PHYSICS 231
Lecture 15: Rotations

Review:

Momentum $p=mv$

Force $F=\Delta p/\Delta t$

Impulse (the change in momentum) $\Delta p= F\Delta t$

Types of collisions

Inelastic collisions

- Momentum is conserved
- Some energy is lost in the collision: KE not conserved
- Perfectly inelastic: the objects stick together.

Conservation of momentum:

$$m_1v_{1i}+m_2v_{2i}=(m_1+m_2)v_f$$

Elastic collisions

- Momentum is conserved
- No energy is lost in the collision: KE conserved

Conservation of momentum:

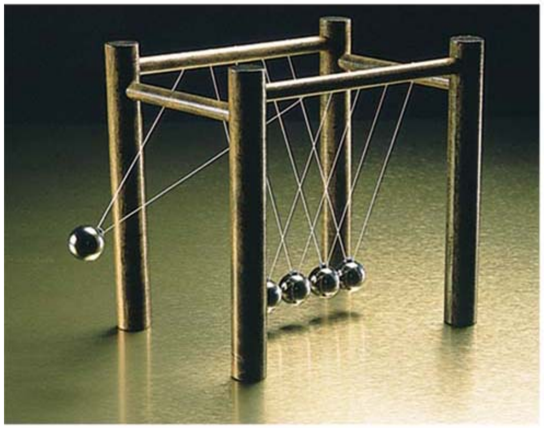
$$m_1v_{1i}+m_2v_{2i}=m_1v_{1f}+m_2v_{2f}$$

Conservation of KE:

$$\tfrac{1}{2}m_1v_{1i}^2+\tfrac{1}{2}m_2v_{2i}^2=\tfrac{1}{2}m_1v_{1f}^2+\tfrac{1}{2}m_2v_{2f}^2$$

$$\boxed{(v_{1i}-v_{2i})=(v_{2f}-v_{1f})}$$

Transporting momentum



For elastic collision of equal masses

$$m_1v_{1i}+m_2v_{2i}=m_1v_{1f}+m_2v_{2f}$$

$$m_2v_{2i}=m_1v_{1f}+m_2v_{2f}$$

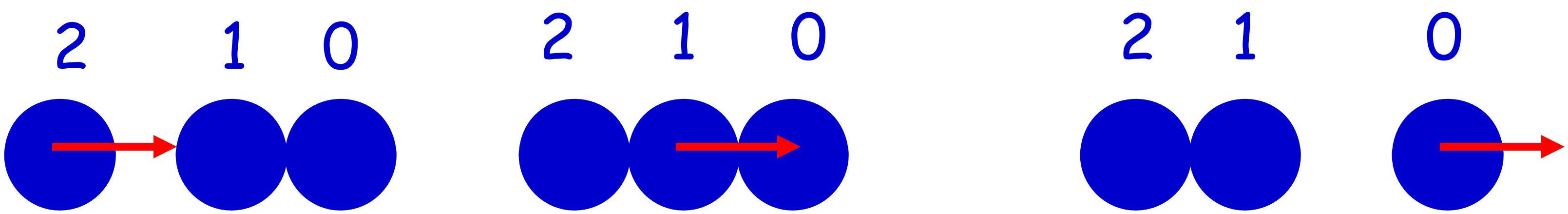
$$v_{2i}=v_{1f}+v_{2f}$$

$$(v_{1i}-v_{2i})=(v_{2f}-v_{1f})$$

$$-v_{2i}= v_{2f}-v_{1f}$$

$v_{2f}=0$

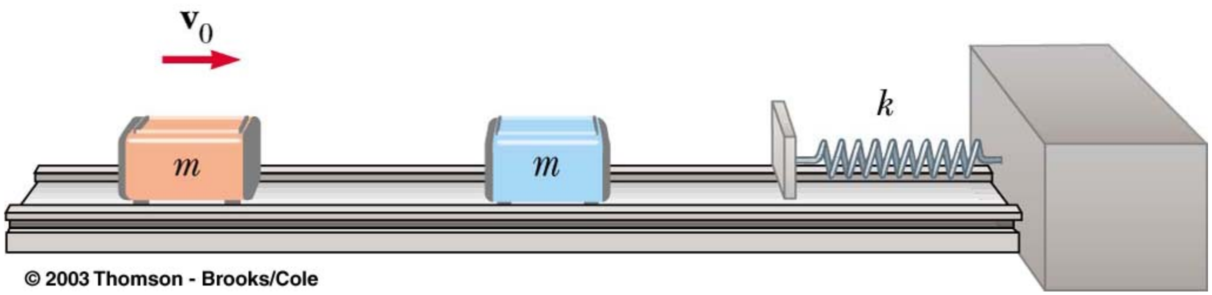
$v_{1f}=v_{2i}$



$V_{2f}=0$
$V_{1f}=V_{2i}$

$V_{1f}=0$
$V_{0f}=V_{1i}$

Carts on a spring track



K=50 N/m
v_0=5.0 m/s
m=0.25 kg

What is the maximum compression of the spring if the carts collide a) elastically and b) inelastically?

A) Conservation of momentum and KE

$m_1v_{1i}+m_2v_{2i}=m_1v_{1f}+m_2v_{2f} \Rightarrow 0.25*5=0.25v_{1f}+0.25v_{2f} \Rightarrow v_{1f}=5-v_{2f}$

$(v_{1i}-v_{2i})=(v_{2f}-v_{1f}) \Rightarrow 5=v_{2f}-v_{1f}$ $v_{1f}=0$ $v_{2f}=5$ m/s

Conservation of energy: $\frac{1}{2}mv^2=\frac{1}{2}kx^2$ $0.5*0.25*5^2=0.5*50x^2$

$x=0.35$ m

B) Conservation of momentum only

$m_1v_{1i}+m_2v_{2i}=(m_1+m_2)v_f \Rightarrow 0.25*5=0.5v_f \Rightarrow v_f=2.5$ m/s

Conservation of energy: $\frac{1}{2}mv^2=\frac{1}{2}kx^2$ $0.5*0.5*2.5^2=0.5*50x^2$

$x=0.25$ m

Part of energy is lost!

quiz (extra credit)

In a system with 2 moving objects when a collision occurs between the objects:

a) the total kinetic energy is always conserved

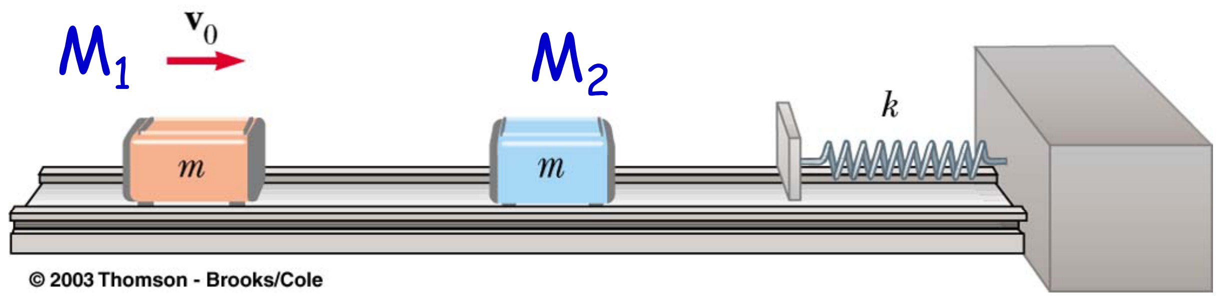
b) the total momentum is always conserved

c) the total kinetic energy and total momentum are always conserved

d) neither the kinetic energy nor the momentum is conserved

For both inelastic and elastic collisions, the momentu conserved.

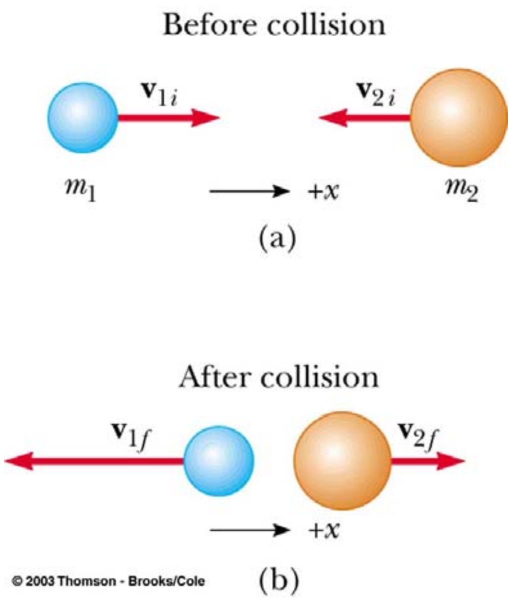
conceptual



right

Given M_1>M_2, and M_2 initially at rest. Just after the collision: M_1 a) moves left b) moves right c) is at rest. The velocity of M_2 will be a) smaller than b) larger than c) the same as the new velocity of M_1. M_2 compresses the spring. The maximum potential energy stored in the spring will be a) equal to b) smaller than c) larger than the kinetic energy of M_2 just after the collision. The maximum potential energy stored in the spring will be a) equal to b) smaller than c) larger than the kinetic energy of M_1 before the collision.

elastic collision of unequal masses



Given $m_2=3m_1$.

What is the velocity of m_1 and m_2 after the collision in terms of the initial velocity of the moving bullet if

a) m_1 is originally at rest

b) m_2 is originally at rest

A) $m_1v_{1i}+m_2v_{2i}=m_1v_{1f}+m_2v_{2f}$

$3m_1v_{2i}=m_1v_{1f}+m_2v_{2f}$

$3v_{2i}=v_{1f}+3v_{2f}$

$(v_{1i}-v_{2i})=(v_{2f}-v_{1f})$

$-v_{2i}= v_{2f}-v_{1f}$

$v_{2f}=v_{2i}/2$

$v_{1f}=3v_{2i}/2$

B) $m_1v_{1i}+m_2v_{2i}=m_1v_{1f}+m_2v_{2f}$

$m_1v_{1i}=m_1v_{1f}+m_2v_{2f}$

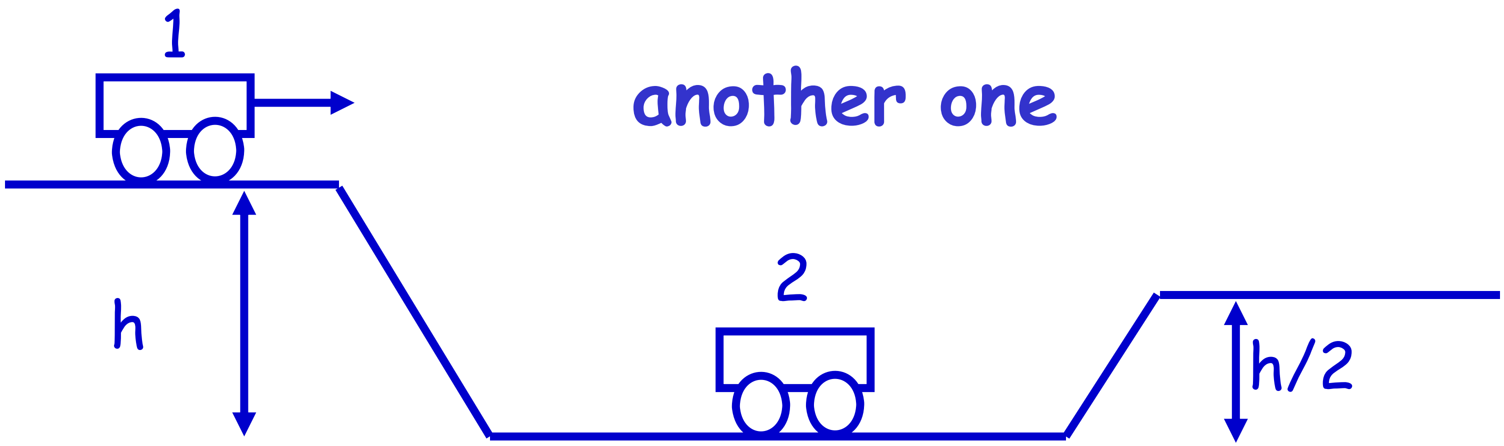
$v_{1i}=v_{1f}+3v_{2f}$

$(v_{1i}-v_{2i})=(v_{2f}-v_{1f})$

$v_{1i}= v_{2f}-v_{1f}$

$v_{2f}=v_{1i}/2$

$v_{1f}=-v_{1i}/2$



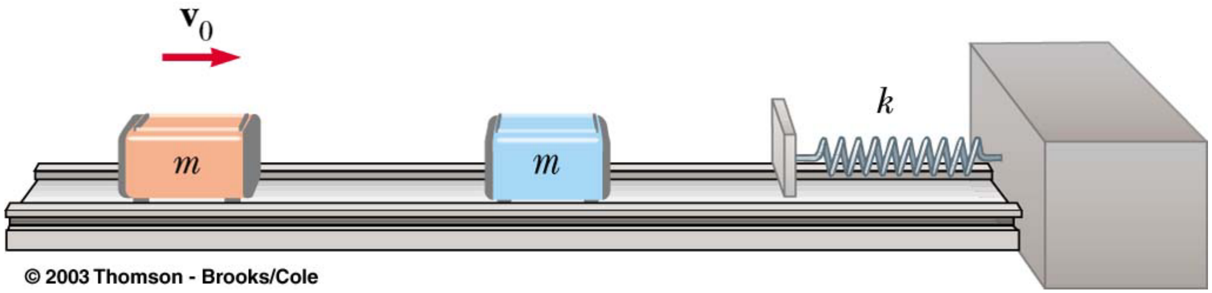
another one

A train wagon (1) is pushed off a hill with height h and couples with wagon (2) which has the same mass (m). Can the combined system move up to the next hill of which the height is half of the height of the first hill?

a) Yes   b) No   c) Don't know

On the first hill: Potential energy is mgh.
If the coupled train makes it up to the next hill:
Pot. E: $2mgh/2=mgh$.
This is possible only if no energy gets lost in collision.
However, collision is perfectly inelastic: no conservation of KE!
So, not possible

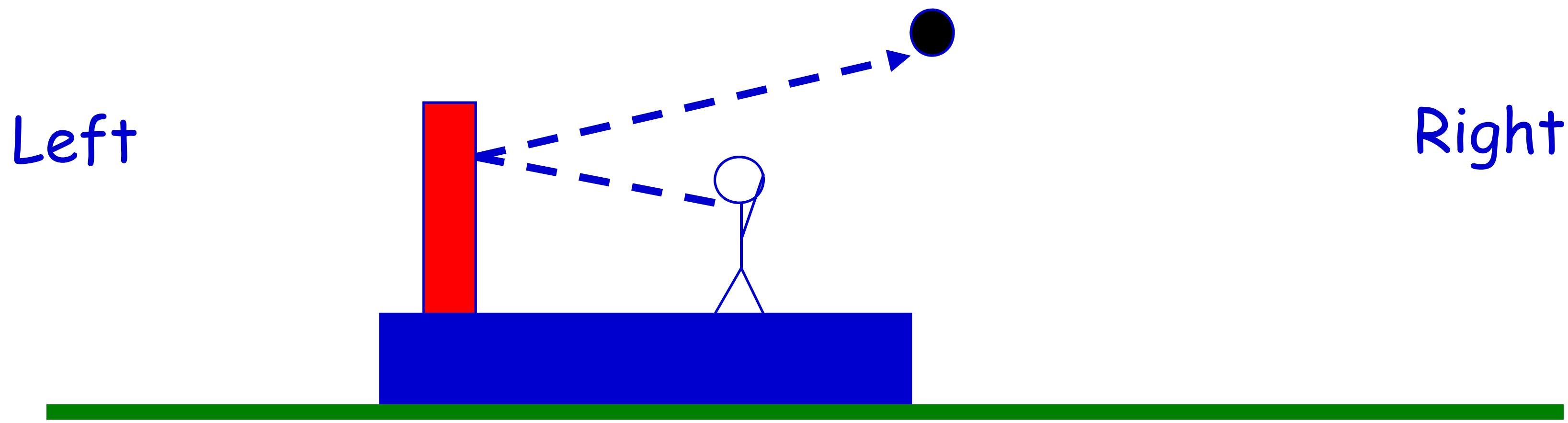
quiz: extra credit



The orange block collides with the blue block (same mass). Which of the following is **not** true?

a) If the collision is elastic, the orange block will stop moving after the collision.
b) If the collision is (perfectly) inelastic, the spring will be compressed less than if the collision is elastic
c) If the collision is (perfectly) inelastic, the maximal potential energy stored in the spring equals the kinetic energy of the orange block before the collision

Quiz: extra credit



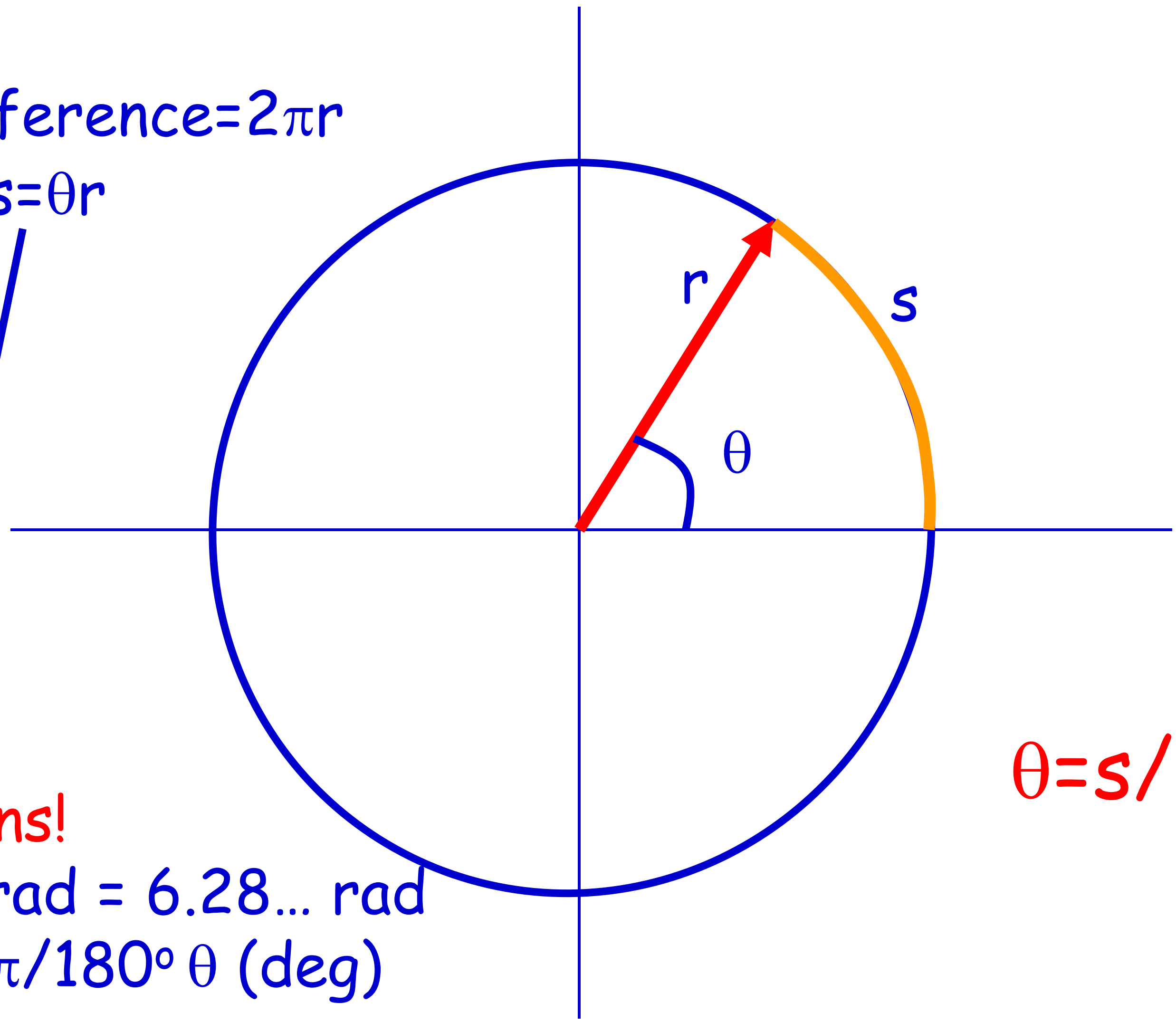
A man is standing on a platform resting on a frictionless surface. At the end of the platform a wall is placed and fixed to the platform.The man throws a ball against the wall and it bounces back over the man as shown in the figure. After the ball has bounced against the wall…

a) the platform moves to the left
b) the platform moves to the right
c) the platform remains at its place

Radians & Radius

Circumference=$2\pi r$

Part $s=\theta r$



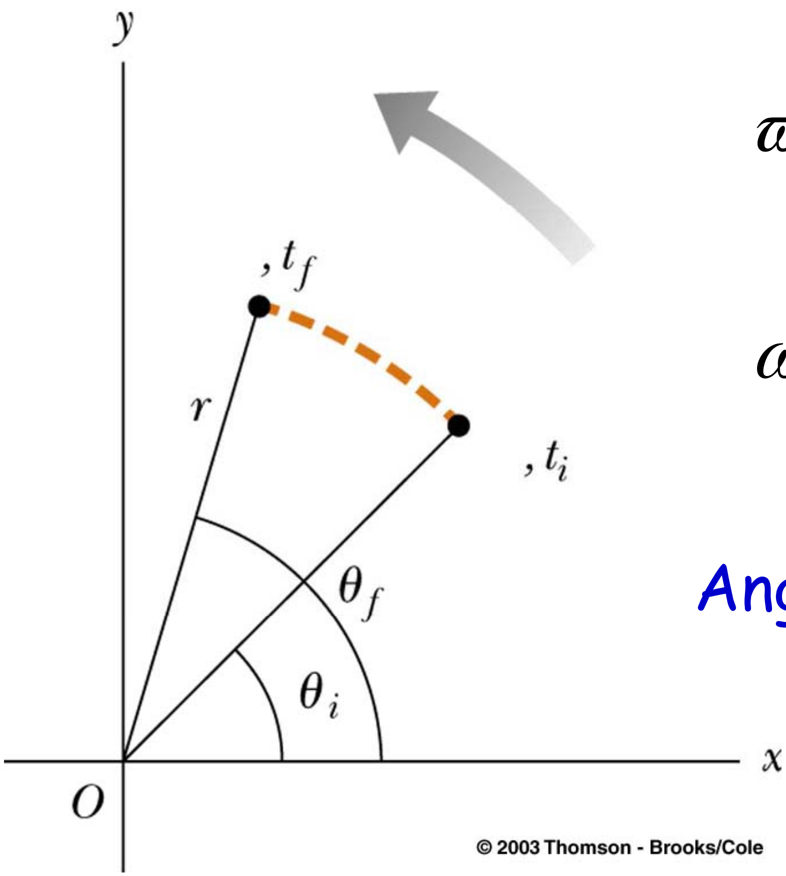
$\theta=s/r$

θ in radians!

$360^{o}=2\pi$ rad = 6.28… rad

θ (rad) = $\pi/180^{o}$ θ (deg)

Angular speed and acceleration



$$\varpi = \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\Delta\theta}{\Delta t}$$

Average angular velocity

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$$

Instantaneous Angular velocity

Angular velocity : rad/s
rev/s
rpm (revolutions per minute)

example

What is the angular velocity of earth around the sun?
Give in rad/s, rev/s and rpm

Answer: in 1 year, the earth makes one full orbit around the sun.

$$\varpi = \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\Delta\theta}{\Delta t}$$

rad/s	rev/s	rpm
= 2π rad/1 year	1 rev/1 year	1 rev/1 year
= 2π rad/(3.2E+7 s)	1 rev/(3.2E+7 s)	1 rev/(5.3E+5 min)
= 2.0E-07 rad/s	3.1E-8 rev/s	1.9E-6 rpm

Angular acceleration

Definition: The change in angular velocity per time unit

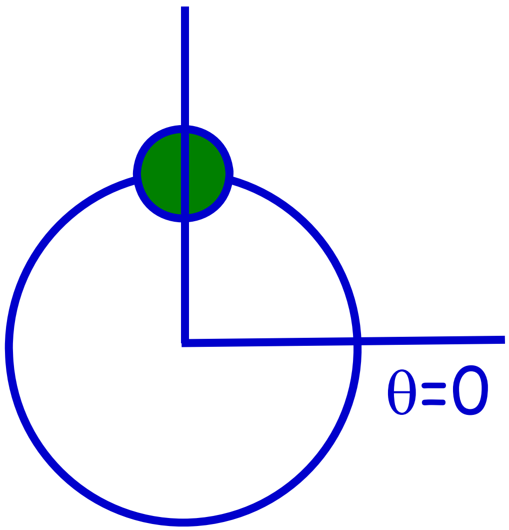
$$\bar{\alpha} = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t}$$ Average angular acceleration

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t}$$ Instantaneous angular acceleration

Unit: rad/s^2

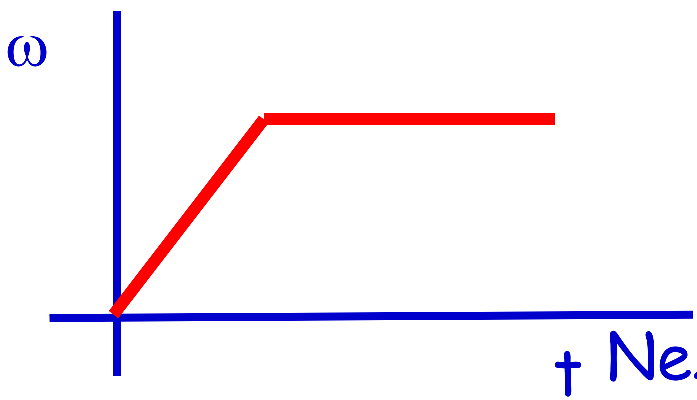
Equations of motion

Linear motion	Angular motion
$X(t)=x(0)+v(0)t+\frac{1}{2}at^2$	$\theta(t)= \theta(0)+\omega(0)t+\frac{1}{2}\alpha t^2$
$V(t)=V(0)+at$	$\omega(t)= \omega(0)+\alpha t$



example

A person is rotating a wheel. The handle is initially at $\theta=90^o$. For 5s the wheel gets an constant angular acceleration of 1 rad/s^2. After that The angular velocity is constant. Through what angle will the wheel have rotated after 10s.



First 5s: $\theta(5)= \theta(0)+\omega(0)t+\frac{1}{2}\alpha t^2$

$= \pi/2+ \quad 0 \quad + \frac{1}{2}1(5)^2$

$= \pi/2+12.5=$14.1 rad ($=806^o$)

$\omega(5)= \omega(0)+\alpha t$

$= 1t= 5$ rad/s

Next 5s: $\theta(5)= 14.1+5t$

= 39.1 rad ($=2240^0=6.2$ rev)

question

An object is rotating over a circle with radius of 2 m. Its speed is 1.5π rad/s. After 4 seconds, how many rotations did the object make?

a) 2
b) 3
c) 4
d) 8
e) 12

$$\theta(t) = \theta(0) + \omega(0)t + \tfrac{1}{2}\alpha t^2$$
$$= 0 + 1.5\pi t + 0$$
$$= 6\pi \text{ rad}$$

2π rad is 1 rotation (360^o), so 3 rotations

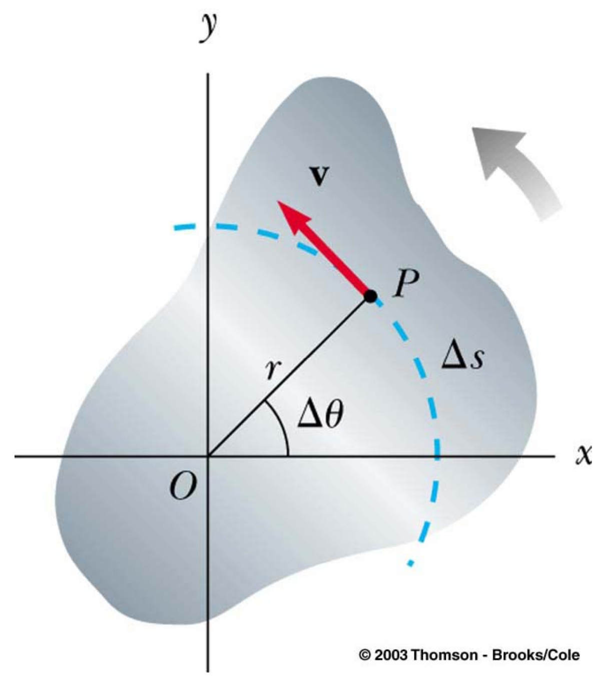
Angular ⟷ Linear velocities

$$\Delta\theta = \frac{\Delta s}{r}$$

$$\frac{\Delta\theta}{\Delta t} = \frac{1}{r}\boxed{\frac{\Delta s}{\Delta t}} \rightarrow V$$

$$\omega = \frac{v}{r}$$

$$v = \omega r$$



V is called the tangential velocity or the linear velocity.

Angular ⟷ linear acceleration

(In Circular Motion)

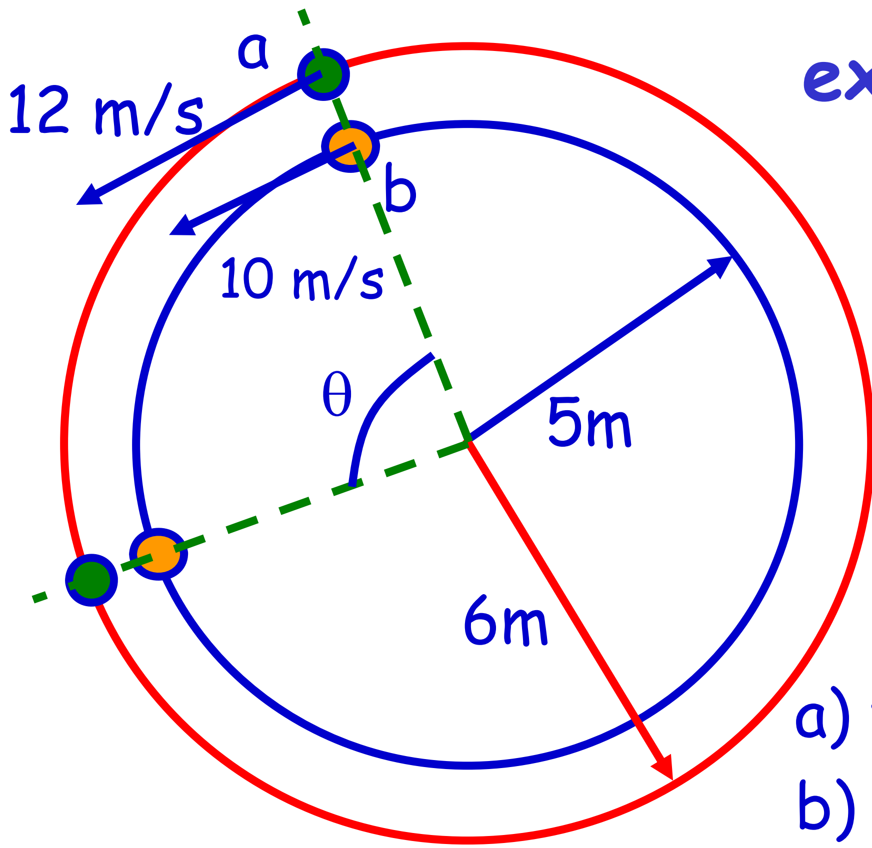
$$v = \omega r$$ Linear velocity

$$\Delta v = \Delta\omega \cdot r$$ Change in velocity

$$\frac{\Delta v}{\Delta t} = \frac{\Delta\omega}{\Delta t} r$$ Change in velocity per time unit

$$a = \alpha r$$ acceleration

The linear acceleration equals the angular acceleration times the radius of the orbit in circular motion.



example

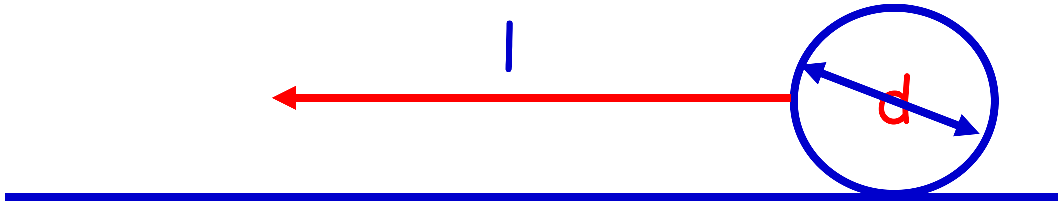
The angular velocity of a is 2 rad/s.
a) What is its tangential velocity?
If b is keeping pace with a,
b) What is its angular velocity?
c) What is its linear velocity?

a) $v = \omega r = 2*6=12$ m/s

b) Must be the same: 2 rad/s

c) $v=\omega r = 2*5=10$ m/s

A rolling coin



ω_0=18 rad/s
α=-1.80 rad/s^2
d=0.02 m

a) For how long does the coin roll?
b) What is the average angular velocity?
c) How far (l)does the coin roll before coming to rest?

a) $\omega(t)= \omega(0)+\alpha t$ 0=18-1.8t t=10 s

b) $\overline{\omega} = (\omega(0)+ \omega(10))/2$=18/2=9 rad/s

c) $\overline{v}=\overline{\omega}r$ = 9*0.01=0.09 m/s l=vt=0.09*10=0.9 m

question

What is the angular speed about the rotational axis of the earth for a person standing on the surface?

a) 7.3×10^{-5} rad/s
b) 3.6×10^{-5} rad/s
c) 6.28×10^{-5} rad/s
d) 3.14×10^{-5} rad/s
e) ????

$$\varpi = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{24 \cdot 3600} = 7.3 \cdot 10^{-5}\ rad/s$$