Grand Unified Theory
SU(5) Model

(Georgi & Glashaw 1974)

1. Local gauge interaction

$$SU(5) \supset SU(3)_C \times SU(2)_L \times U(1)_Y$$

Contains only one gauge coupling.

<u>Note</u> ① The SM gauge interactions of quarks and leptons are completely fixed by their gauge charges.

② There are $\underline{15} = \underline{10} + \underline{5}^*$ left-handed two-component fermion fields in each family.

$\Rightarrow$ SU(5) is to unify these 15 fermion fields.

$$\underline{5}^*: \quad \left(d_r^c, d_b^c, d_g^c, e^-, -\nu_e \right)_L$$

$$\underline{10}: \quad \left(\begin{array}{ccc|cc} 0 & u_g^c & -u_b^c & u_r & d_r \\ -u_g^c & 0 & u_r^c & u_b & d_b \\ u_b^c & -u_r^c & 0 & u_g & d_g \\ \hline -u_r & -u_b & -u_g & 0 & e^+ \\ -d_r & -d_b & -d_g & -e^+ & 0 \end{array} \right)_L$$

where the charged conjugate field

$$\psi^c = C\gamma^0\psi^* = i\gamma^2\psi^*$$

$$(\psi_R)^c = (\psi^c)_L \equiv \psi^c_L$$

2. In SU(5), the electric charge Q for $\underline{5}$-representation

$$Q(\psi_i) = \begin{pmatrix} -\frac{1}{3} & & & & \\ & -\frac{1}{3} & & & \\ & & -\frac{1}{3} & & \\ & & & 1 & \\ & & & & 0 \end{pmatrix},$$

(Q is traceless, as one of the 24 generators)

where

$\underline{5}$: $(\psi_i)_R = (d_r, d_b, d_g, e^+, -\nu_e^c)_R$

$\Rightarrow$ $\boxed{3Q_d + Q_{e^+} = 0}$ $\Rightarrow$ $\boxed{Q_d = -\frac{1}{3}}$

(Traceless Condition)

explains why quarks carry fractional charge.

3. SU(5) has $5^2-1=24$ generators, hence there are 24 gauge boson's.

$\Rightarrow$

$$\underline{24} = (\underline{8},\underline{1}) + (\underline{1},\underline{3}) + (\underline{1},\underline{1}) + (\underline{3},\underline{2}) + (\underline{3}^*,\underline{2})$$

8 gluons $\quad$ $W^\pm, W^0$ $\quad$ B $\quad$ 12 new heavy gauge bosons,

Z, γ

X & Y

The gauge boson masses are generated by Spontaneous Symmetry breaking mechanism:

$$SU(5) \xrightarrow{\langle \Phi \rangle} SU(3)\times SU(2)\times U(1)_Y \xrightarrow{\langle \phi \rangle} SU(3)\times U(1)_{em}$$

(X, Y gain mass.) $\sim 10^{15}$ GeV

($W^{\pm}, Z$ gain mass) $\sim 10^{2}$ GeV

4. Gauge Coupling unification & $\sin^2\theta_W$

At the unification scale M_X:

$$D_\mu = \partial_\mu + i g_5 \sum_{a=0}^{23} A^a_\mu \frac{\lambda^a}{2}$$

Compared to

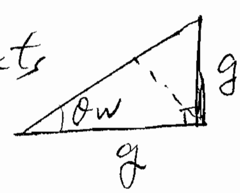
$$D_\mu = \partial_\mu + i g_s \sum_{a=1}^{8} G^a_\mu \left(\frac{\lambda^a}{2}\right) + i g \sum_{j=1}^{3} W^j_\mu \left(\frac{\tau^j}{2}\right) + i g' B_\mu \left(\frac{Y}{2}\right)$$

$\Rightarrow \quad g_5 = g_3 = g_2 = g_1 \qquad$ with $g_3 \equiv g_s, \quad g_2 \equiv g.$

Since $Y = -\sqrt{\frac{5}{3}}\,\lambda^0$, so $\quad g' = -\sqrt{\frac{3}{5}}\, g_1$

$$\Rightarrow \quad \sin^2\theta_W \equiv \frac{g'^2}{g^2 + g'^2} = \frac{3}{8} \quad (\text{at } M_X \text{ scale})$$

By renormalization equation, SU(5) predicts



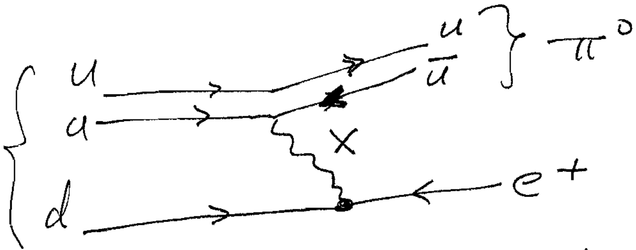
$$\sin^2\theta_W = \frac{3}{8} - \frac{55\,\alpha(\mu)}{24\pi} \ln\left(\frac{M_X}{\mu}\right)$$

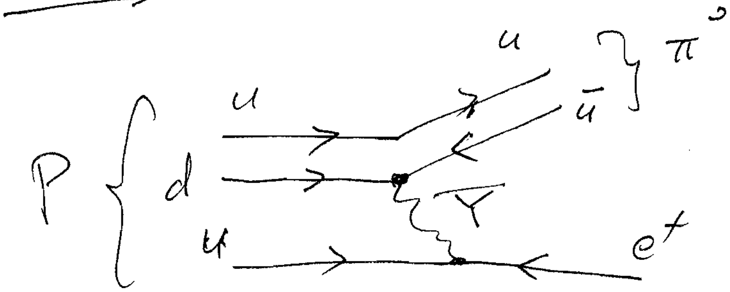
$\simeq 0.21 \qquad$ for $M_X \simeq 4\times 10^{14}$ GeV, $\mu \simeq M_W = 80$ GeV

5. Proton decay

$p \to e^+ \pi^0$

$(n \to e^+ \pi^-)$





1) X and Y violate Baryon number (They are "leptoquarks" or "diquarks".)

2) Lifetime $\tau_p \sim \frac{1}{\Gamma_p}$ with $\Gamma_p \sim \frac{\alpha_5^2 m_p^5}{M_X^4}$

$\Rightarrow$ ($\tau_p \sim 10^{30}$ years) for non-susy SU(5)

3) SU(5) model has been excluded by Super-Kamiokande

$\tau(p \to e^+\pi^0) > 5 \times 10^{33}$ years (79.3 ktyr exposure)

$\tau(n \to e^+\pi^-) > 5 \times 10^{33}$ yrs (61 ktyr)

$\tau(p \to K^+\bar{\nu}) > 1.6 \times 10^{33}$ yrs (79.3 ktyr)

$\tau(n \to K^0\bar{\nu}) > 1.7 \times 10^{32}$ yrs (61 ktyr)

$\Rightarrow$ It also rules out "minimal" susy SU(5), but not susy SO(10), etc.

6. Fermion masses and Yukawa Unification

1) SU(5) model predicts

$$\frac{m_b(\mu)}{m_\tau(\mu)} \simeq 3 \quad \left(\text{at } \mu \simeq 10\ \text{GeV}\right), \quad \left(\begin{array}{l} m_b \simeq 4.5\ \text{GeV} \\ m_\tau \simeq 1.7\ \text{GeV} \end{array}\right)$$

from the SU(5) GUT scale relation $\lambda_b = \lambda_\tau$.

2) In SUSY SO(10), the t-b-τ Yukawa coupling unification is possible for $\tan\beta \sim (40-50)$, with

$$m_t = \lambda_t v_u \qquad v_u = \frac{v}{\sqrt{2}} \sin\beta \equiv \langle H_u^0 \rangle$$

$$m_b = \lambda_b v_d \qquad v_d = \frac{v}{\sqrt{2}} \cos\beta \equiv \langle H_d^0 \rangle$$

$$m_\tau = \lambda_\tau v_d, \qquad v \simeq 246\ \text{GeV}$$