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The Parton Structure of the Nucleon and Precision Determination of the Weinberg Angle in Neutrino Scattering

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A recently completed next-to-leading-order program to calculate neutrino cross sections, including power-suppressed mass correction terms, has been applied to evaluate the Paschos-Wolfenstein relation, in order to assess quantitatively the validity and significance of the NuTeV anomaly. In particular, we carefully study the shift of $\sin^2\theta_{\rm W}$ obtained in calculations by using a new generation of PDF sets that allow $s(x) \neq \bar{s}(x)$, enabled by recent neutrino dimuon data from CCFR and NuTeV, as compared to the previous $s = \bar{s}$ parton distribution functions such as CTEQ6M. The extracted value of $\sin^2\theta_{\rm W}$ is closely correlated with the strangeness asymmetry momentum integral $[S^-] \equiv \int_0^1 x[s(x) - \bar{s}(x)]dx$. Since $[S^-]$ has been found to be positive and relatively large for the new PDF sets, this implies a sizable negative shift of $\sin^2\theta_{\rm W}$ in the direction to bridge the difference between the NuTeV and Standard Model (SM) values of $\sin^2\theta_{\rm W}$. The precise implications for the NuTeV anomaly will need to be determined after including detector-dependent corrections; however, the results of our study suggest that the new dimuon data, the Weinberg angle measurement, and other global data sets used in QCD parton structure analysis can all be consistent within the SM.

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An important open question in particle physics in recent years has been the significance of the "NuTeV anomaly"—a $3\,\sigma$ deviation of the measured $\sin^2\theta_{\rm W}$ (0.2277 ± 0.0004) reported in Ref. [1] from the "standard model value" ($0.2227 \pm 0.0013 \pm 0.0009$, based on the world average of other measurements [2]). Possible sources of the NuTeV anomaly within and beyond the standard model have been systematically examined in [3]. No consistent picture has yet emerged in spite of an extensive literature [4–8] on this subject. The measurement in Ref. [1] was based on a correlated fit to the ratios of charged and neutral current (CC & NC) interactions in sign-selected neutrino and anti-neutrino scattering events on a (primarily) iron target at Fermilab. This procedure is closely related to measuring the Paschos-Wolfenstein ratio [9], which provides the theoretical underpinning of the analysis. For an isoscalar target, the parton model relates the Paschos-Wolfenstein ratio R^- to the Weinberg angle by

$$R^- \equiv \frac{\sigma^{\nu}_{\rm NC} - \sigma^{\bar{\nu}}_{\rm NC}}{\sigma^{\nu}_{\rm CC} - \sigma^{\bar{\nu}}_{\rm CC}} \simeq \frac{1}{2} - \sin^2\theta_{\rm W} \quad . \tag{1}$$

Beyond the parton model, the corrections from higher-order QCD as well as gluon and sea-quark contributions have been expected to be relatively small for the ratio R^- (being a ratio of differences of cross sections). But at the accuracy required to test the consistency of the SM, one needs a more quantitative analysis based on a thorough revision of the relevant perturbative cross sections [10] and non-perturbative parton distribution functions [11]—similar to previous combined perturbative/non-perturbative re-analyses [12, 13] of challenges to the QCD formalism. We report here the impact of the combined results of Refs. [10, 11] on the "NuTeV anomaly".

A particularly elusive source of uncertainty so far [3, 4] has been that associated with a possible strangeness asymmetry in the parton structure of the nucleon. A positive (negative) asymmetry, as measured by the momentum integral $[S^-] = \int x[s(x) - \bar{s}(x)]\,dx$, would reduce (increase) the NuTeV discrepancy. A devoted global analysis of parton distribution functions [11] has now indicated that this asymmetry is positive and of order 1–$3{\times}10^{-3}$. To quantify the impact of this new finding on R^-, we combine it with a recently completed state-of-the-art program for calculating neutrino cross sections in next-to-leading-order QCD, including target mass and charm mass corrections, by two of the authors [10]. We first summarize this calculation.

At sufficiently high neutrino energy, the total neutrino cross section

$$\sigma^{\nu} \equiv \sigma^{\nu N \to lX} = \int d^3 p_l \; \frac{d^3\sigma^{\nu N \to lX}}{d^3 p_l} \qquad (2)$$

can be calculated in QCD perturbation theory—in contrast to the electromagnetic case where the massless photon propagator leads to the dominance of non-perturbative photoproduction events over deep inelastic scattering and to a divergent phase space integral. The differential cross section in Eq. (2) factorizes into a sum of convolutions of parton distribution functions and partonic cross sections

$$d^3\sigma^{\nu N \to lX} = \sum_{f=q,g} f \otimes d^3\sigma^{\nu f \to lX} \quad . \qquad (3)$$

Ref. [10] performed this calculation at NLO accuracy, also taking into account target and charm mass effects, as well as non-isoscalarity of the target material. Ref. [10] used previously available parton distributions $\{f(x,Q)\}$ [14, 15]—all of which assume $s \leftrightarrow \bar{s}$ symmetry inside the nucleon. This study confirmed the smallness of the higher order corrections to R^- in general. (The same conclusion is reached by the NLO and NNLO moment analyses of [3, 5].) It was also shown that the experimental necessity of non-monochromatic neutrino and anti-neutrino beams, which do not have exactly the same profile, and typical cuts in the hadronic event energy do not alter Eq. (1) substantially. However, corrections to Eq. (1) that were not explored in [10] arise if there is significant violation of either of the following symmetries

- isospin; e.g. $u_p(x) = d_n(x)$
- $s(x) = \bar{s}(x)$

for the nucleon partonic structure.

There is no obvious reason to expect larger isospin violations [6] at the partonic level than the tiny effects observed in low energy pion nucleon scattering. The fact that the strange quark mass $m_s = \mathcal{O}(\Lambda_{\rm QCD})$ sets a scale below the applicability of pQCD inevitably renders the strange quark PDF a non-perturbative component of the nucleon bound state. Except for the sum rule in Eq. (6) below, there is no fundamental nor even merely approximate symmetry that relates the strange quark PDF $s(x)$ to the anti-quark PDF $\bar{s}(x)$.

In the analysis of Ref. [1], the parton distribution functions employed in Eq. (3) are those of an iron target. We base our calculation throughout on the parton densities of an unbound nucleon, which is legitimate as long as we calculate relative shifts between $[S^-] = 0$ and $[S^-] \neq 0$ PDFs consistently. Apart from the fact that the experimental information on nuclear PDFs [16] is relatively scarce, we thereby avoid the classification of nuclear effects in terms of twist (τ) and the corresponding complications: Nuclear PDFs only account for leading twist 2 ($\tau = 2$) effects whereas higher twists ($\tau > 2$), whether they relate to nuclear modifications or not, are generally difficult to account for. In the present context of extracting the weak couplings of quark flavors, standard phenomenological parametrizations like

$$d\sigma(x,Q^2) = d\sigma^{\tau=2}(x,Q^2)\left[1 + \frac{\mathrm{HT}(x)}{Q^2}\right] \qquad (4)$$

make too-strong assumptions and must be rejected as model dependent—e.g. they exclude flavor non-diagonal contributions such as could arise from "cat's ears" diagrams.

If the scale dependence of the parton distributions is neglected, i.e. $f(x,Q) \simeq f(x)$, and in the approximation of overlooking experimental cuts, the total cross section in Eq. (2) is sensitive to the second Mellin moment integrals $\int dx\, x\, f(x)$ of the PDFs [3, 5] (corresponding to the local quark operator matrix elements of spin 2). Making the further approximation of an isoscalar target, and in the limit of a negligible charm quark mass, a strange sea asymmetry contributes at LO as [3]

$$R^- \simeq \frac{1}{2} - \sin^2\theta_{\rm W} - \left(\frac{1}{2} - \frac{7}{6}\sin^2\theta_{\rm W}\right)\frac{[S^-]}{[Q^-]} \qquad (5)$$

with isoscalar up or down quark distributions $q(x)$ contributing via $[Q^-] = \sum \int x[q(x) - \bar{q}(x)]dx$. The quantum numbers of the nucleon (in any isospin state) dictate the strict sum rule

$$\left[s^-\right] \equiv \int dx \;\; [s(x) - \bar{s}(x)] = 0 \qquad (6)$$

for the first moment integral such that a non-zero $[s(x) - \bar{s}(x)]$ is bound to oscillate in x. In Ref. [11] a global analysis of PDF-related data has been performed that includes the dimuon production cross sections in Ref. [17]. The dimuon data signal a semi-leptonic charm decay in $W^+ s \to c$ and $W^- \bar{s} \to \bar{c}$ events and put constraints on the non-singlet asymmetry $[s(x) - \bar{s}(x)]$ as well as on $[s(x) + \bar{s}(x)]$. Fig. 1 shows a representative fit from Ref. [11] that obeys the sum rule of Eq. (6): the negative $[s(x) - \bar{s}(x)]$ at low-x is compensated by positive $[s(x) - \bar{s}(x)]$ at large-x; this leads to positivity of the second moment integral:

$$\left[S^-\right] \equiv \int dx \; x \;\; [s(x) - \bar{s}(x)] > 0 \quad . \qquad (7)$$

The same trend had previously been observed in a fit to global neutrino cross section data [4]. (This kind of behavior was anticipated by a dynamical model calculation [18], on the basis of baryon-meson fluctuations of the nucleon light-cone wave function.[23])

By including the CCFR-NuTeV dimuon data (which are directly sensitive to $s(x)$ and $\bar{s}(x)$), and by exploring the full allowed parameter space in a global QCD

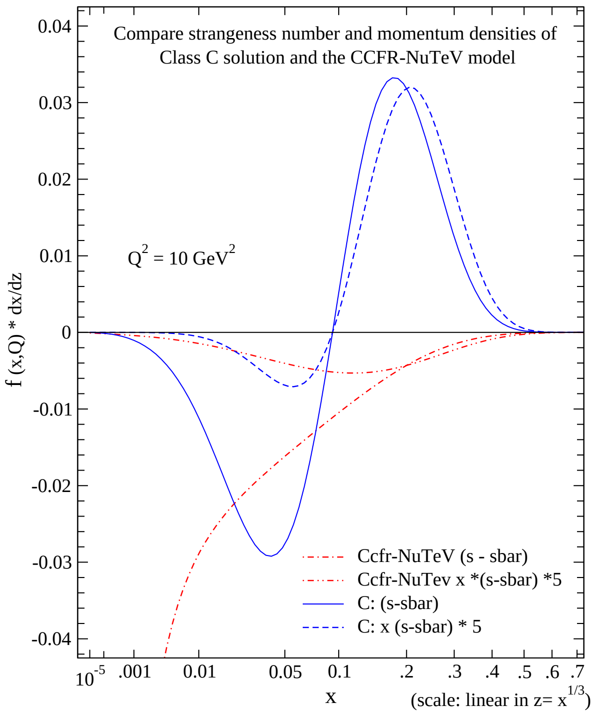

FIG. 1: Comparison of the number and momentum distributions of the strangeness non-singlet asymmetry $s-\bar{s}$. The curves labelled C are obtained from one of the preferred fits of [11]). The dot-dashed curved are calculated from the CCFR-NuTeV model. The role of number and momentum sum rules can be seen from a comparison of the two cases.

analysis, Ref. [11] now presents a general picture of the strangeness sector of nucleon structure. It shows that the strong interplay between the existing experimental constraints and the global theoretical constraints, such as Eq.(6), places robust limits on acceptable values of the strangeness asymmetry momentum integral $[S^-]$: $0.001 < [S^-] < 0.003$ (or $0 < [S^-] < 0.004$, depending on how stringent the criterion is set), but a sizable negative $[S^-]$ is strongly disfavored by both dimuon and other inclusive data.

We quantify the impact of the PDFs of Ref. [11] on the Paschos-Wolfenstein relation in Eq. (1) by employing the NLO neutrino cross section calculations of Ref. [10]. These include, first of all, NLO QCD corrections, charm quark effects, and target mass corrections. They are needed because it is impossible to implement experimental cuts on Q^2 in NC neutrino events, which leads to a non-negligible contribution from low Q values to the integral in Eq. (2)—e.g., about 5% from $Q^2 < 1$ GeV^2 for $\sigma^{\bar{\nu}}_{\rm CC}$ and from $Q^2 < 2$ GeV^2 for $\sigma^{\nu}_{\rm CC}$. Other corrections included are non-isoscalarity of the target material (iron), an energy average over the neutrino and anti-neutrino flux spectra and the cuts in hadronic energy (20 GeV $< yE_\nu <$ 180 GeV for lepton inelasticity y) as in the experimental analysis [1].

fit	$[S^-]\times 100$	$\chi^2_{\rm dimuon}$	$\chi^2_{\rm inclusiveI}$	δR^-
B$^+$	0.540	**1.30**	0.98	-0.0065
A	0.312	1.02	0.97	-0.0037
B	0.160	*1.00*	*1.00*	-0.0019
C	0.103	1.01	1.03	-0.0012
B$^-$	-0.177	**1.26**	**1.09**	0.0023

TABLE I: Shifts in R^-, calculated with PDF sets of Ref. [11] (with non-zero $[S^-]$) compared to the value with the CTEQ6M set ($[S^-]=0$), are given in the last column. The quality of these new fits is gauged by the relative χ^2 values (normalized to that of the reference set "B") for the dimuon data set [17] and for the subset of global data set which have some sensitivity to $s(x)-\bar{s}(x)$ (labeled "inclusive I"). See [11] for details.

Following the notation in Ref. [11], the PDF sets A,B,C are representative good fits that span the allowed parameter space. They all have $s(x)\neq\bar{s}(x)$, and $[S^-]>0$. In our calculations, we employ these PDFs consistently; i.e., the full set $\{f(x,Q)\}$ of PDFs, not just the strange distributions.

Our calculation of R^-, compared to the R^- obtained using the CTEQ6 PDFs [14], yields

$$\delta R^- \equiv R^-_{\{\rm A,B,C,B^+,B^-\}} - R^-_{\rm CTEQ6}, \quad (8)$$

values of which are given in Table I along with a summary of the underlying PDFs. We show not only the preferred fit values for the sets A, B, C but also results for fits B$^\pm$ that have been obtained from the Lagrange multiplier method by pushing the limits of the allowed $[S^-]$ value in both directions beyond the preferred range, as described in [11]. The quality of the fits is indicated by the relative χ^2 values, which are normalized to the reference solution "B". Thus, the values in the B-row are 1.0 (italized) by definition. The three preferred sets A, B, C are comparable in quality; the extreme sets B$^+$ and B$^-$ are clearly disfavored. Ref. [11] suggests that a reasonable range for $[S^-]$, consistent with current experimental and theoretical constraints, is 1–3$\times 10^{-3}$. While a value of 0 (no strangeness asymmetry) is not necessarily excluded, a negative $[S^-]$ is definitely disfavored.

In an experiment that could measure R^- directly, the values of δR^- in Table I would correspond to a negative shift in measured $\sin^2\theta_{\rm W}$ based on an analysis with $[S^-]=0$ (cf. Eq.(1)),

$$\delta R^- \simeq -\delta \sin^2\theta_{\rm W} \ . \quad (9)$$

We find that the shift in R^-—calculated as an average over (anti-)neutrino energy according to the flux spectra—is relatively insensitive to the incident neutrino energy. The results for δR^- in Table I are also not modified significantly when the cut on yE_ν is eliminated. This suggests that the incorporation of other detector effects [5, 19] which make the analysis in Ref. [1] somewhat more involved than a direct measurement of R^- will not significantly impact the importance of the $[S^-]$ contribution to $\sin^2\theta_W$[24].

The shift based on the central fit B can bridge up to about $1.5\,\sigma$ of the overall $3\,\sigma$ discrepancy between the NuTeV result and the world average of other measurements of $\sin^2\theta_W$. For fits with higher $[S^-]$, such as "C", it is possible to reduce the discrepancy to within $1\,\sigma$.

In conclusion, the new global analysis [11], which for the first time includes neutrino dimuon data, considerably strengthens the case for a positive $[S^-] > 0$, which previously has been made both on phenomenological [4] and dynamical model [18] bases. Ref. [11] showed that this conclusion follows rather generally from the interplay between the dimuon and other inclusive ("inclusive I") data on the one hand, and the theoretical constraints of sum rules on the other hand. These experimental and theoretical inputs require that $[s(x) - \bar{s}(x)] > 0$ for a substantial part of the large x region. This new non-perturbative feature of the nucleon sea shifts the value of $\sin^2\theta_W$ which was extracted from neutrino DIS data under the assumption that $[s(x) - \bar{s}(x)] = 0$ [3]. The results of this study, stated in the previous paragraph, suggest that the new dimuon data, the Weinberg angle measurement, and other global data sets used in QCD parton structure analysis can all be consistent within the standard model of particle physics.

While our study is model-independent, we have neglected isospin violations and (higher-twist) nuclear modifications. These may occur. They will be very hard to constrain in a model-independent global analysis, though, because there is no process to signal either of the two [6]. More input on $[s(x) - \bar{s}(x)]$ would, of course, be helpful in pinning down the contribution of strangeness asymmetry to δR^-. Measurements of associated production of charmed jets and $W^\pm$-bosons at the Tevatron, at RHIC or at the future LHC would increase our knowledge of $s(x)$ and $\bar{s}(x)$. Compared to determining $[s(x) + \bar{s}(x)]$ in the same process (cf. [20]), the situation for $[s(x) - \bar{s}(x)]$ may be more favorable in that the "valence" density is less diluted by evolution to the high scale $Q \sim M_W$ than the predominantly singlet $[s(x) + \bar{s}(x)]$, which is concentrated in the small-x region. Meanwhile, since $[s(x) - \bar{s}(x)]$ is a non-singlet quantity, it should also be feasible to study it on the lattice by extending techniques in [21]. We expect fascinating developments along these lines.

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