

I. LECTURE 1: INTRODUCTION TO PHYSICS

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A. The Goal of Physics

What is physics? What is physics *about*?

It is easy to answer those questions for other sciences. What is chemistry about? Chemistry is about atoms and molecules, and their reactions. What is biology about? Biology is about life. What is geology about? Geology is about the Earth. But what is physics about? What is the *subject* of physics?

My answer is that physics is about the basic laws of nature, expressed in mathematics.

The goal of physics is to learn how nature works. Experimental physics tests our current understanding of nature, and seeks to discover new phenomena to extend our knowledge. Theoretical physics puts our knowledge of nature into precise mathematical equations, and analyses those mathematical theories to describe natural phenomena.

So physics is about a lot of different things. We observe various phenomena to study the laws of nature. Motion, forces, waves, electricity and magnetism, light, atoms, and many other natural phenomena, all belong to physics, because they all teach us something about how nature works.

In these lectures we will study some of the basic laws of nature: Newton's laws of motion in mechanics, the conservation laws of energy and momentum, Maxwell's equations of electric and magnetic fields, the principles of quantum theory. These are all fundamental aspects of nature, that we express in mathematical terms.

B. Goals for a physics course in high school

High-school physics courses serve several purposes, not only for students who will go on to study science or engineering in college, but also for those who do not pursue a scientific career. But here I want to describe ways that I think a high-school physics course can help those students who do go on to take physics in college. I must admit it's sort of arrogant for me to state goals for a high-school course, because I've never taught in a high school. On the other hand, I've known enough physics students at MSU to have some ideas about what prior skills will help students who take physics in college.

Physics is an important part of the education of any science student, or engineering student. At MSU, a student majoring in any science is required to complete 2 semesters of college physics; an engineering major is required to complete 2 semesters of physics with calculus. Any background from high school that helps the student in his or her college physics courses is valuable to the student.

Here's my *personal* list of goals for a high school physics course, which I believe will help students who go on to take physics in college:

1. knowledge of the concepts of physics;
2. laboratory experience: taking data, keeping a lab notebook, and writing lab reports;
3. math skills, especially *applied* math, including
 - (a) how to calculate with units,
 - (b) how to calculate with variables, *i.e.*, *algebra*,
 - (c) vectors and trigonometry,
 - (d) how to plot graphs,
 - (e) knowledge of basic functions (sin, cos, exp, log),
 - (f) how to solve story problems, where the equation is not given.

Of course these goals are *the ideal*, and it would be very rare to achieve all of them in any real course. But any of these would be valuable to a student taking physics in college.

In my personal experience, the biggest difference between students who do well in a college physics course, and those who struggle through the course, is point 3 above: math skills. I wonder how many times I've had a student tell me "I understand the concepts, but I just can't do the problems".

C. Physics and Mathematics

Physics is an experimental science, and physics is also a mathematical science. Theory and experiment progress together in physics, inextricably connected. The success of physics comes from the precision of its theories, and the theories are *mathematical*. The beauty of physics is that simple mathematical relations describe nature. In our workshop we will study some of these mathematical relations. We will not use very advanced

mathematics like calculus, but we will definitely include some of the math skills listed in the previous section.

The role of mathematics in physics has a long history. The first great mathematical physicist was Archimedes, who lived in Syracuse, on the island of Sicily, from 287 to 212 BC. (Syracuse was an independent Greek city, but was conquered by Rome at the end of Archimedes' life. In fact, Archimedes acted as military engineer for the King of Syracuse, and invented military devices that were used in the defence of the city against Rome. His military inventions were very effective, according to Roman history. (See Plutarch's life of Marcellus.) It took the Roman army and navy over 2 years to break the defence of Syracuse. Archimedes was killed by a Roman soldier during the fall of Syracuse.)

In physics Archimedes is remembered for several discoveries: the theory of levers ("Give me a place to stand and I will move the earth"), density ("Eureka") and buoyancy (Archimedes' Principle). But Archimedes was first a mathematician. Some of his mathematical discoveries were: the calculation of π , the formula for the area of a circle, the mathematics of spheres and cylinders, and the spiral curve. So you see, physics and mathematics have been connected since the beginning of physics. (According to history, Archimedes was studying a geometric proof when he was killed by a Roman soldier.)

Physics, and other sciences, underwent a revolution during the 17th century. One important aspect of that revolution was the application of mathematics to describe the physical world. I want to review two of the major figures in that scientific revolution – Galileo and Newton.

D. Galileo

Physics, as we practice it today, began with Galileo (1564-1642). Galileo was probably the most remarkable individual in the history of science. He revolutionized both astronomy and physics in his time. In astronomy, he overthrew the ancient theory of Ptolemy (which got him into a lot of trouble with the Inquisition in Rome!). In physics, he overthrew the ancient theories of Aristotle.

But Galileo was first a mathematician. He worked as professor of mathematics at the universities of Pisa and Padua for over 20 years, before he became famous for his astronomical discoveries. During that time he was often involved in disputes with the "natural philosophers" (*i.e.*, scientists), who adhered rigidly

to the teachings of Aristotle. The Aristotelians dismissed Galileo's criticisms of their science, because he was "only a mathematician".

Galileo did not invent the telescope, but he did make telescopes, shortly after he learned of the invention, using designs that he worked out by mathematical analysis. He made the best telescopes of his day. Because he had the best telescopes, he made the most significant discoveries in astronomy; for example:

- craters and mountains on the moon,
- faint stars,
- that the Milky Way consists of "innumerable stars",
- four moons of Jupiter,
- the phases of Venus,
- sunspots and the motion of sunspots.

It seems that every time Galileo turned his telescope to another object in the sky, he made an important new discovery. He was considered greater than Columbus: Columbus had discovered new lands on this world, but Galileo discovered whole new worlds! In the end, his discoveries provided the proof of the Copernican theory. However, for arguing in favor of Copernicus, Galileo was accused of heresy, put on trial, sentenced to house arrest, and ordered to desist from discussing cosmology.

Galileo became famous because of his astronomical discoveries. But he also made important discoveries in physics (which was called natural philosophy in Galileo's time). Let's consider some of his physics discoveries:

- The pendulum is *isochronous*.
(That is, the period of swinging is constant, independent of amplitude; this is approximately true for small amplitudes.) This was Galileo's first discovery in science, made while he was a medical student in Pisa. Notice that it is a *quantitative* statement. The well-known story is that he made this discovery by observing a swinging lamp in the cathedral, and timing its swing using his pulse.
- All objects fall with the same constant acceleration, except for the effect of air resistance.
Galileo was professor of mathematics at Pisa, and there's always been a story that he demonstrated this discovery by dropping objects from the leaning tower. The story may or may not be true, but

in any case it took great genius to overlook the differences between falling objects, and to realize that the essential fact is that the acceleration would be the same if air resistance could be eliminated. This discovery may seem a small matter today, but at the time it was significant: It contradicted Aristotle, and, as Galileo knew, it was the beginning of the collapse of the Aristotelian theory of physics.

- The law of *inertia*.

That is, an isolated object at rest remains at rest, and an isolated object in motion remains in motion. This law is in complete disagreement with Aristotle's theory. Aristotle taught that the natural state of any mass is to be at rest, so an object in motion naturally slows down and comes to rest. Again, it took the great genius of Galileo to look beyond the everyday experience that moving objects do come to rest, and to realize that *friction*, a force exerted by some other object, is what causes a moving object to slow down and come to rest; but a truly isolated object would move forever. He came to this startling conclusion by experiments with balls rolling down and up inclined planes. This is brilliant thinking, to reach such a far-reaching statement from such simple observations!

- The mathematics of constant acceleration.

Again from observing balls rolling on inclined planes, Galileo determined that for constant acceleration the distance traveled in successive constant time intervals grows by increments that are precisely the odd integers. The distances after fixed time intervals are 0, 1, 4, 9, 16, 25, ... (in some unit of length) and so the increments are 1, 3, 5, 7, 9, ... (in the same unit). These were the first experiments in science that required *accurate timing*. Galileo used the amount of water flowing out of a large basin to time the motion of the ball. (There were no accurate clocks in Galileo's day; indeed, his discovery that the pendulum is isochronous was the principle for the invention of accurate clocks.)

Some equations The velocity and distance traveled, of an object with constant acceleration a , are, as functions of time t

$$v = v_0 + at$$

$$d = v_0t + \frac{1}{2}at^2$$

where v_0 is the velocity at $t = 0$.

The height of a falling object as a function of time t is, neglecting the effect of air resistance,

$$h = h_0 + v_0t - \frac{1}{2}gt^2$$

where h_0 and v_0 are the initial height and velocity; the acceleration is $a = -g$, negative indicating downward.

Now, back to the story:

During the last years of his life, while under house arrest, Galileo wrote his great book on physics, *Two New Sciences*. (He had this book published in Leyden, out of caution after what had happened to him earlier when he published the *Dialogue*, on the Copernican system, in Italy.) The two new sciences were *motion* and *strength of materials*.

The point of my review of Galileo is this: he was a mathematician, and he used mathematics to describe nature. He was also an experimenter, and the purpose of his experiments was always to reach quantitative, *i.e.* mathematical, results. Mathematics has always been important in physics, since the very beginning of the subject.

Finally, here's what Galileo himself wrote about mathematics and nature, from *The Assayer*; ("Sarsi" refers to an astronomer who had challenged Galileo's authority, and whom Galileo attacked mercilessly in *The Assayer*):

'In Sarsi I seem to discern the firm belief that in philosophizing one must support oneself upon the opinion of some celebrated author ... Well, Sarsi, that is not how matters stand. Philosophy is written in this grand book, the universe, which stands continually open to our gaze. But the book cannot be understood unless one first learns to comprehend the letters in which it is composed. It is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures without which it is humanly impossible to understand a single word of it; without these one wanders about in a dark labyrinth.'

E. Newton

The greatest mathematical physicist was Isaac Newton (1642-1727). He was born the same year that Galileo died. He carried on Galileo's approach to physics, using mathematics, and completed the revolution that Galileo started.

Newton published just two books – *Opticks* and the *Principia*. The two books are very different. In fact, looking at the two together it is hard to believe they were both written by the same person, because they are so different. *Opticks* concerns Newton's experiments with light, *e.g.* his famous demonstration that white light is a superposition of all the colors. This book is descriptive, not mathematical; there are few calculations or equations. In contrast, the *Principia* (the English translation of the title is *Mathematical Principles of Natural Philosophy*) is filled with mathematics. The subject is presented in terms of definitions, theorems, proofs, geometric constructions, and equations. On the surface the content looks like pure mathematics. But the *Principia* is a book about nature. In it Newton solved the *premier* science problem of his day, which was to understand the motion of the planets. In order to solve this problem, Newton had to discover the general laws of motion and of gravitation. And to analyse these physical laws, he had to develop a new branch of mathematics, which we now call *Calculus*.

Newton deduced, from Kepler's empirical laws of planetary orbits, the existence of a force of attraction \vec{F} between the planet and the sun, that is inversely proportional to the square of the separation, $\vec{F} \propto -\hat{r}/r^2$.

At that time there was an existing theory of planetary motion, formulated by Descartes, which would compete with Newton's theory of gravitational attraction. Descartes' theory postulated that the planets are carried around the sun by vortices in an unseen, interplanetary, fluid-like medium. But the Cartesian theory could make no *quantitative* statements about the motion. The mathematical accuracy of Newton's theory demolished the vortex theory. More generally, Newton's method – using mathematics to describe the laws of nature – became the basis for most later developments in physics.

We'll study Newton's laws of motion in the next few lectures. For today, let's look at Newton's laws in his own words (translated from the Latin of the *Principia*):

1. "Every body perseveres in its state of resting or of moving uniformly straight ahead except insofar as it is compelled by impressed forces to change

that state."

2. "A change in motion is proportional to the motive force impressed, and takes place along a straight line on which that force is impressed."
3. "To any action there is always an opposite and equal reaction; in other words, the actions of two bodies upon each other are always equal and are opposite in direction."

The *Principia* was published in 1687, and we still use these same laws of motion in mechanics today. This is an idea that has stood the test of time! For example, when NASA needs to calculate the trajectory of a satellite, they use Newton's laws of motion.

My point in reviewing Newton is this: Newton was first a mathematician. He trained himself in mathematics as a student. He was professor of mathematics at Cambridge University for over 20 years. His mathematical interests included infinite series, tangents and areas of curves, and the theory of motion. These interests led to his development of calculus.

One story will suffice to indicate Newton's constant interest in mathematics. The Swiss mathematician Johan Bernoulli posed the problem of the *brachistochrone*: find the shape of that curve, connecting two given points, such that an object, sliding down the curve under the influence of gravity, would travel from the higher point to the lower point in the least time. (In Greek, brachist=shortest, chronos=time) Bernoulli challenged all the mathematicians in Europe to find the shape of the curve.

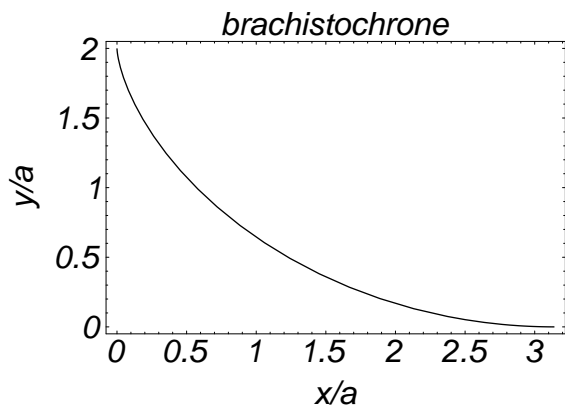


FIG. 1. The brachistochrone

He sent a copy of the problem to Newton, believing that Newton would not be able to solve the problem. (Bernoulli took the side of Leibniz in the dispute between Newton and Leibniz over who had invented calculus.) This challenge occurred after Newton had already established his fame as the greatest scientist of the day, and he did not need to prove his ability to the world. Nevertheless, Newton stayed up all night solving the problem, the same day that he received it. And he sent the solution back to Bernoulli *anonymously*, a sort of an insult, implying that the problem wasn't even worth enough to put his name on it. Bernoulli received 3 solutions to his problem: from Leibniz, from de l'Hopital, and from an anonymous person in England. He said, in a famous quote, that he recognized Newton's work "as the lion is recognized by his paw".

(The solution of the problem is that the brachistochrone curve is a cycloid.)

Newton did have other interests besides mathematics, *e.g.*, optics, alchemy, and theology. But his first interest was mathematics, and his greatest discovery was the mathematical theory of motion, especially applied to astronomy.

Physics is a mathematical science as well as an experimental science. The purpose of the lectures in this workshop is to study some examples of *theory* in physics.

Exercises for Lecture 1

Use the equations on page 3 to solve these problems.

1. Consider a car moving with a constant speed of 60 mph.
 - (a) Calculate how many feet it travels in 1 second.
 - (b) Calculate how many seconds it takes to travel 100 yards.
2. (a) A drag strip race car "goes from 0 to 60 in 6 seconds". Assume uniform acceleration. Calculate how many feet the car travels during the 6 seconds.
 - (b) Compare the car's acceleration to g .
3. A diver jumps from a 10 m platform. Calculate how much time she has, during which she must execute all her flips and twists, before she enters the water.