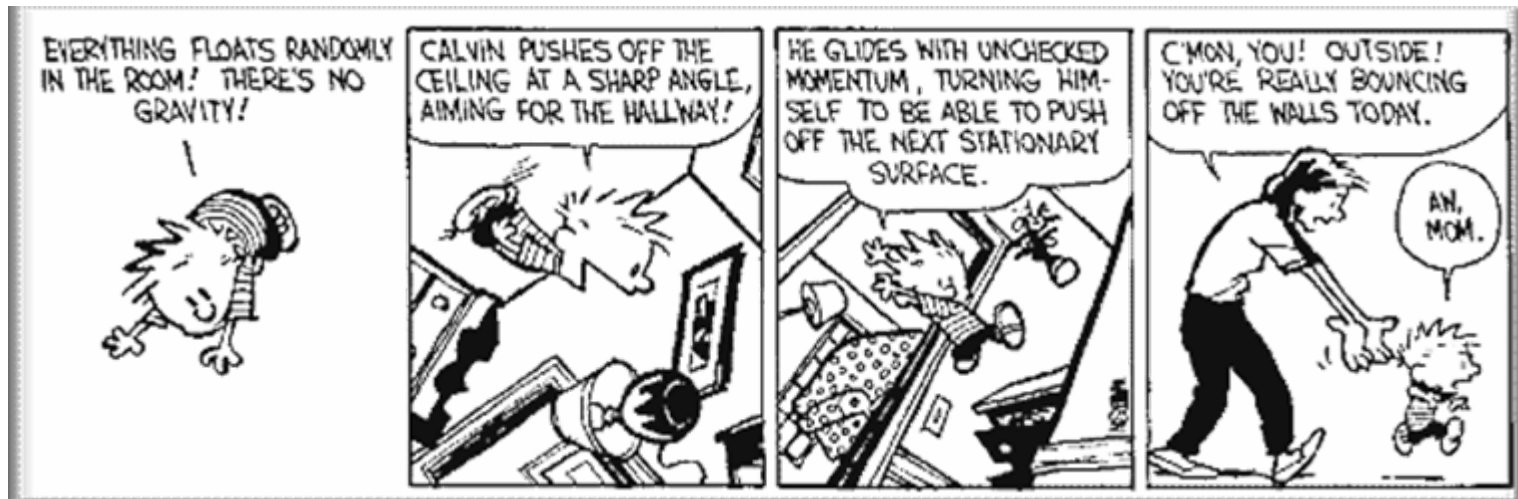
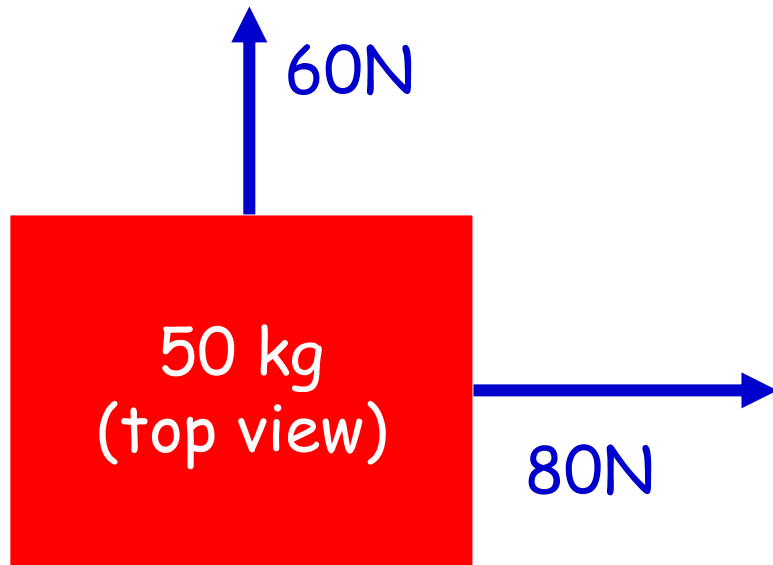


PHYSICS 231

Lecture 10: Too much work!





Two persons are dragging a box over a floor. Assuming there is no friction, what is the acceleration of the crate?

- a) 1.2 m/s^2
- b) 1.6 m/s^2
- c) 2.0 m/s^2
- d) 2.8 m/s^2
- e) no acceleration whatsoever

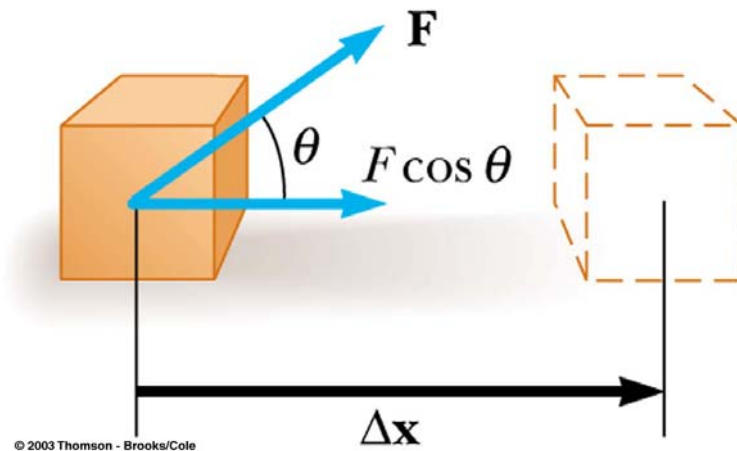
WORK

Work: 'Transfer of energy'

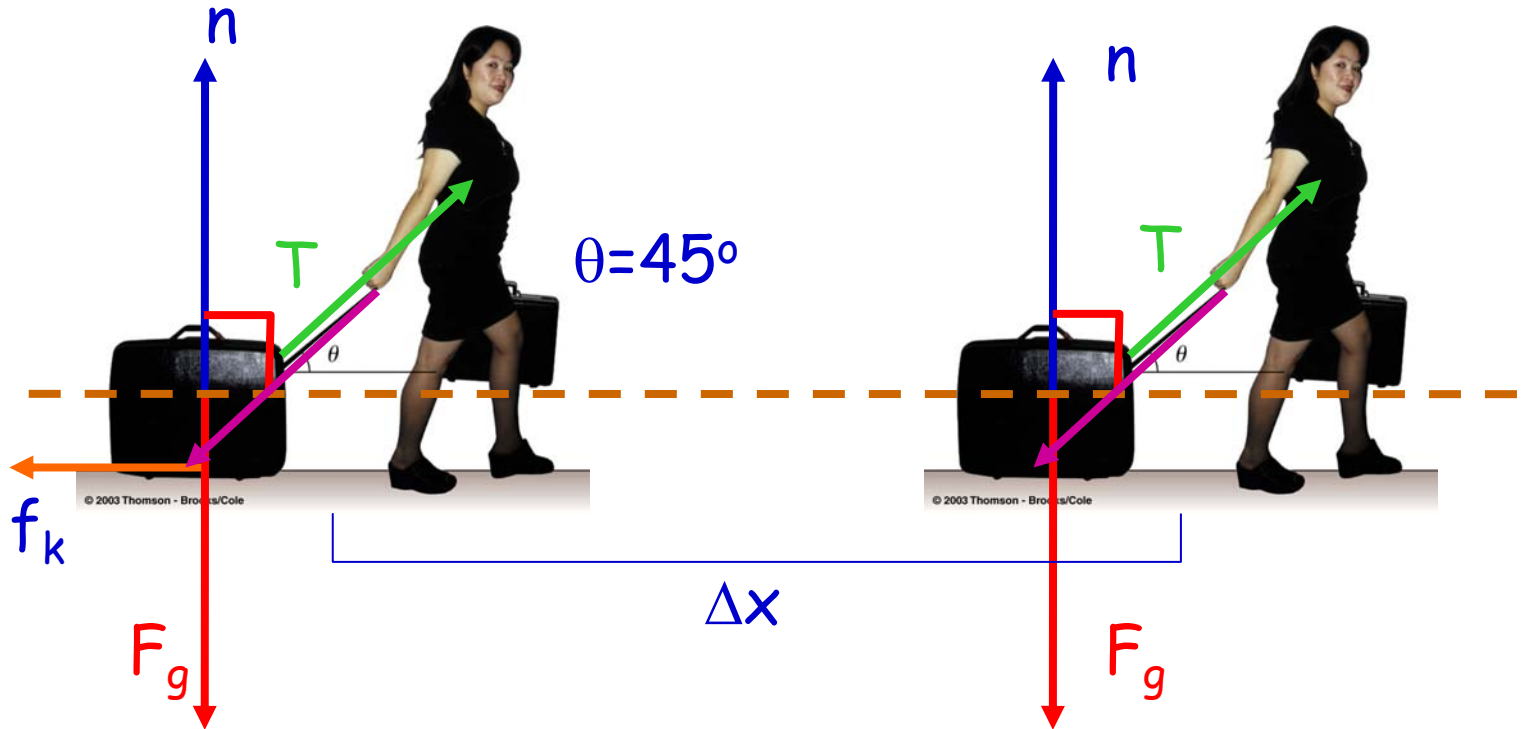
Quantitatively: The work W done by a constant force on an object is the product of the force along the direction of displacement and the magnitude of displacement.

$$W = (F \cos \theta) \Delta x$$

Units: = N m = Joule



An example



The work done by the person on the suitcase: $W=(T\cos 45^\circ)\Delta x$

The work done by F_g on the suitcase: $W=(F_g\cos 270^\circ)\Delta x=0$

The work done by n on the suitcase: $W=(F_g\cos 90^\circ)\Delta x=0$

The work done by friction on the suitcase: $W=(f_k\cos 180^\circ)\Delta x=-u_k n\Delta x$

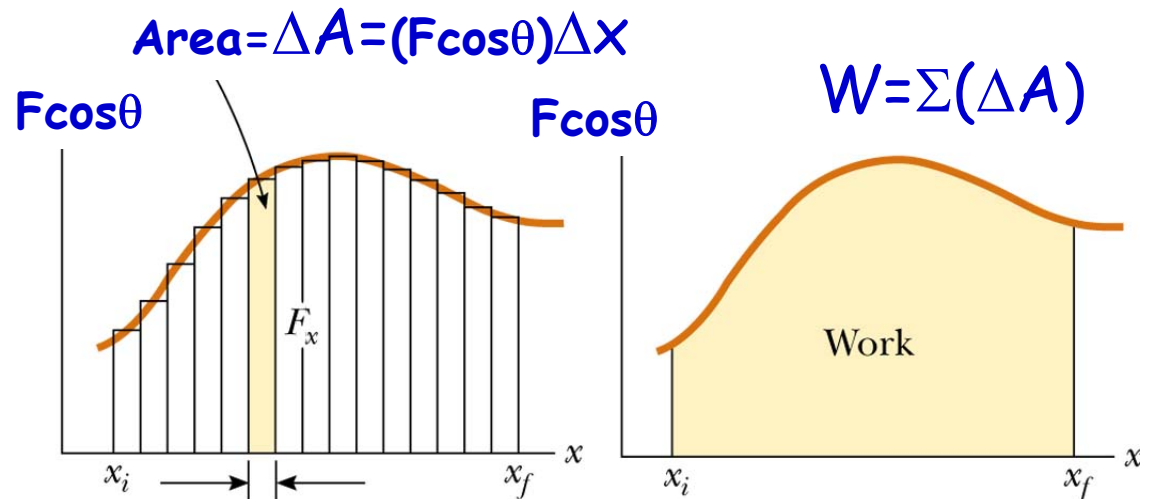
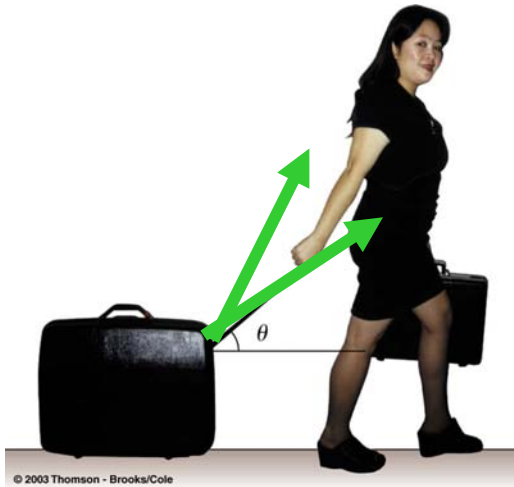
The work done by the suitcase on the person: $W=(T\cos 225^\circ)\Delta x$

opposite

Non-constant force

$W=(F\cos\theta)\Delta x$: what if $F\cos\theta$ not constant while covering Δx ?

Example: what if θ changes while dragging the suitcase?



The work done is the same as the **area** under the graph of $F\cos\theta$ versus x

Power: The rate of energy transfer

Work (the amount of energy transfer) is independent of time.

$$W = (F \cos \theta) \Delta x \dots \text{no time in here!}$$

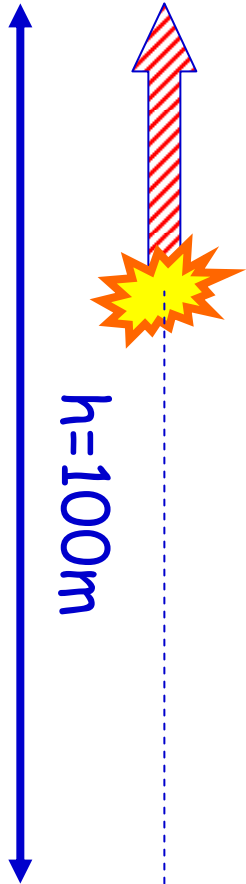
To measure how fast we transfer the energy we define:

Power(P) = $W / \Delta t$ (J/s = Watt) (think about horsepower etc).

$$P = (F \cos \theta) \Delta x / \Delta t = (F \cos \theta) v_{\text{average}}$$

Example

A toy-rocket of 5.0 kg, after the initial acceleration stage, travels 100 m in 2 seconds. What is the work done by the engine? What is the power of the engine?



$$W=(F\cos\theta)\Delta h=m_{\text{rocket}}g \Delta h=4905 \text{ J}$$

(Force by engine must balance gravity!)

$$P=W/\Delta t=4905/2=2453 \text{ W (=3.3 horsepower)}$$

Or

$$1 \text{ hp} = 750 \text{ W}$$

$$P=(F\cos\theta)v=mgv=5.0 \cdot 9.81 \cdot 100/2=2453 \text{ W}$$

Potential Energy

Potential energy (PE): energy associated with the position of an object within some system.

Gravitational potential energy: Consider the work done by the gravity in case of the rocket:

$$W_{\text{gravity}} = F_g \cos(180^\circ) \Delta h = -mg \Delta h = -(mgh_f - mgh_i) = mgh_i - mgh_f \\ = PE_i - PE_f$$

The 'system' is the gravitational field of the earth.

$$PE = mgh$$

Since we are usually interested in the change in gravitational potential energy, we can choose the ground level ($h=0$) in a convenient way.

Another rocket

A toy rocket (5kg) is launched from rest and reaches a height of 100 m within 2 seconds. What is the work done by the engine during acceleration?

$$h(t) = h(0) + v_0 t + 0.5 a t^2 \quad 100 = 0.5 a 2^2 \quad \text{so } a = 50 \text{ m/s}^2$$

$$V(t) = V(0) + a t \quad V(2) = 0 + 50 * 2 = 100 \text{ m/s}$$

$$\text{Force by engine} = (50 + 9.81)m = 59.81 * 5 = 299.05 \text{ N}$$

(9.81 m/s² due to balancing of gravitation)

$$W = F \Delta h = 299.05 * 100 = 29905 \text{ J}$$

Change in potential energy:

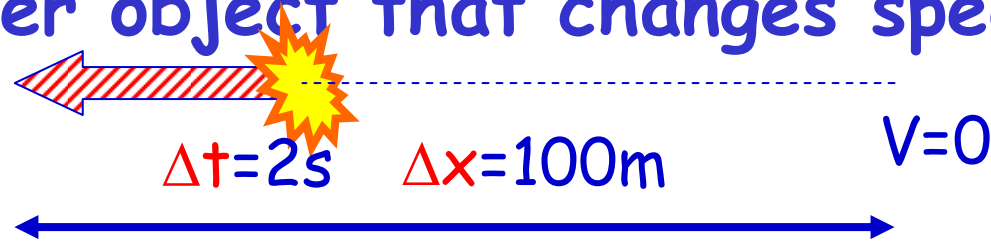
$$PE_f - PE_i = mgh(2s) - mgh(0) = 4905 - 0 = 4905 \text{ J}$$

Where did all the work (29905 - 4905 = 25000 J) go?

Into the acceleration: energy of motion (kinetic energy)

Kinetic energy

Consider object that changes speed only



a) $W = F\Delta x = (ma)\Delta x$... used Newton's second law

b) $v = v_0 + at$ so $t = (v - v_0)/a$

c) $x = x_0 + v_0t + 0.5at^2$ so $x - x_0 = \Delta x = v_0t + 0.5at^2$

Combine b) & c)

d) $a\Delta x = (v^2 - v_0^2)/2$

Combine a) & d)

$$W = \frac{1}{2}m(v^2 - v_0^2)$$

Kinetic energy: $KE = \frac{1}{2}mv^2$

When work is done on an object and the only change is its speed: The work done is equal to the change in KE:

$$W = KE_{\text{final}} - KE_{\text{initial}}$$

Rocket:

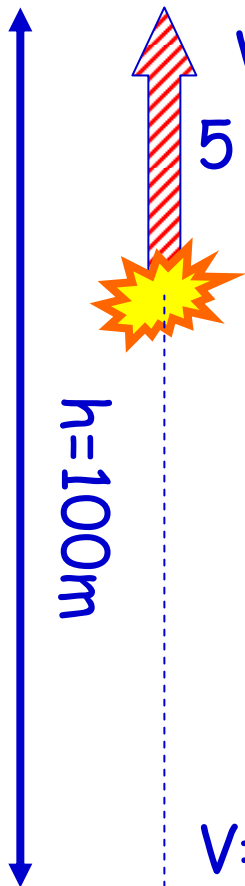
$$W = \frac{1}{2}5(100^2 - 0^2) \\ = 25000 \text{ J!!}$$

That was missing!

Conservation of mechanical energy

Mechanical energy = potential energy + kinetic energy

In a **closed system**, mechanical energy is conserved*



$$V=100 \quad ME=mgh+\frac{1}{2}mv^2=\text{constant}$$

5 kg

What about the accelerating rocket?

At launch: $ME=5*9.81*0+\frac{1}{2}5*0^2=0$

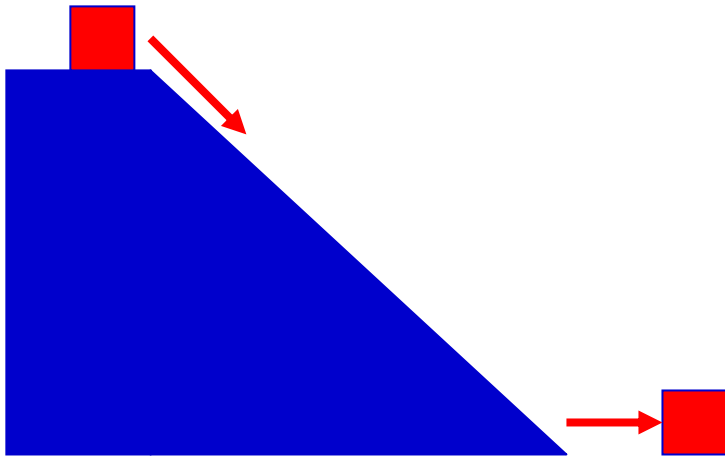
At 100 m height: $ME=5*9.81*100+\frac{1}{2}5*100^2=29905$

We did **not** consider a closed system! (Fuel burning)

* There is an additional condition, see next lecture

V=0

question



In the absence of friction, which energy-time diagram is correct?

- potential energy
- total energy
- kinetic energy

C

