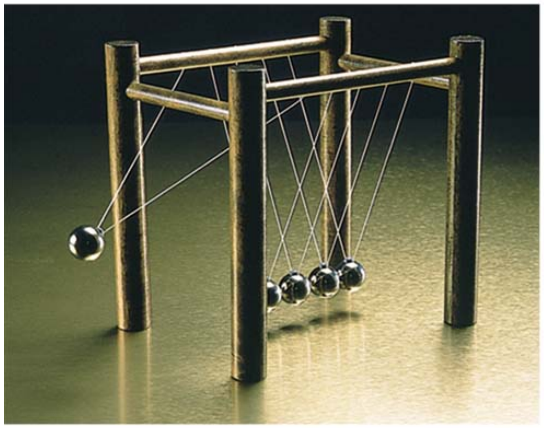
PHYSICS 231
Lecture 14: Collisions II: The momentum strikes back.



Conservation of Momentum

$F_{21}\Delta t = m_1v_{1f}-m_1v_{1i}$
$F_{12}\Delta t = m_2v_{2f}-m_2v_{2i}$

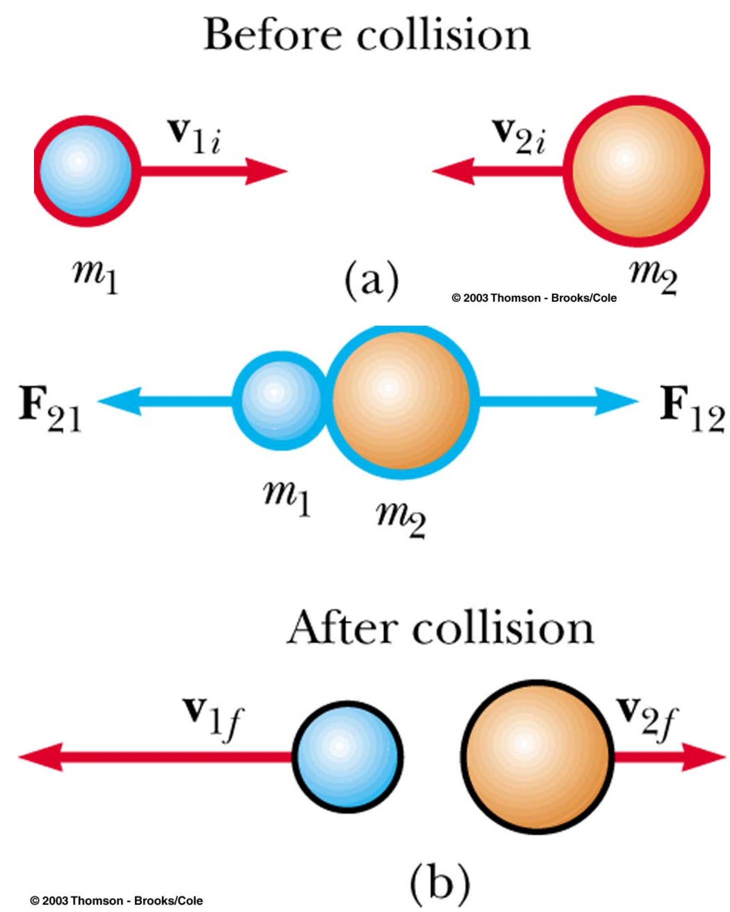
Newton's 3rd law:
$F_{12}=-F_{21}$

$(m_1v_{1f}-m_1v_{1i})=-(m_2v_{2f}-m_2v_{2i})$

Rewrite:

$m_1v_{1i}+m_2v_{2i}=m_1v_{1f}+m_2v_{2f}$
$p_{1i}+p_{2i}=p_{1f}+p_{2f}$

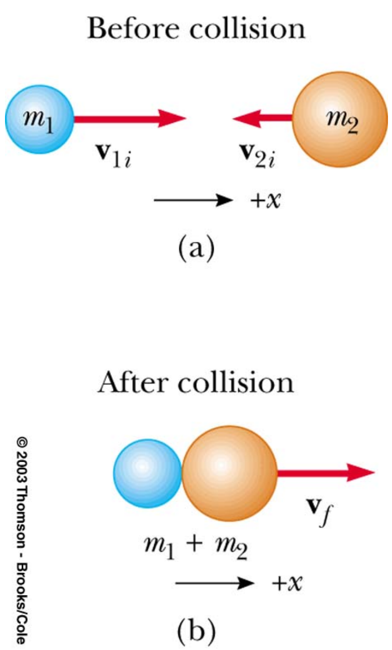
CONSERVATION OF MOMENTUM

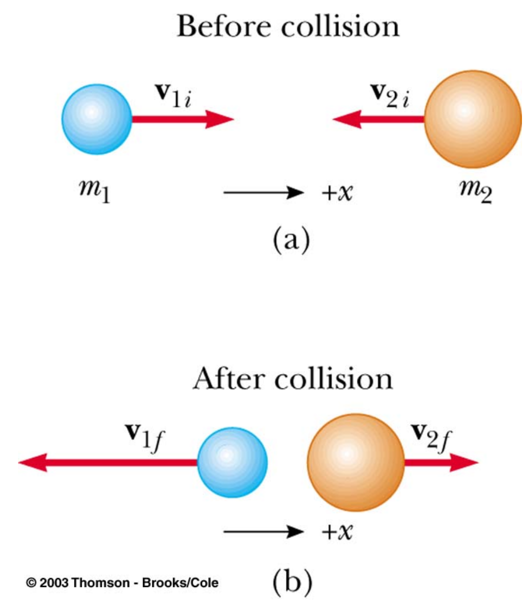


CLOSED SYSTEM!

Types of collisions

Inelastic collisions	Elastic collisions
•Momentum is conserved	•Momentum is conserved
•Some energy is lost in the collision: KE not conserved	•No energy is lost in the collision: KE conserved
•Perfectly inelastic: the objects stick together.	





Perfectly inelastic collisions

Conservation of P: $m_1v_{1i}+m_2v_{2i}=m_1v_{1f}+m_2v_{2f}$

After the collision m_1 and m_2 form one new object with mass $M=m_1+m_2$

$$m_1v_{1i}+m_2v_{2i}=v_f(m_1+m_2)$$
$$v_f=(m_1v_{1i}+m_2v_{2i})/(m_1+m_2)$$

Demo: perfect inelastic collision on airtrack

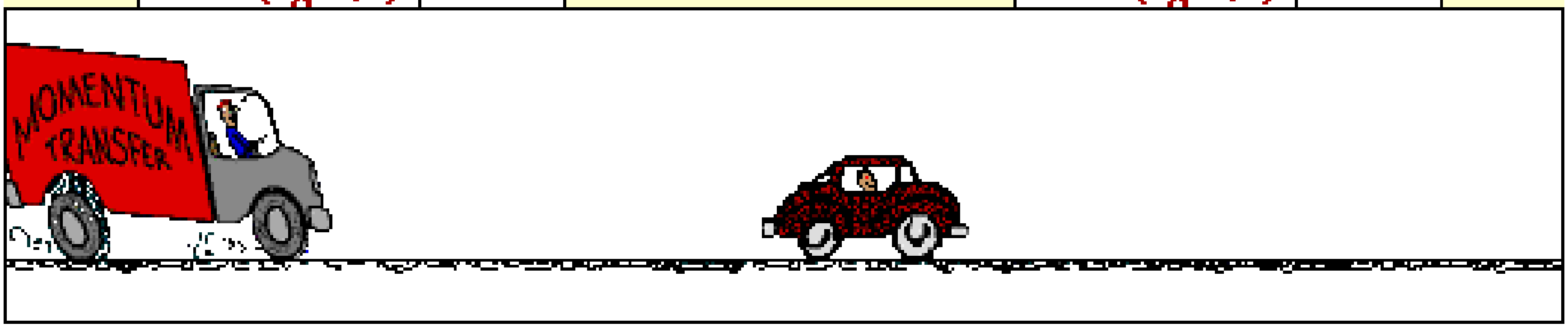
inelastic collision

Truck

mass (kg)	3000
vel. (m/s)	20.0
mom. (kg m/s)	60 000

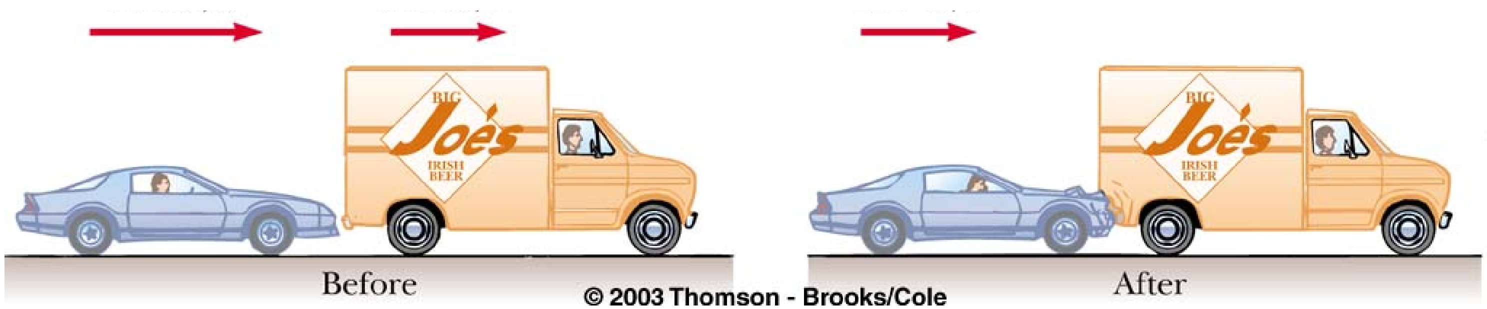
Car

mass (kg)	1000
vel. (m/s)	0.0
mom. (kg m/s)	0



Perfect inelastic collision: an example

50 m/s   20 m/s   25 m/s



A car collides into the back of a truck and their bumpers get stuck. What is the ratio of the mass of the truck and the car? ($m_{truck}=c*m_{car}$) What is the fraction of KE lost?

$m_1v_{1i}+m_2v_{2i}=v_f(m_1+m_2)$ $50m_c+20c*m_c=25(m_c+c*m_c)$

so $c=25m_c/5m_c=5$

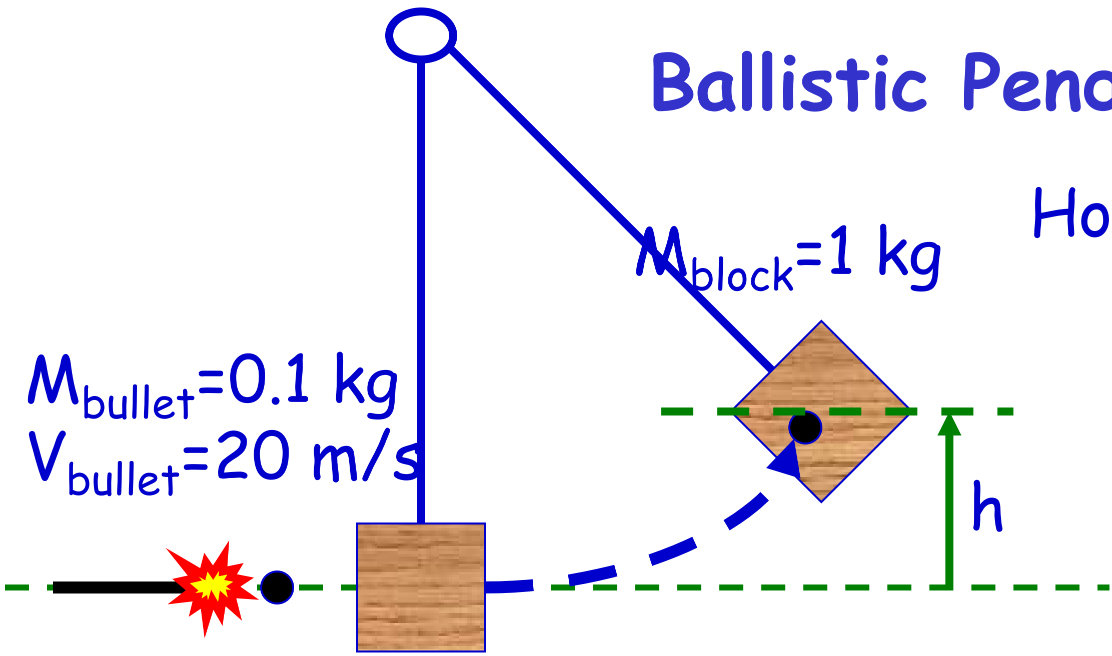
Before collision: $KE_i=\frac{1}{2}m_c50^2+\frac{1}{2}5m_c20^2$

After collision: $KE_f=\frac{1}{2}6m_c25^2$

Ratio: $KE_f/KE_i= (6*25^2)/(50^2+5*20^2)=0.83$

17% of the KE is lost (damage to cars!)

Ballistic Pendulum



How high will the block go?

There are 2 stages:

- The collision
- The Swing of the block

<u>The collision</u> The bullet gets stuck in the block (perfect inelastic collision). Use conservation of momentum.

$m_1 v_{1i} + m_2 v_{2i} = v_f(m_1+m_2)$ so: $0.1*20+1*0=v_f(0.1+1)$ $v_f=1.8$ m/s

<u>The swing of the block</u> Use conservation of Mechanical energy.

$(mgh+\frac{1}{2}mv^2)_{\text{start of swing}} = (mgh+\frac{1}{2}mv^2)_{\text{at highest point}}$

$0+\frac{1}{2}1.1(1.8)^2=1.1*9.81*h$ so $h=0.17$ m

Why can't we use Conservation of ME right from the start??

Elastic collisions

Conservation of momentum: $m_1v_{1i}+m_2v_{2i}=m_1v_{1f}+m_2v_{2f}$

Conservation of KE: $\frac{1}{2}m_1v_{1i}^2+\frac{1}{2}m_2v_{2i}^2=\frac{1}{2}m_1v_{1f}^2+\frac{1}{2}m_2v_{2f}^2$

Rewrite conservation of KE:

a) $m_1(v_{1i}-v_{1f})(v_{1i}+v_{1f}) = m_2(v_{2f}-v_{2i})(v_{2f}+v_{2i})$

Rewrite conservation of P :

b) $m_1(v_{1i}-v_{1f}) = m_2(v_{2f}-v_{2i})$

Divide a) by b): $(v_{1i}+v_{1f})=(v_{2f}+v_{2i})$

rewrite: $(v_{1i}-v_{2i})=(v_{2f}-v_{1f})$

Use in problems

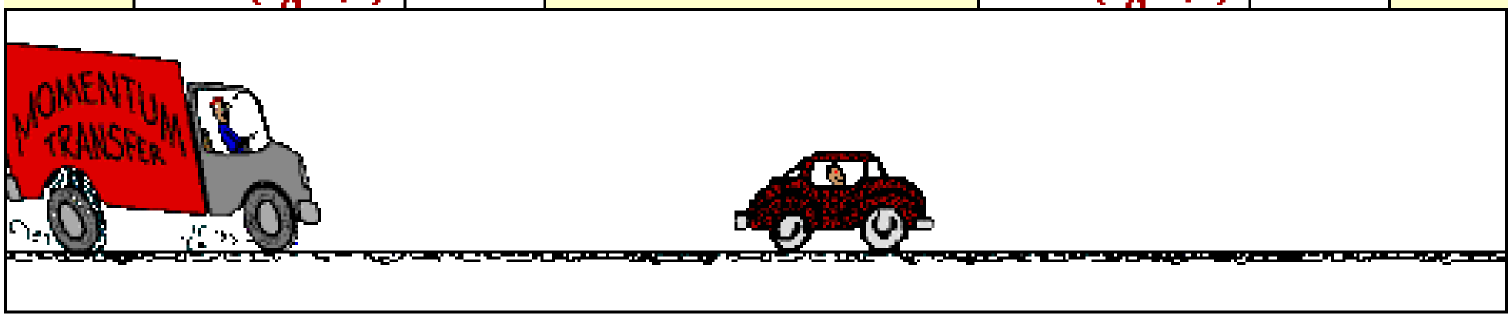
elastic collisions

Truck

mass (kg)	3000
vel. (m/s)	20.0
mom. (kg m/s)	60 000

Car

mass (kg)	1000
vel. (m/s)	0.0
mom. (kg m/s)	0

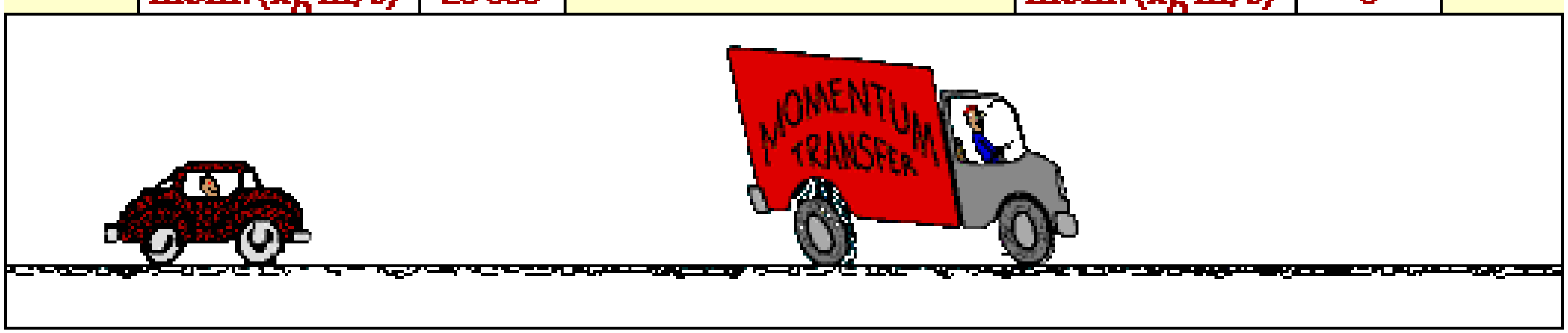


Car

mass (kg)	1000
vel. (m/s)	20.0
mom. (kg m/s)	20 000

Truck

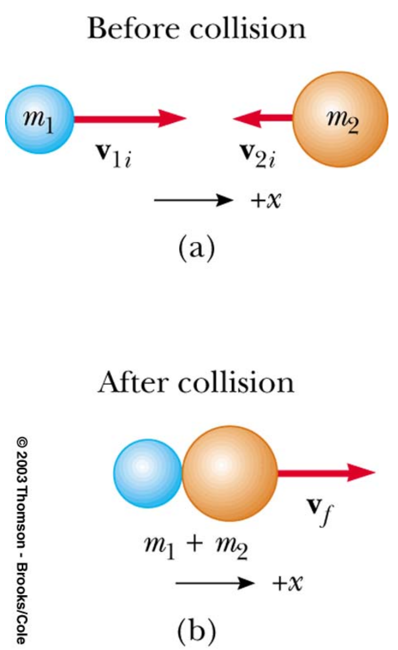
mass (kg)	3000
vel. (m/s)	0.0
mom. (kg m/s)	0



Types of collisions

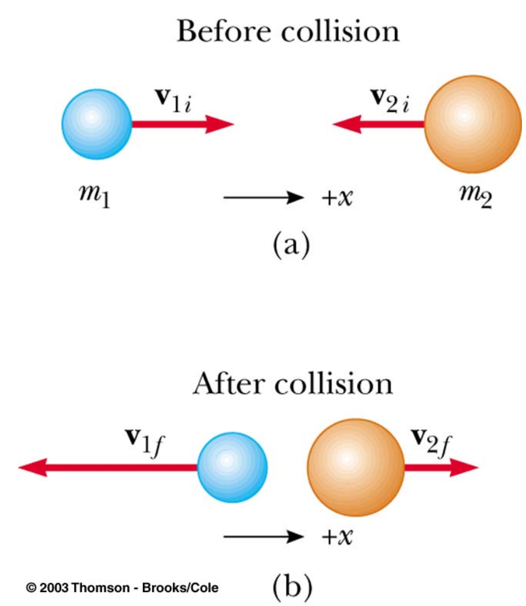
Inelastic collisions

- Momentum is conserved
- Some energy is lost in the collision: KE not conserved
- Perfectly inelastic: the objects stick together.



Elastic collisions

- Momentum is conserved
- No energy is lost in the collision: KE conserved



Review:

Momentum $p=mv$

Force $F=\Delta p/\Delta t$

Impulse (the change in momentum) $\Delta p= F\Delta t$

Types of collisions

Inelastic collisions

- Momentum is conserved
- Some energy is lost in the collision: KE not conserved
- Perfectly inelastic: the objects stick together.

Conservation of momentum:

$m_1v_{1i}+m_2v_{2i}=(m_1+m_2)v_f$

Elastic collisions

- Momentum is conserved
- No energy is lost in the collision: KE conserved

Conservation of momentum:

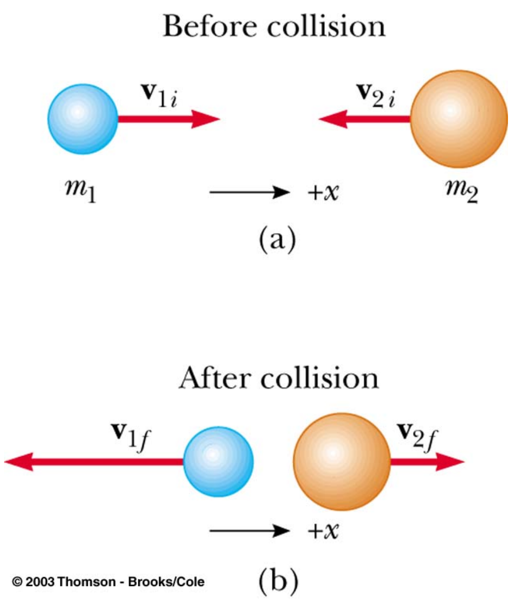
$m_1v_{1i}+m_2v_{2i}=m_1v_{1f}+m_2v_{2f}$

Conservation of KE:

$\frac{1}{2}m_1v_{1i}^2+\frac{1}{2}m_2v_{2i}^2=\frac{1}{2}m_1v_{1f}^2+\frac{1}{2}m_2v_{2f}^2$

$(v_{1i}-v_{2i})=(v_{2f}-v_{1f})$

elastic collision of equal masses



Given $m_2=m_1$.
What is the velocity of m_1 and m_2 after the collision in terms of the initial velocity of m_2 if m_1 is originally at rest?

$$m_1v_{1i}+m_2v_{2i}=m_1v_{1f}+m_2v_{2f}$$

$$m_2v_{2i}=m_1v_{1f}+m_2v_{2f}$$

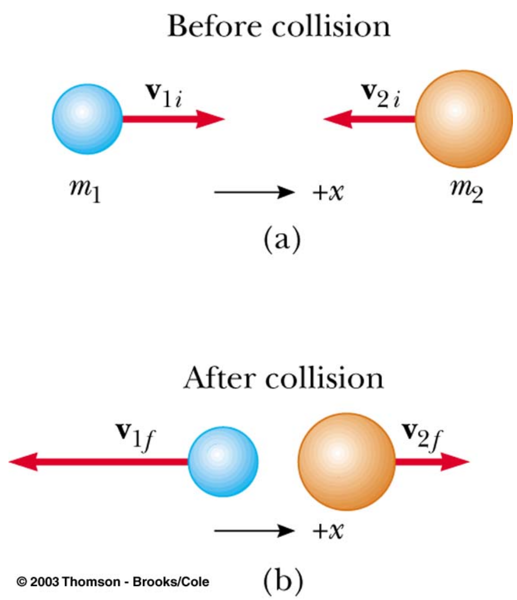
$$v_{2i}=v_{1f}+v_{2f}$$

$$(v_{1i}-v_{2i})=(v_{2f}-v_{1f})$$

$$-v_{2i}= v_{2f}-v_{1f}$$

$$v_{2f}=0$$
$$v_{1f}=v_{2i}$$

elastic collision of unequal masses



Given $m_2=3m_1$.
What is the velocity of m_1 and m_2 after the collision in terms of the initial velocity of the moving bullet if

a) m_1 is originally at rest

b) m_2 is originally at rest

A) $m_1v_{1i}+m_2v_{2i}=m_1v_{1f}+m_2v_{2f}$

$3m_1v_{2i}=m_1v_{1f}+m_2v_{2f}$

$3v_{2i}=v_{1f}+3v_{2f}$

$(v_{1i}-v_{2i})=(v_{2f}-v_{1f})$

$-v_{2i}= v_{2f}-v_{1f}$

$v_{2f}=v_{2i}/2$
$v_{1f}=3v_{2i}/2$

B) $m_1v_{1i}+m_2v_{2i}=m_1v_{1f}+m_2v_{2f}$

$m_1v_{1i}=m_1v_{1f}+m_2v_{2f}$

$v_{1i}=v_{1f}+3v_{2f}$

$(v_{1i}-v_{2i})=(v_{2f}-v_{1f})$

$v_{1i}= v_{2f}-v_{1f}$

$v_{2f}=v_{1i}/2$
$v_{1f}=-v_{1i}/2$

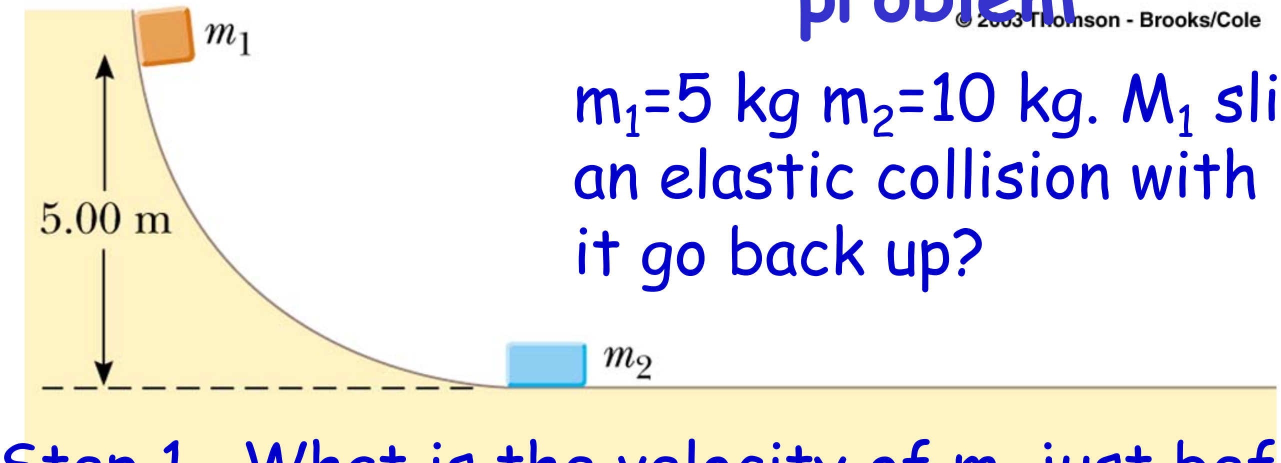
question

The kinetic energy of an object is quadrupled. Its momentum will change by a factor of…

a) 0
b) 2
c) 4
d) 8
e) 16

$E_{kin}=0.5mv^2$ if E_{kin}x4 then v x 2
p=mv if v x 2, then p x 2

problem



m_1=5 kg m_2=10 kg. M_1 slides down and makes an elastic collision with m_2. How high does it go back up?

Step 1. What is the velocity of m_1 just before it hits m_2?

Conservation of ME: $(m_1gh_1+0.5mv^2)_{start}=(m_1gh_1+0.5mv^2)_{bottom}$

$5*9.81*5+0=0+0.5*5*v^2$ so v_{1i}=9.9 m/s

Step 2. Collision: Elastic so conservation of momentum AND KE.

- $m_1v_{1i}+m_2v_{2i}=m_1v_{1f}+m_2v_{2f} \Rightarrow 5*9.9+0=5*v_{1f}+10v_{2f} \Rightarrow v_{2f}=4.95-0.5*v_{1f}$
- $(v_{1i}-v_{2i})=(v_{2f}-v_{1f}) \Rightarrow 9.9-0=v_{2f}-v_{1f}$ $\Rightarrow \underline{v_{2f}=9.9+v_{1f}}$ -

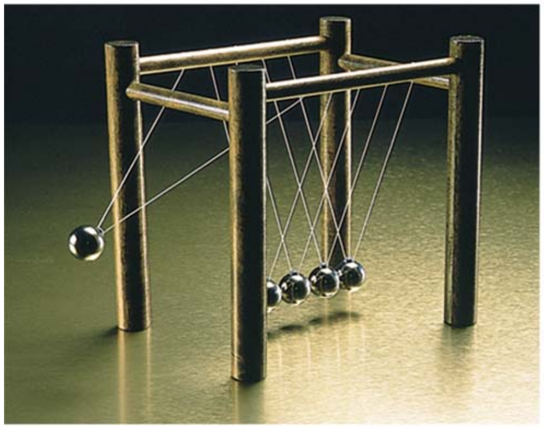
$v_{1f}=-3.3m/s \Leftarrow 0 =-4.95-1.5V_{1f}$

Step 3. M_1 moves back up; use conservation of ME again.

$(m_1gh_1+0.5mv^2)_{bottom}=(m_1gh_1+0.5mv^2)_{final}$

$0 +0.5*5*(-3.3)^2=5*9.81*h_1+0$ h=0.55 m

Transporting momentum



For elastic collision of equal masses

$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$

$m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$

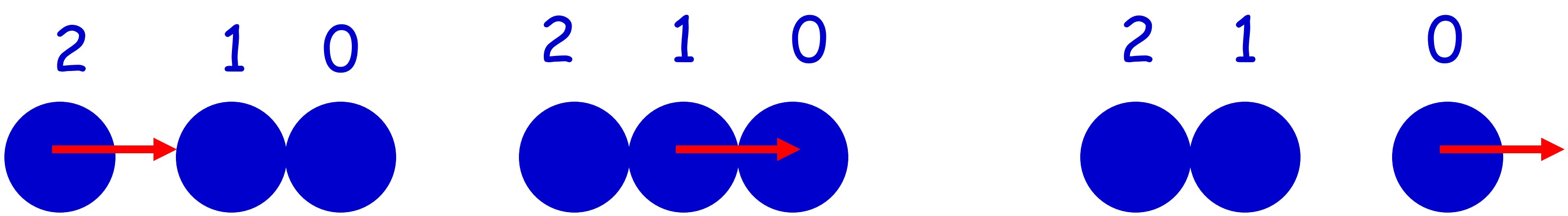
$v_{2i} = v_{1f} + v_{2f}$

$(v_{1i} - v_{2i}) = (v_{2f} - v_{1f})$

$-v_{2i} = v_{2f} - v_{1f}$

$v_{2f} = 0$

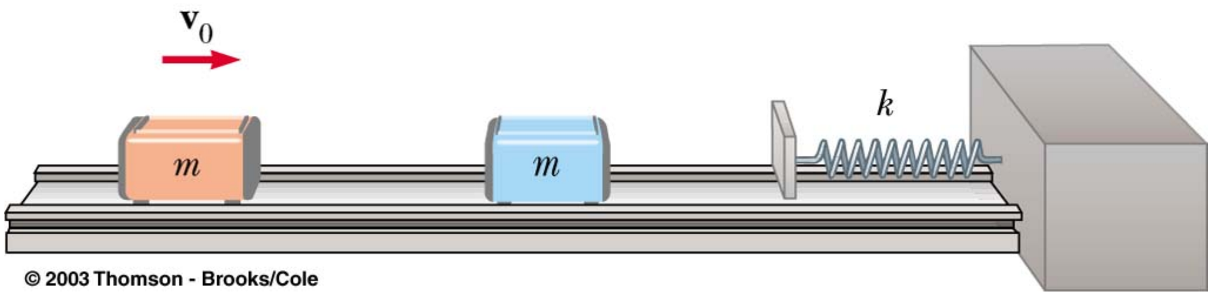
$v_{1f} = v_{2i}$



$V_{2f} = 0$
$V_{1f} = V_{2i}$

$V_{1f} = 0$
$V_{0f} = V_{1i}$

Carts on a spring track



K=50 N/m
v_0=5.0 m/s
m=0.25 kg

What is the maximum compression of the spring if the carts collide a) elastically and b) inelastically?

A) Conservation of momentum and KE

$m_1v_{1i}+m_2v_{2i}=m_1v_{1f}+m_2v_{2f} \Rightarrow 0.25*5=0.25v_{1f}+0.25v_{2f} \Rightarrow v_{1f}=5-v_{2f}$

$(v_{1i}-v_{2i})=(v_{2f}-v_{1f}) \Rightarrow 5=v_{2f}-v_{1f}$ $v_{1f}=0$ $v_{2f}=5$ m/s

Conservation of energy: $\frac{1}{2}mv^2=\frac{1}{2}kx^2$ $0.5*0.25*5^2=0.5*50x^2$

x=0.35 m We could have skipped collision part!!

B) Conservation of momentum only

$m_1v_{1i}+m_2v_{2i}=(m_1+m_2)v_f \Rightarrow 0.25*5=0.5v_f \Rightarrow v_f=2.5$ m/s

Conservation of energy: $\frac{1}{2}mv^2=\frac{1}{2}kx^2$ $0.5*0.5*2.5^2=0.5*50x^2$

x=0.25 m Part of energy is lost!

quiz (extra credit)

In a system with 2 moving objects when a collision occurs between the objects:

a) the total kinetic energy is always conserved
b) the total momentum is always conserved
c) the total kinetic energy and total momentum are always conserved
d) neither the kinetic energy nor the momentum is conserved

For both inelastic and elastic collisions, the momentum is conserved.

Impact of a meteorite

Estimate what happens if a 1-km radius meteorite collides with earth: a) Is the orbit of earth around the sun changed?

b) how much energy is released?

Assume: meteorite has same density as earth, the collision is inelastic and the meteorite's v is 10 km/s (relative to earth)

A) Earth's mass: 6E+24 kg radius: 6E+6 m

density=mass/volume=$M/(4/3\pi r^3)$=6.6E+3 kg/m^3

mass meteorite: $4/3\pi(1000)^3$*6.6E+3=2.8E+13 kg

Conservation of momentum: $m_e v_e + m_m v_m = (m_e + m_m) v_{me}$

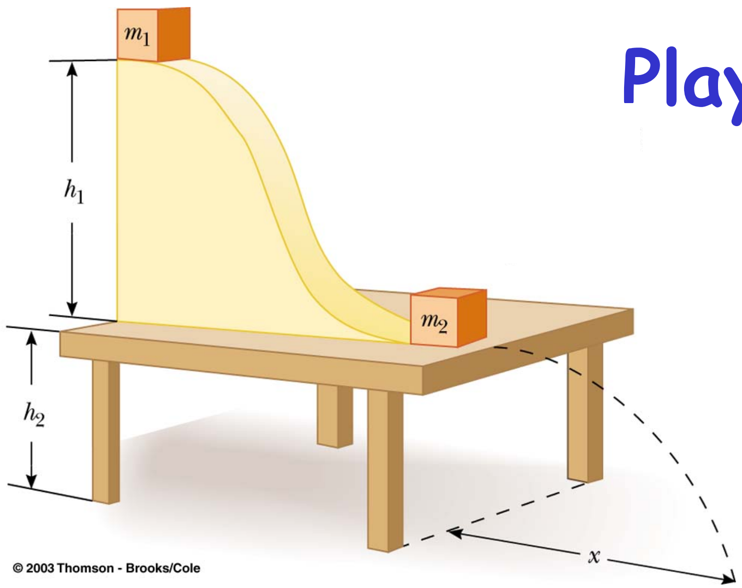
(2.8E+13)(1E+4)=(6E+24)v_{me} so v_{me}=4.7E-08 m/s (no change)

B) Energy=Kinetic energy loss: $(\frac{1}{2}m_e v_e^2 + \frac{1}{2}m_m v_m^2) - (\frac{1}{2}m_{m+e} v_{me}^2)$

0.5(2.8E+13)(1E+4)^2-0.5(6E+24)(4.7E-08)^2=1.4E+21 J

Largest nuclear bomb existing: 100 megaton TNT=4.2E+17 J

Energy release: 3.3E+3 nuclear bombs!!!!!

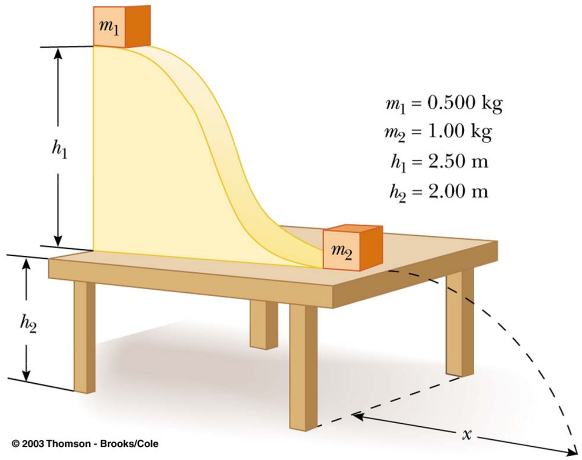


Playing with blocks

m_1=0.5 kg   collision is elastic
m_2=1.0 kg
h_1=2.5 m
h_2=2.0 m

A) determine the velocity of the blocks after the collision
b) how far back up the track does m_1 travel?
C) how far away from the bottom of the table does m_2 land
d) how far away from the bottom of the table does m_1 land

Determine the velocity of the blocks after the collision



m_1=0.5 kg   collision is elastic
m_2=1.0 kg
h_1=2.5 m
h_2=2.0 m

Step 1: determine velocity of m_1 at the bottom of the slide

Conservation of ME $(mgh+\frac{1}{2}mv^2)_{top} = (mgh+\frac{1}{2}mv^2)_{bottom}$

$0.5*9.81*2.5+0=0+0.5*0.5*v^2$

so: v_1=7.0 m/s

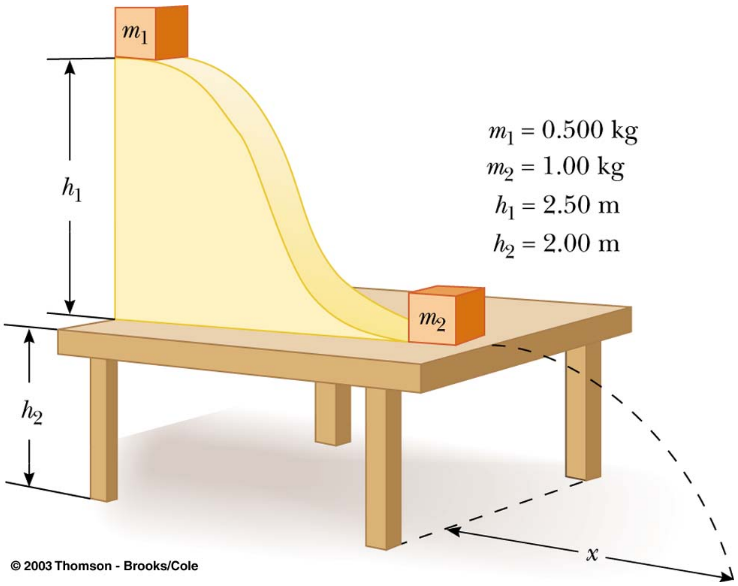
Step 2: Conservation of momentum and KE in elastic collision

$m_1v_{1i}+m_2v_{2i}=m_1v_{1f}+m_2v_{2f}$ so $0.5*7+0=0.5v_{1f}+v_{2f}$

$(v_{1i}-v_{2i})=(v_{2f}-v_{1f})$ so $7.0-0=v_{2f}-v_{1f}$

Combine these equations and find: v_{1f}=-2.3 m/s v_{2f}=4.7 m/s

How far back up does m_1 go after the collision?



m_1=0.5 kg   collision is elastic
m_2=1.0 kg
h_1=2.5 m
h_2=2.0 m
v_{1f}=-2.3 m/s v_{2f}=4.7 m/s

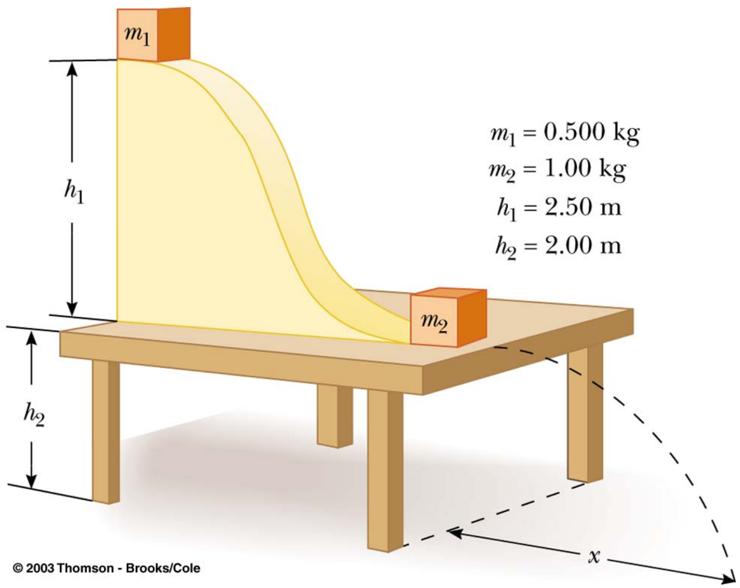
Use conservation of ME:

$(mgh+\frac{1}{2}mv^2)_{bottom} = (mgh+\frac{1}{2}mv^2)_{back\ up\ slide}$

$0+0.5*0.5*(-2.3)^2=0.5*9.81*h+0$

h=0.27 m

How far away from the table does m_2 land?

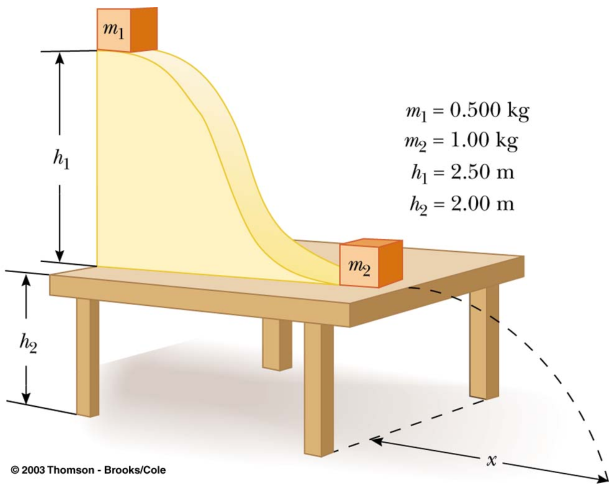


m_1=0.5 kg   collision is elastic
m_2=1.0 kg
h_1=2.5 m
h_2=2.0 m
v_{1f}=-2.3 m/s v_{2f}=4.7 m/s
h_1=0.27 m (after collision back up)

This is a parabolic motion with initial horizontal velocity.

Horizontal	vertical
$x(t)=x(0)+v_x(0)t+\frac{1}{2}at^2$	$y(t)=y(0)+v_y(0)t-\frac{1}{2}gt^2$
$x(t)=4.7t$	$0=2.0-0.5*9.81*t^2$
$x(0.63)=2.96$ m ←	$t=0.63$ s

How far away from the table does m_1 land?



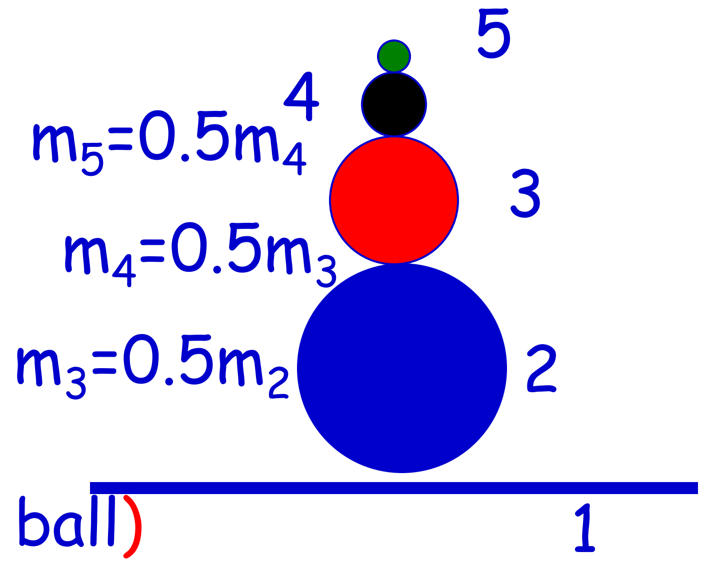
m_1=0.5 kg   collision is elastic
m_2=1.0 kg
h_1=2.5 m
h_2=2.0 m
v_{1f}=-2.3 m/s v_{2f}=4.7 m/s
h_1=0.27 m (after collision back up)
x_2=2.96 m

Use conservation of ME: m_1 has $-v_{1f}$=2.3 m/s when it returns back at the bottom of the slide.

This is a parabolic motion with initial horizontal velocity.

Horizontal	vertical
$x(t)=x(0)+v_x(0)t+\frac{1}{2}at^2$	$y(t)=y(0)+v_y(0)t-\frac{1}{2}gt^2$
$x(t)=2.3t$	$0=2.0-0.5*9.81*t^2$
$x_1(0.63)=1.45$ m ←	$t=0.63$ s

Ballistic balls



Consider only the lowest ball first.

$X(t)=1.5-0.5*9.8*t^2=0$ so $t=0.55$ s

$V(t)=-9.8t$ so $V(0.55)=-5.4$ m/s

Collision with earth:

- $m_1v_{1i}+m_2v_{2i}=m_1v_{1f}+m_2v_{2f}$ (1: earth 2: ball)
- $v_{1i}-v_{2i}=v_{2f}-v_{1f}$ $v_{1i}=0$

$v_{2f}=(m_2-m_1)v_{2i}/(m_1+m_2)$ $m_1 >> m_2$ so $v_{2f}=-v_{2i}=5.4$ m/s

Consider the collision of ball m(=n+1) with ball n

- $m_nv_{ni}+m_mv_{mi}=m_nv_{nf}+m_mv_{mf}$
- $(v_{ni}-v_{mi})=(v_{nf}-v_{mf})$

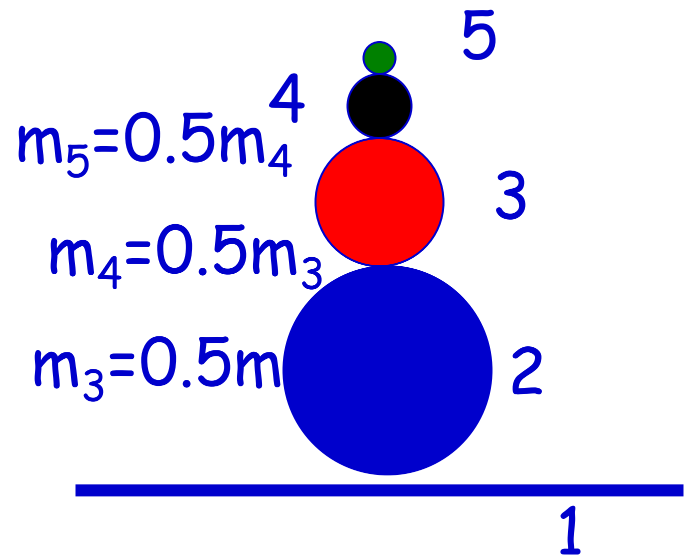
$v_{mf}=[2m_nv_{ni}+(m_m-m_n)v_{mi}]/[m_n+m_m]$ & $m_m=0.5m_n$

so $v_{mf}=[2m_nv_{ni}-0.5m_nv_{mi}]/[1.5m_n]=[1.33v_{ni}-0.33v_{mi}]$

- $V_{3f}=1.33*5.4-0.33(-5.4)=9.0$ m/s
- $V_{4f}=1.33*9.-0.33(-5.4)=13.7$ m/s
- $v_{5f}=1.33*13.7-0.33(-5.4)=20.$ m/s

Ballistic balls II

Highest point:
$v(t)=20.-9.8t=0$ so $t=2.0$ s
$x(t)=20t-0.5*9.8*2.0^2=20.$ m !!!



The next joyride: Rotational motion!