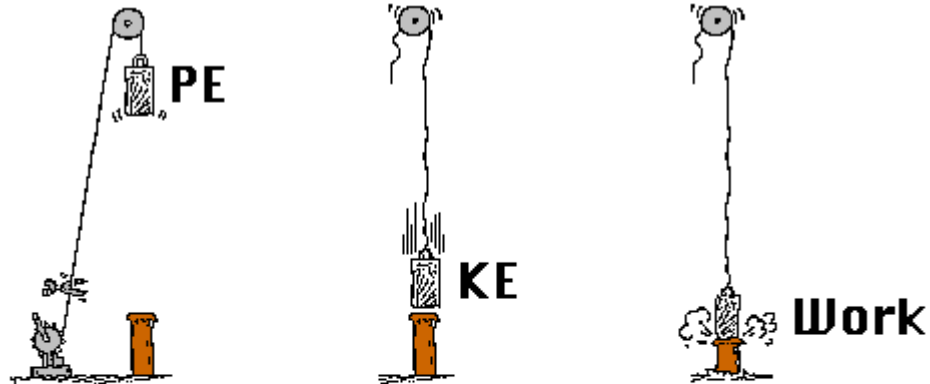


PHYSICS 231

Lecture 13: Keeping momentum



The ram of a pile-driver possesses mechanical energy – the ability to do work. When held at a height, it possesses mechanical energy in the form of potential energy. As it falls it possesses mechanical energy in the form of kinetic energy. As it strikes the spike, it applies a force to displace the spike – i.e., it does work on the spike.

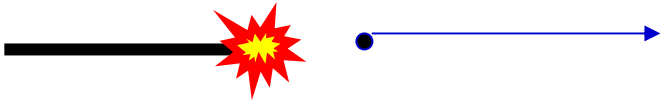
quiz (extra credit)



At which point are the **kinetic** and **potential** energy highest, respectively?

- a) A and B
- b) D and E
- c) D and A
- d) B and A
- e) D and D

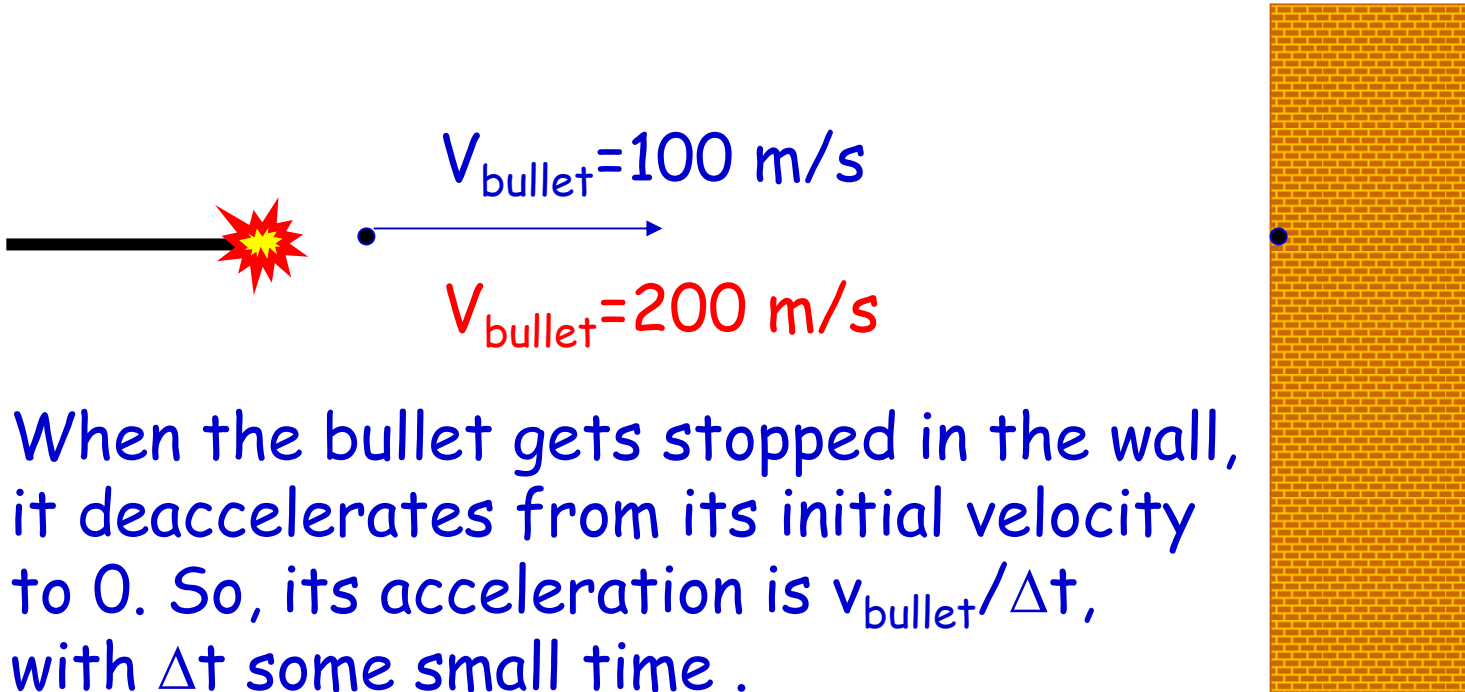
Momentum & Collisions



When a bullet hits the wall, its velocity is very much reduced. The wall does not move, although the force on the bullet is the same as the force on the wall (Newton's 3rd law: $F_{\text{wall-bullet}} = -F_{\text{bullet-wall}}$).

$$\begin{array}{l} F_{\text{wall-bullet}} = m_{\text{bullet}} a_{\text{bullet}} \\ F_{\text{bullet-wall}} = m_{\text{wall}} a_{\text{wall}} \end{array} \quad \rightarrow \quad M_{\text{bullet}} \ll M_{\text{wall}}$$
$$|a_{\text{bullet}}| \gg a_{\text{wall}}$$

Is it only the mass???



When the bullet gets stopped in the wall, it deaccelerates from its initial velocity to 0. So, its acceleration is $v_{\text{bullet}}/\Delta t$, with Δt some small time .

$$\text{Second law: } F_{\text{wall-bullet}} = m_{\text{bullet}} a_{\text{bullet}} = m_{\text{bullet}} v_{\text{bullet}} / \Delta t$$

More general...and formal.

$$F=ma$$

Newton's 2nd law

$$F=m\Delta v/\Delta t$$

$$a=\Delta v/\Delta t$$

$$F=m(v_{\text{final}}-v_{\text{initial}})/\Delta t$$

Define $p=mv$

p : momentum (kgm/s)

$$F=(p_{\text{final}}-p_{\text{initial}})/\Delta t$$

$$F=\Delta p/\Delta t$$

The net force acting on an object equals the change in momentum (Δp) in a certain time period (Δt).

Since **velocity is a vector**, **momentum is also a vector**, pointing in the same direction as v .

Impulse

$$F = \Delta p / \Delta t$$

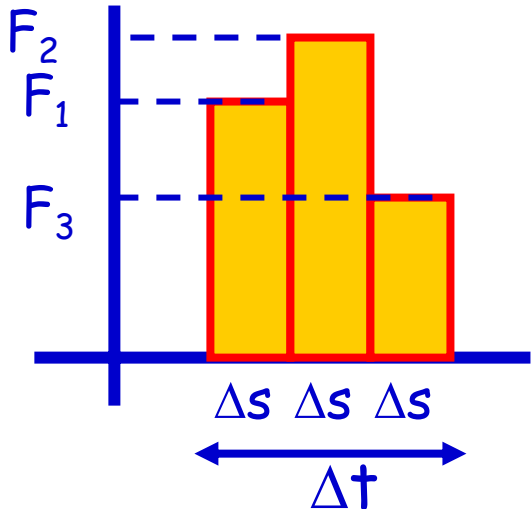
Force = change in (mv) per time period (Δt).

$$\Delta p = F \Delta t$$

The change in momentum equals the **force** acting on the object times **how long you apply** the force.

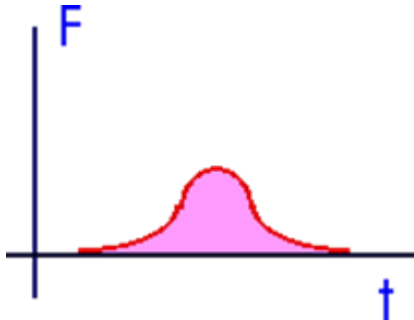
Definition: $\Delta p = \text{Impulse}$

What if the force is not constant within the time period Δt ?

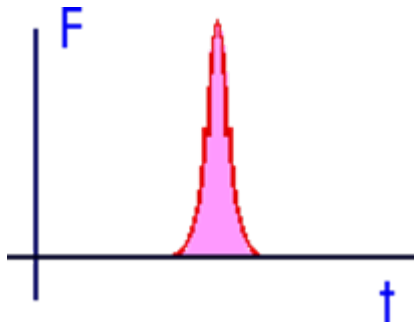


$$\begin{aligned} \Delta p &= F \Delta t = (F_1 \Delta s + F_2 \Delta s + F_3 \Delta s) = \\ &= \Delta t (F_1 \Delta s + F_2 \Delta s + F_3 \Delta s) / \Delta t \\ &= \Delta t F_{\text{average}} \end{aligned}$$

$$\Delta p = F_{\text{average}} \Delta t$$



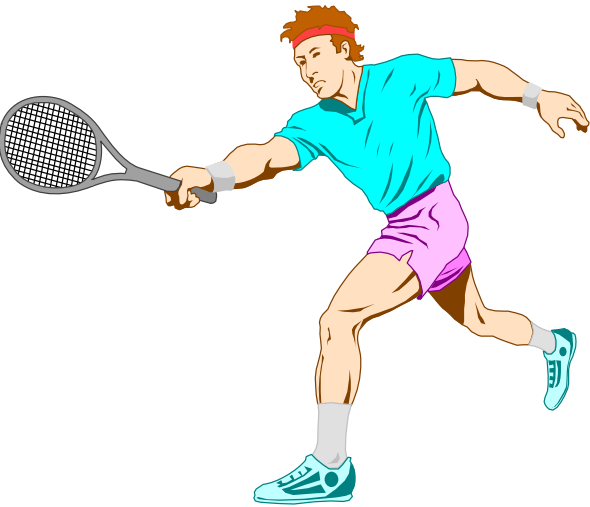
car hitting haystack



car hitting wall

The change in momentum (impulse) is the same, but the force reaches a much higher value when the car hits a wall!

Some examples



A tennis player receives a shot approaching him (horizontally) with 50m/s and returns the ball in the opposite direction with 40m/s. The mass of the ball is 0.060 kg.

A) What is the impulse delivered by the ball to the racket?

B) What is the work done by the racket on the ball?

A) Impulse=change in momentum (Δp).

$$\Delta p = m(v_{\text{final}} - v_{\text{initial}}) = 0.060(-40 - 50) = -5.4 \text{ kgm/s}$$

$$\begin{aligned} \text{B) } W &= KE_{\text{final}} - KE_{\text{initial}} = \frac{1}{2}mv_{\text{final}}^2 - \frac{1}{2}mv_{\text{initial}}^2 && \text{(no PE!)} \\ &= \frac{1}{2}0.060([-40]^2 - [50]^2) = -27 \text{ J} \end{aligned}$$

Child safety

A friend claims that it is safe to go on a car trip with your child without a child seat since he can hold onto your 12kg child even if the car makes a frontal collision (lasting 0.05s and causing the vehicle to stop completely) at $v=50$ km/h (about 30 miles/h). Is he to be trusted?

$F=\Delta p/\Delta t$ force=impulse per time period

$$=m(v_f-v_i)/\Delta t$$

$$v_f=0 \text{ and } v_i=50\text{km/h}=13.9 \text{ m/s}$$

$$m=12\text{kg } \Delta t=0.05\text{s}$$

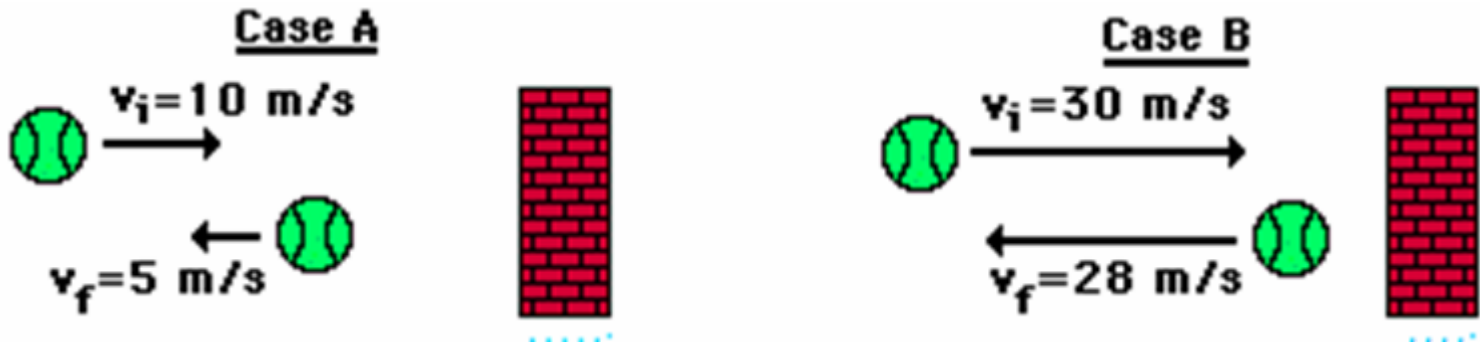
$$F=12(-13.9)/0.05=3336 \text{ N}$$

This force corresponds to lifting a mass of 340 kg or about 680 pounds!

DON'T TRUST THIS GUY!



question



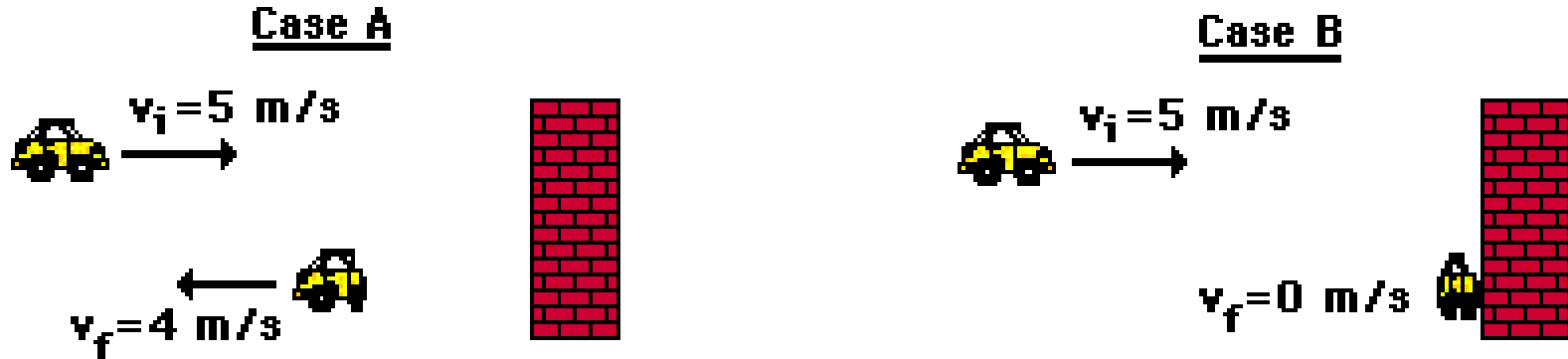
The velocity change is largest in case: A **B**

The acceleration is largest in case: A **B**

The momentum change is largest in case: A **B**

The impulse is largest in case: A **B**

accident



Consider the cases A and B in which a car crashes into a wall. In which case is the likelihood that the passengers are severely hurt greater?

A: the change in impulse is largest (A: $9m_{\text{car}}$ B: $5m_{\text{car}}$) and thus the force acting on the passengers largest

Momentum $p=mv$

$$F=\Delta p/\Delta t$$

Impulse (the change in momentum) $\Delta p= F\Delta t$

Demo: the 'safe' bungee jumper

Conservation of Momentum

$$F_{21}\Delta t = m_1v_{1f} - m_1v_{1i}$$
$$F_{12}\Delta t = m_2v_{2f} - m_2v_{2i}$$

Newton's 3rd law:

$$F_{12} = -F_{21}$$

$$(m_1v_{1f} - m_1v_{1i}) = -(m_2v_{2f} - m_2v_{2i})$$

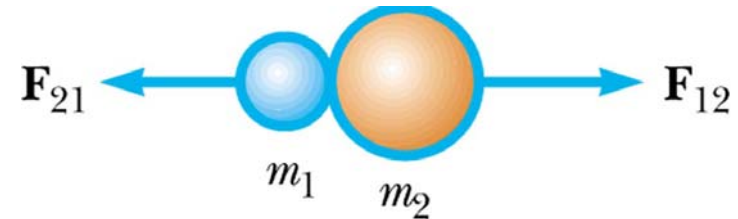
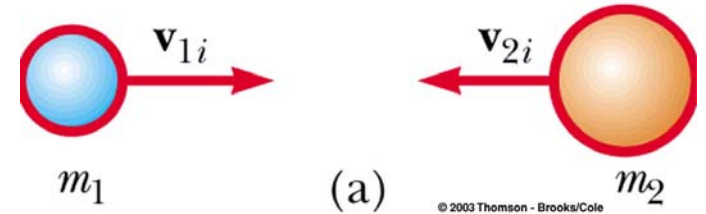
Rewrite:

$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$$

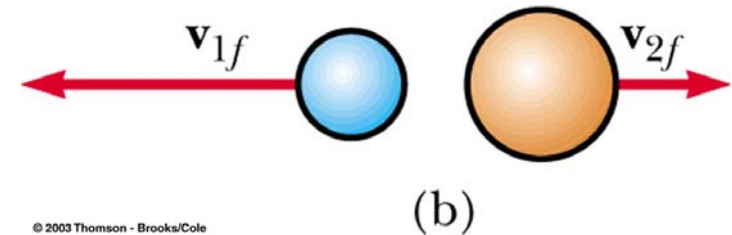
$$p_{1i} + p_{2i} = p_{1f} + p_{2f}$$

CONSERVATION OF MOMENTUM

Before collision



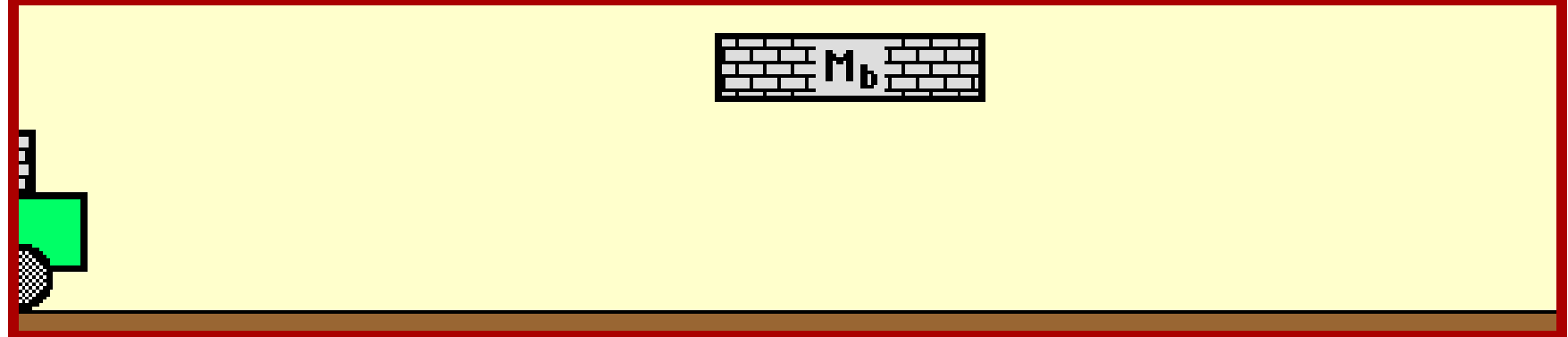
After collision



CLOSED SYSTEM!

momentum conservation

| Loaded Cart | | Dropped Brick | |
|----------------|------|----------------|-----|
| Mass (kg) | 3.0 | Mass (kg) | 2.0 |
| Vel. (cm/s) | 50.0 | Vel. (cm/s) | 0.0 |
| Mom. (kg cm/s) | 150 | Mom. (kg cm/s) | 0 |



The diagram shows a green cart on the left side of a yellow track. A brick, labeled M_b , is positioned in the center of the track. The cart has a black wheel and a black handle. The brick is a grey rectangular block with a brick pattern.

Moving in space



An astronaut (100 kg) is drifting away from the spaceship with $v=0.2$ m/s. To get back he throws a wrench (2 kg) in the direction away from the ship. With what velocity does he need to throw the wrench to move with $v=0.1$ m/s towards the ship?

- a) 0.1 m/s b) 0.2 m/s c) 5 m/s d) 16 m/s
e) this will never work?

Initial momentum: $m_{ai}v_{ai}+m_{wi}v_{wi} = 100*0.2+2*0.2=20.4$ kgm/s

After throw: $m_{af}v_{af}+m_{wf}v_{wf}=100*(-0.1)+2*v_{wf}$ kgm/s

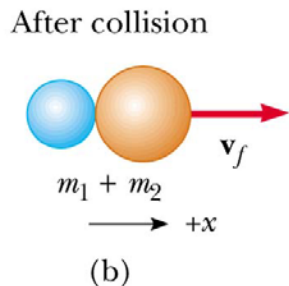
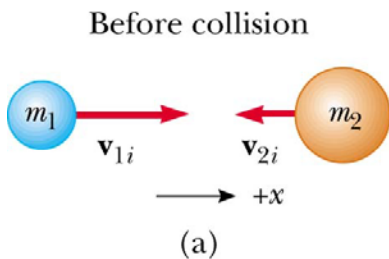
Conservation of momentum: $m_{ai}v_{ai}+m_{wi}v_{wi} = m_{af}v_{af}+m_{wf}v_{wf}$

$$20.4 = -10 + 2*v_{wf} \quad v_{wf} = 15.7 \text{ m/s}$$

Types of collisions

Inelastic collisions

- Momentum is conserved
- Some energy is lost in the collision: KE not conserved
- Perfectly inelastic: the objects stick together.



Elastic collisions

- Momentum is conserved
- No energy is lost in the collision: KE conserved

