnetic core. **Fig. 8** The alternating current driven in the primary coil by the input EMF  $V_1(t)$  creates a strong alternating magnetic field  $\mathbf{B}(t)$ , according to Ampère's law. Because of the ferromagnetism,  $\mathbf{B}$  extends around the toroid. Then the changing magnetic flux through the secondary coil induces an EMF by Faraday's law,  $\mathcal{E} = -d\Phi/dt$ ; this EMF is the output voltage  $V_2$ . If  $N_1$  is the number of turns of wire in the primary coil and  $N_2$  is the number of turns in the secondary coil, then the ratio of the voltages is  $V_2/V_1 = N_2/N_1$ .

Transformers are a crucial part of the electric power grid. An electric power plant creates a 3-phase alternating EMF supplied to the grid. But the electric appliances that will use the power are located far from the power plant. Therefore a current must occur over very long transmission lines. The amount of lost power  $P_{\text{lost}}$ , dissipated in resistance of the transmission wires, is given by the formula  $P_{\text{lost}} = I^2 R$  where  $I$  = current and  $R$  = resistance of the line. The power supplied to users is  $P_{\text{used}} = IV$ , where  $V$  is the transmitted EMF. The ratio of power lost to power used is

$$\frac{P_{\text{lost}}}{P_{\text{used}}} = \frac{(P_{\text{used}}/V)^2 R}{P_{\text{used}}} = \frac{P_{\text{used}} R}{V^2},$$

i.e., inversely proportional to the square of the line voltage. Therefore the transmission of electric power is most efficient if the voltage  $V$  is high.

A typical power-plant generator might produce an alternating EMF with amplitude 168kV, i.e.,

root mean square EMF 120 kV.<sup>5</sup> But that voltage would be changed in a step-up transformer at the power plant to a very high voltage, e.g., 345 kV. High-voltage transmission lines carry the electric power across large distances. Then step-down transformers are required at the end of a transmission line in order to reduce the voltage for applications. A transformer substation might reduce the voltage to 4.8 kV for overhead lines into residential areas. A final transformer mounted on the utility pole reduces the voltage to 110 V for the wires to a single house.

## VI The Maxwell Equations of the Electromagnetic Field

The mathematical theory of electromagnetism was developed and published in 1864 by James Clerk Maxwell. He described the known electric and magnetic effects in terms of four equations relating the electric and magnetic fields and their sources—charged particles and electric currents. The development of this theory was a supreme achievement in the history of science. Maxwell’s theory is still used today by physicists and electrical engineers. The theory was further developed in the 20th century to account for the quantum theory of light. But even in quantum electrodynamics Maxwell’s equations remain valid although their interpretation is somewhat different from the classical theory. In any case, Maxwell’s theory continues today to be an essential part of theoretical physics.

A knowledge of calculus and vectors is necessary for a full understanding of the Maxwell equations. However, the essential structure of the theory can be understood without going into the mathematical details. Each equation is expressed most powerfully as a partial differential equation relating variations of the electric and magnetic fields, with respect to variations of position or time, and the charge and current densities in space.

**Gauss’s law.** Gauss’s law, written as a field equation, is  $\nabla \cdot \mathbf{E} = \rho/\epsilon_0$ . (The symbol  $\nabla \cdot$  denotes the divergence operator.) Here  $\rho(\mathbf{x}, t)$  is the charge per unit volume in the neighborhood of  $\mathbf{x}$  at time  $t$ ;  $\epsilon_0$  is a constant of nature equal to  $8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2$ . Gauss’s law relates the electric field  $\mathbf{E}(\mathbf{x}, t)$  and the charge density. The solution for a charged particle  $q$  at rest is  $\mathbf{E}(\mathbf{x}) = kqe/r^2$

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<sup>5</sup>The unit kV, kilovolt, is 1000 volts.

where  $r$  is the distance from the charge to  $\mathbf{x}$ ,  $\mathbf{e}$  is the direction vector, and  $k = 1/(4\pi\epsilon_0)$ ; this is the familiar inverse square law of electrostatics. Electric field lines diverge at a point charge.

**Gauss's law for magnetism.** The analogous equation for the magnetic field is  $\nabla \cdot \mathbf{B} = 0$ . There are no magnetic monopoles—particles that act as a point source of  $\mathbf{B}(\mathbf{x}, t)$ . Unlike the electric field lines, which may terminate on charges, the magnetic field lines always form closed curves because magnetic charges do not exist. There is no divergence of magnetic field lines.

**Faraday's law.** The field equation that describes Faraday's law of electromagnetic induction is  $\nabla \times \mathbf{E} = -\partial\mathbf{B}/\partial t$ . The quantity  $\nabla \times \mathbf{E}$ , called the curl of  $\mathbf{E}(\mathbf{x}, t)$ , determines the way that the vector field  $\mathbf{E}$  curls around each direction in space. Also,  $\partial\mathbf{B}/\partial t$  is the rate of change of the magnetic field. This field equation expresses the fact that a magnetic field that varies in time implies an electric field that curls around the change of the magnetic field. It is equivalent to Faraday's statement that the rate of change of magnetic field flux through a surface  $S$  is equal to an electromotive force (EMF) around the boundary curve of  $S$ .

**The Ampère-Maxwell law.** In a system of steady electric currents the magnetic field is constant in time and curls around the current in directions defined by the right-hand rule. The field equation that expresses this field  $\mathbf{B}$  (Ampère's law) is  $\nabla \times \mathbf{B} = \mu_0\mathbf{J}$ , where  $\mathbf{J}(\mathbf{x})$  is the current per unit area at  $\mathbf{x}$  and  $\mu_0$  is a constant equal to  $4\pi \times 10^{-7} \text{ Tm/A}$ .<sup>6</sup> But Ampère's law is incomplete, because it does not apply to systems in which the currents and fields vary in time. Maxwell deduced from mathematical considerations a generalization of Ampère's law,

$$\nabla \times \mathbf{B} = \mu_0\mathbf{J} + \mu_0\epsilon_0 \frac{\partial\mathbf{E}}{\partial t},$$

in which the second term on the right side is called the displacement current. The displacement current is a necessary term in order for the system of four partial differential equations to be self-consistent. The Ampère-Maxwell law implies that  $\mathbf{B}$  curls around either electric current ( $\mathbf{J}$ ) or

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<sup>6</sup>The units are T = tesla for magnetic field and A = ampere for electric current.

changing electric field ( $\partial\mathbf{E}/\partial t$ ). The latter case is analogous to electromagnetic induction but with  $\mathbf{E}$  and  $\mathbf{B}$  reversed; a rate of change in one field induces circulation in the other field.

Maxwell's introduction of the displacement current was a daring theoretical prediction. At that time there was no experimental evidence for the existence of displacement current. Laboratory effects predicted by the displacement current are very small and their observation was not possible with the apparatus available at that time. However, the Maxwell equations, including the displacement current, make a striking prediction—that light consists of electromagnetic waves. The fact that Maxwell's theory explains the properties of light, and other forms of electromagnetic radiation, provides the evidence for the existence of the displacement current.

## VII Electromagnetic Waves

Electromagnetism includes phenomena involving charges, currents, magnets, and the fields  $\mathbf{E}$  and  $\mathbf{B}$ . Light is also an electromagnetic phenomenon. Light and other forms of radiation are described by field theory as electromagnetic waves. Therefore optics—the science of light—is a part of electromagnetism.

The Maxwell equations describe the behavior of the electric field  $\mathbf{E}(\mathbf{x}, t)$  and magnetic field  $\mathbf{B}(\mathbf{x}, t)$ . The time-dependent fields influence each other, even in a vacuum where no charge or current is present, through Faraday's law ( $\nabla \times \mathbf{E} = -\partial\mathbf{B}/\partial t$ ) and the displacement current ( $\nabla \times \mathbf{B} = \mu_0\epsilon_0\partial\mathbf{E}/\partial t$ ). The four field equations are all satisfied in vacuum if both  $\mathbf{E}(\mathbf{x}, t)$  and  $\mathbf{B}(\mathbf{x}, t)$  are waves traveling in space. However, the electric and magnetic waves are co-dependent, i.e., intricately related in several ways.

Figure 9 shows a snapshot in time of the fields for a section of an electromagnetic wave in vacuum.

**Fig. 9** The field vectors are shown at points along a line parallel to the direction of propagation of the wave ( $z$  axis). The wavelength is constant (at least for the section of wave shown). The field vectors  $\mathbf{E}$  and  $\mathbf{B}$  vary in magnitude as a function of position along the line of propagation. The

magnitudes are related by  $B = E/c$  where  $c$  is the wave speed.<sup>7</sup> The directions of  $\mathbf{E}$  and  $\mathbf{B}$  are orthogonal, in the  $x$  and  $y$  directions respectively. The field components oscillate sinusoidally with  $z$ . An electromagnetic wave is a transverse wave because the fields oscillate in directions orthogonal to the direction of propagation.

Figure 9 shows the field vectors along only a single line parallel to the direction of propagation  $\hat{\mathbf{z}}$ . The full wave fills a three-dimensional volume. In a linearly polarized plane wave the field vectors are constant on planar wave fronts orthogonal to  $\hat{\mathbf{z}}$ , e.g., with  $\mathbf{E}$  and  $\mathbf{B}$  pointing in the  $\hat{\mathbf{x}}$  and  $\hat{\mathbf{y}}$  directions, respectively, uniformly over the wave front. All properties of polarized light are then described by the wave theory. As another 3D example, a short radio antenna radiates a spherical wave; far from the antenna  $\mathbf{E}$  and  $\mathbf{B}$  are orthogonal to one another on spherical wave fronts.

A wave is a structure that is extended in space. The parts of the wave also vary in time. Figure 9 is a snapshot showing the wave as a function of position at an instant of time. As time passes, the fields at any fixed position  $\mathbf{x}$  will change. For a linearly polarized plane wave,  $\mathbf{E}$  at  $\mathbf{x}$  oscillates in time between orientations parallel and antiparallel to a fixed polarization direction ( $\hat{\mathbf{x}}$ ) which is orthogonal to the line of propagation ( $\hat{\mathbf{z}}$ );  $\mathbf{B}$  varies similarly in a direction ( $\hat{\mathbf{y}}$ ) orthogonal to both  $\mathbf{E}$  and the line of propagation. The combined variations in space and time imply that the wave structure moves as a whole in the direction of propagation. So a snapshot at a later time would look just the same as Fig. 9 except translated by some distance in the direction of propagation. In other words, the shape is constant but the positions of the nodes (points where  $\mathbf{E} = 0$  and  $\mathbf{B} = 0$ ) move in space.

**The electromagnetic spectrum.** Electromagnetic waves have an infinite range of wavelength  $\lambda$ . Visible light has  $\lambda$  from 400 nm (violet) to 700 nm (red). Ultraviolet and infrared light have shorter and longer wavelengths, respectively. Microwaves have  $\lambda$  from 1 mm to 1 m, with important technological uses, e.g., communications and radar. X-rays have very short wavelengths,  $\lambda < 1$  nm. All these forms of electromagnetic radiation have the same wave speed in vacuum, which can be

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<sup>7</sup>The speed of light in vacuum is  $c = 2.998 \times 10^8$  m/s.

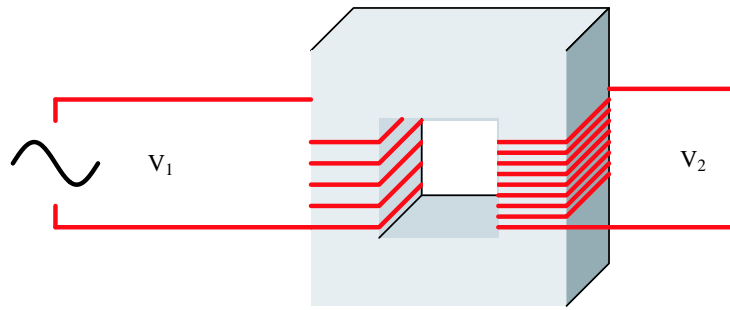


Figure 8: The design principle of a transformer. Two coils of conducting wire are wrapped around an iron core. The number of turns is different in the two coils, so the output EMF  $V_2$  is different from the input EMF  $V_1$ .

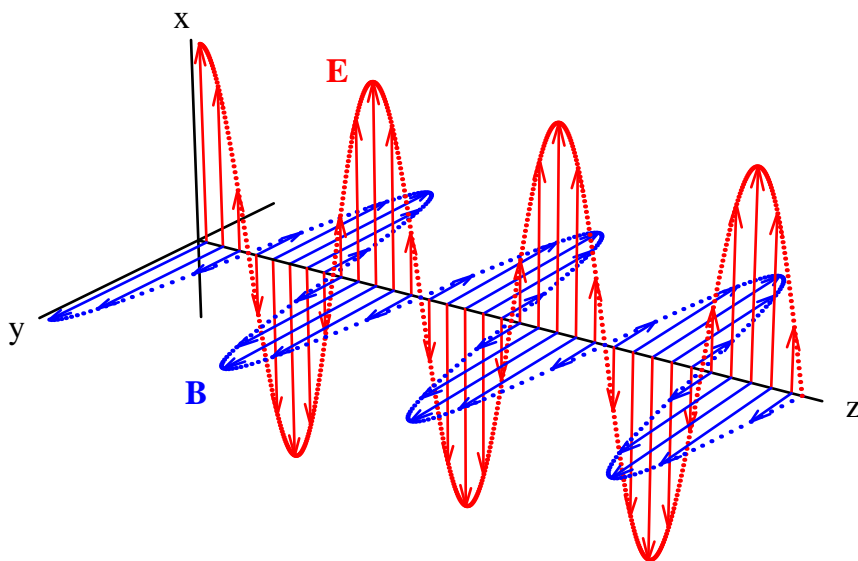


Figure 9: Electric and magnetic field vectors in a section of a linearly polarized electromagnetic wave.

evaluated from the field theory,  $c = 1/\sqrt{\mu_0\epsilon_0} = 2.998 \times 10^8$  m/s.

## VIII Summary

Our scientific understanding of electricity and magnetism has developed over a period of centuries. Phenomena that we understand today as examples of electricity, magnetism, or electromagnetism have been observed since ancient times: Lightning is an electric discharge in the atmosphere. A lodestone attracts iron. Light consists of electromagnetic waves. Such common phenomena, while easily observed, were unexplained until the scientific revolution. Today we are able to accurately describe and explain all variety of electromagnetic phenomena, based on the mathematical theory of fields. With this knowledge has come the ability to create technologies, such as the distribution of electric power, that affect our daily lives.

## IX Bibliography

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## Figures

The figures have been included in the text above. In the next pages, the drawings are shown separately.

Figure 1

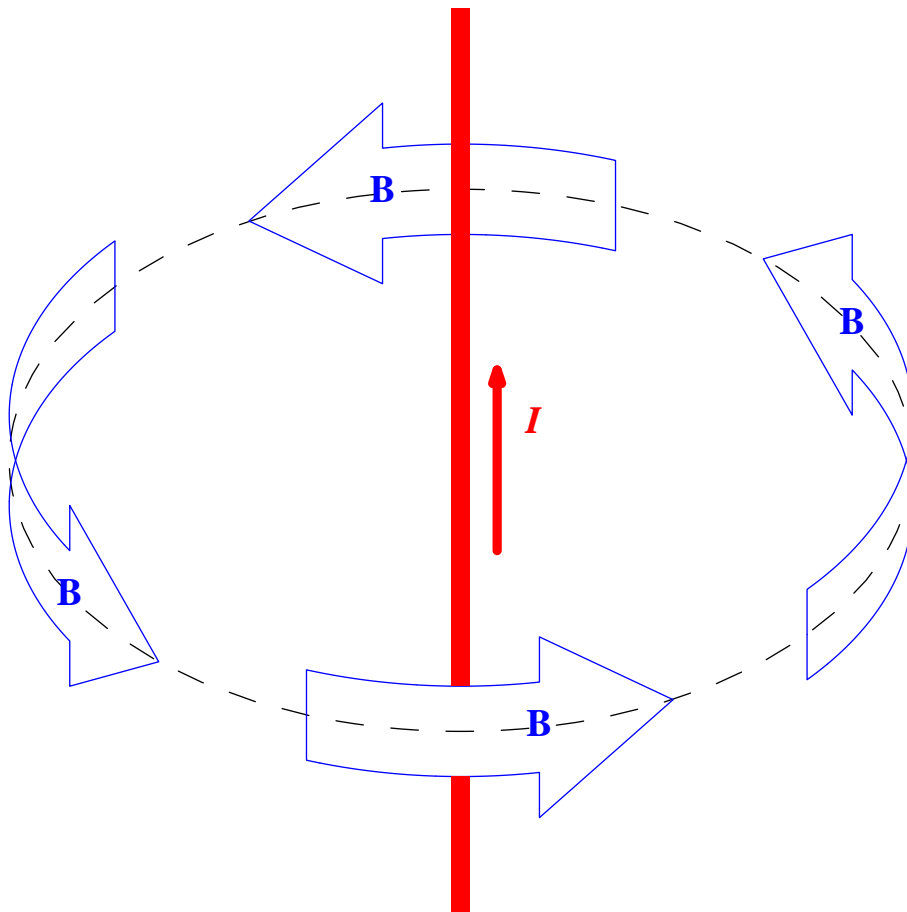


Figure 2

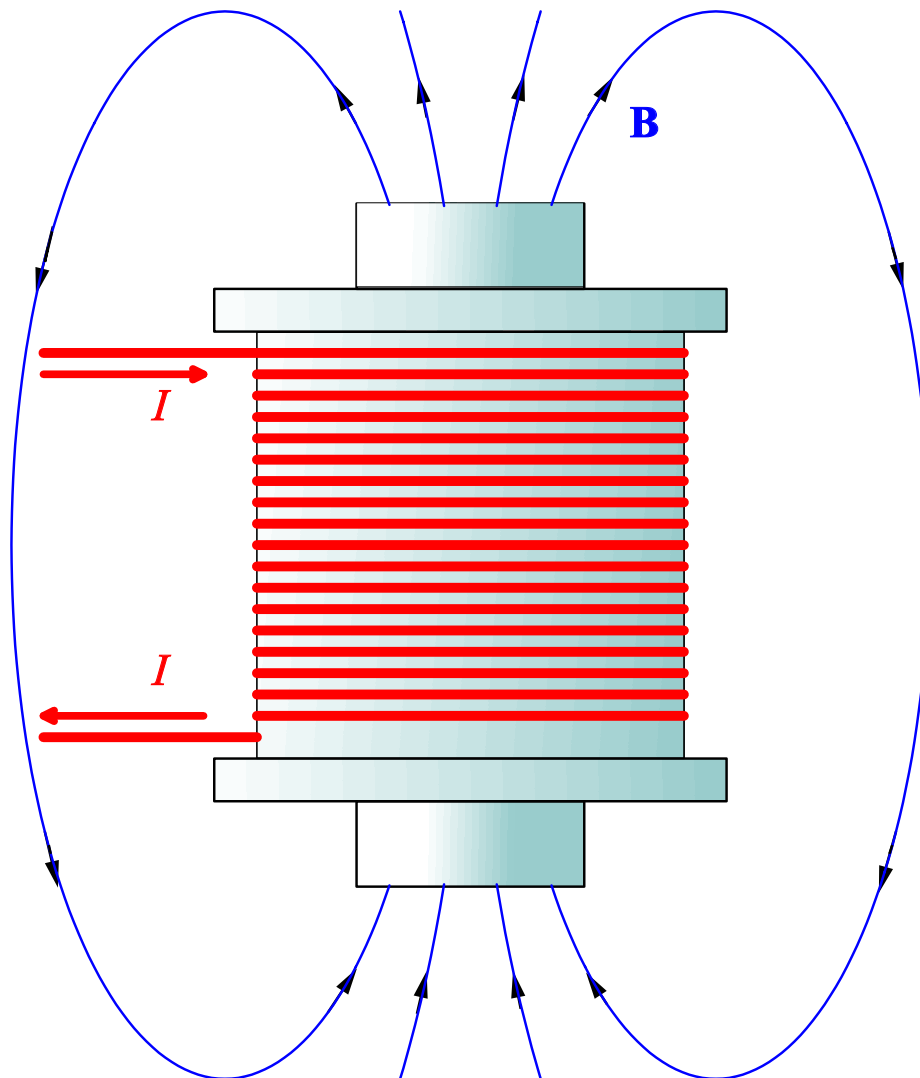


Figure 3

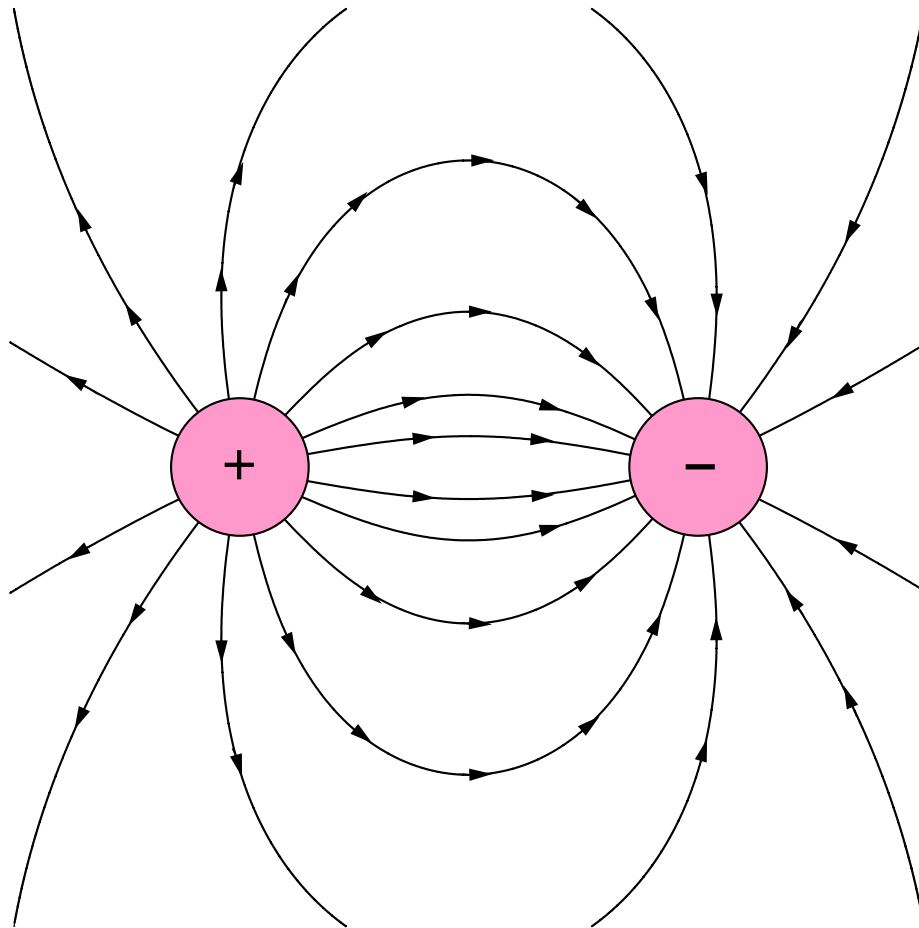


Figure 4a

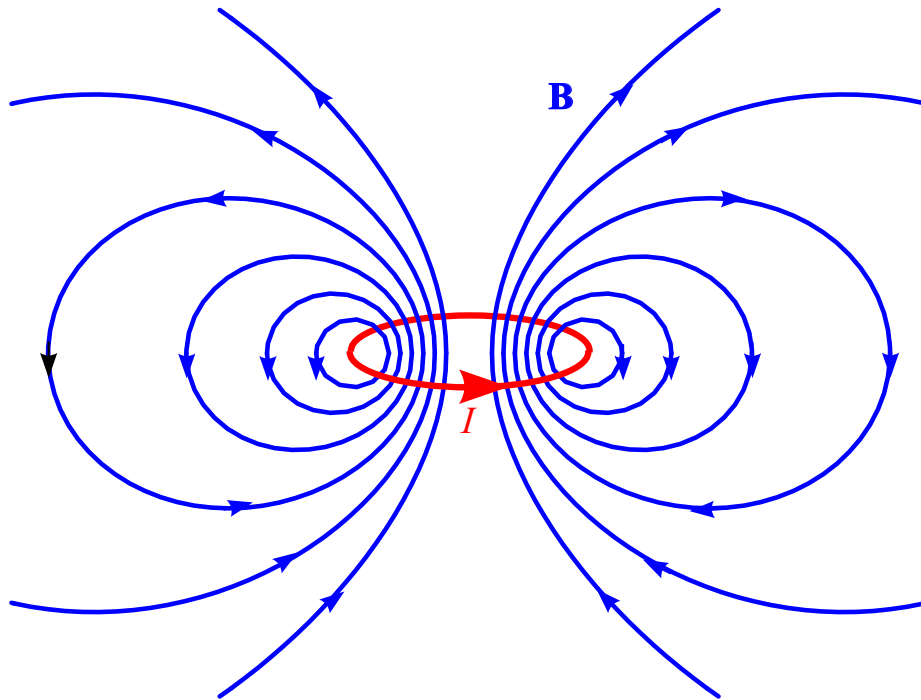


Figure 4b

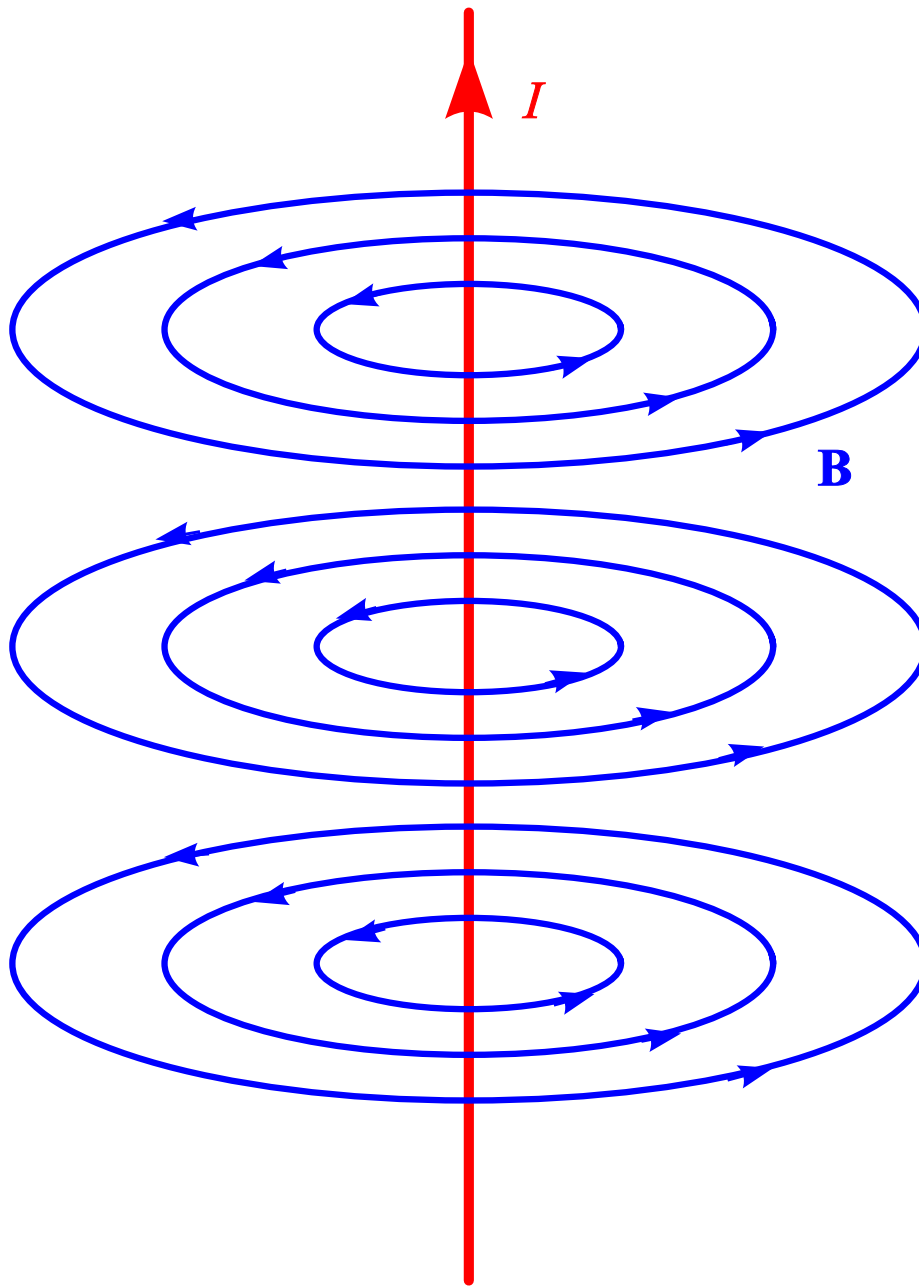


Figure 5

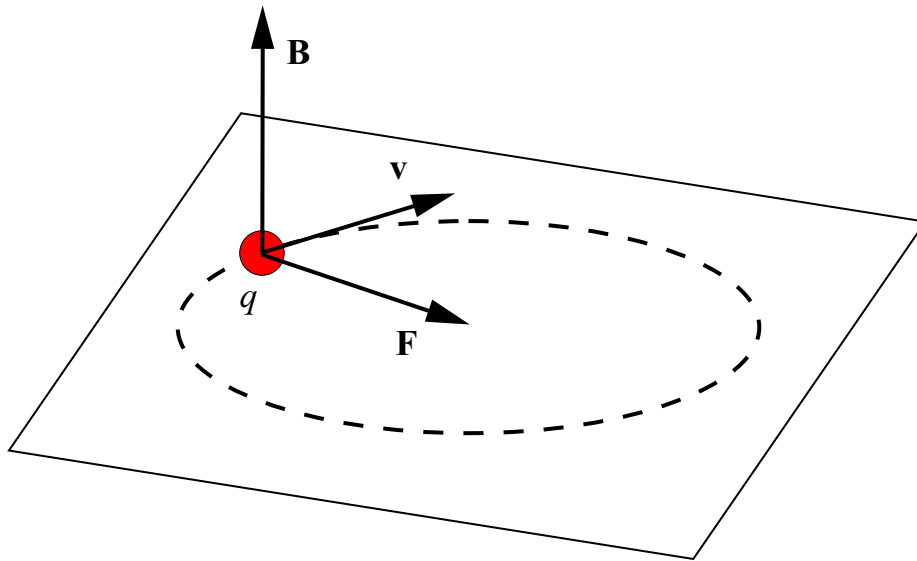


Figure 6

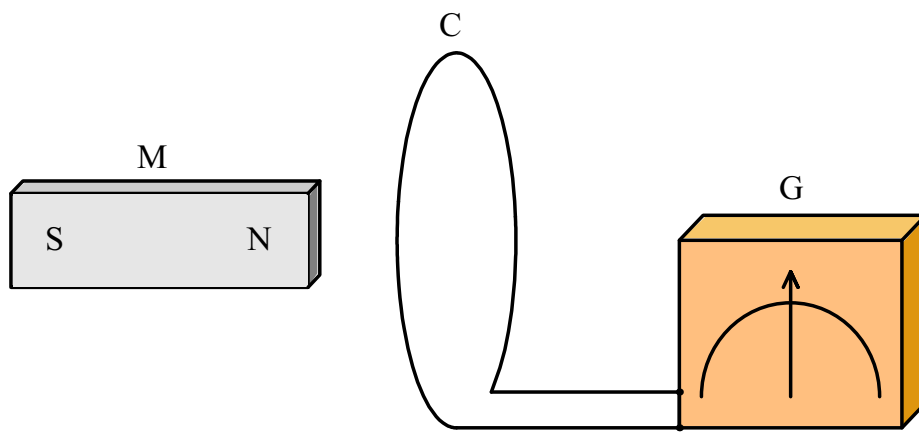


Figure 7

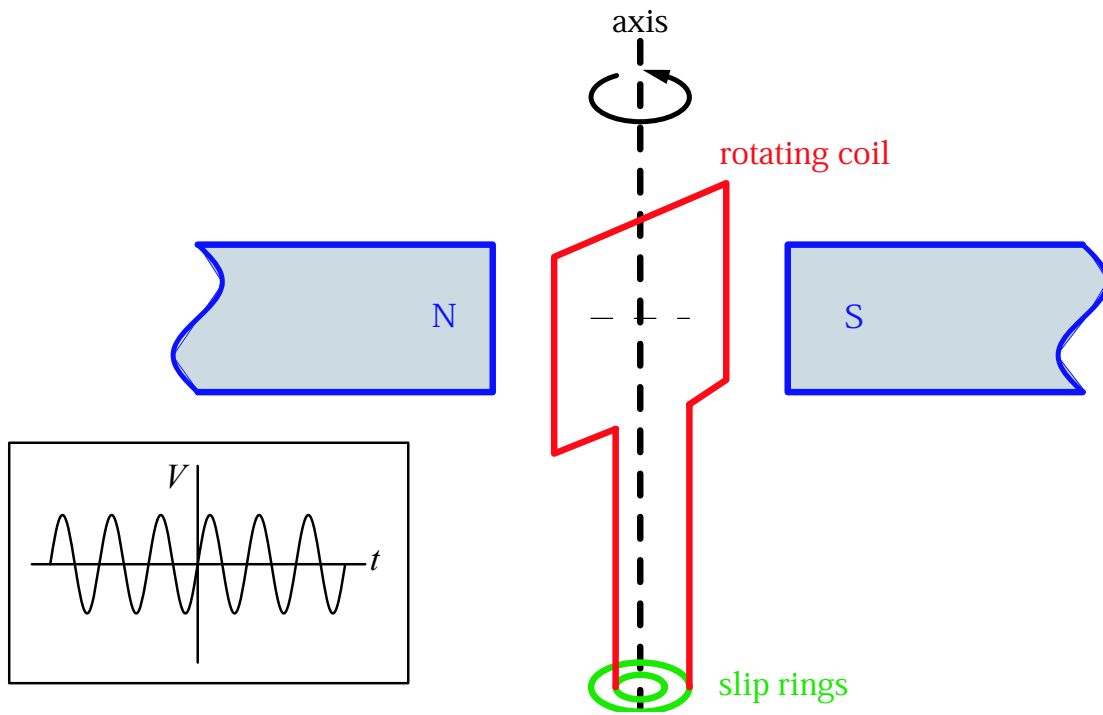


Figure 8

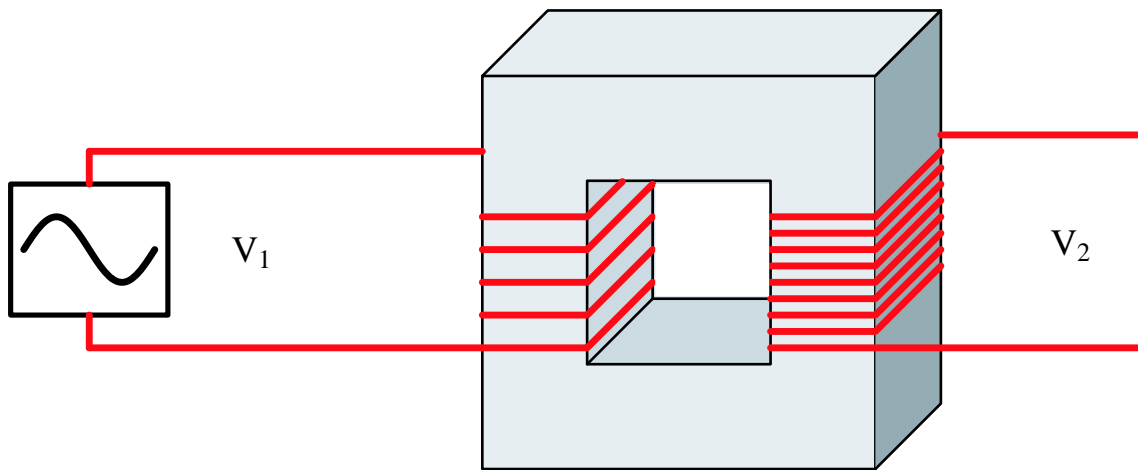


Figure 9

