

PHYSICS 231

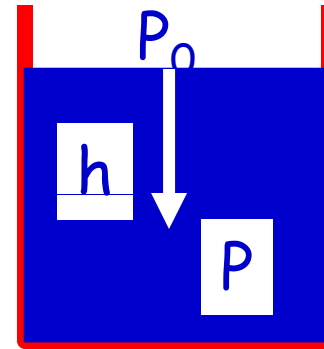
Lecture 22: fluids and viscous flow



Pressure at depth h

$$P = P_0 + \rho_{\text{fluid}}gh$$

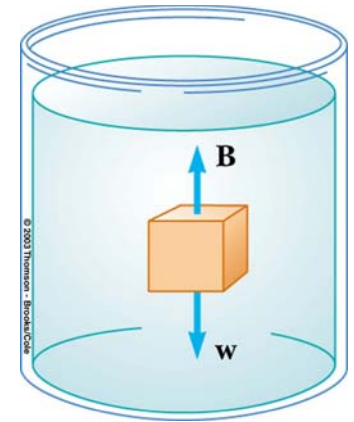
h : distance between liquid surface and the point where you measure P



Buoyant force for submerged object

$$B = \rho_{\text{fluid}}V_{\text{object}}g = M_{\text{fluid}}g = w_{\text{fluid}}$$

The buoyant force equals the weight of the amount of water that can be put in the volume taken by the object. $\rho_{\text{object}} (> \text{ or } =) \rho_{\text{fluid}}$
If object is not moving: $B = w_{\text{object}}$ $\rho_{\text{object}} = \rho_{\text{fluid}}$



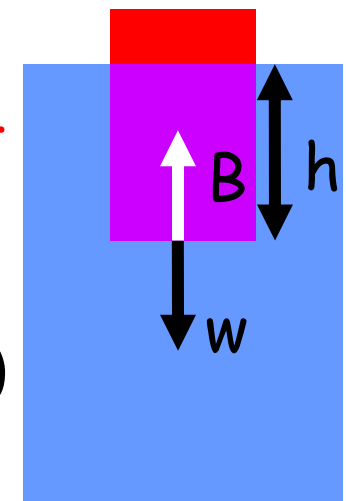
Buoyant force for floating object

The buoyant force equals the weight of the amount of water that can be put in the part of the volume of the object that is under water. ($B = w_{\text{object}}$)

$$\rho_{\text{object}}V_{\text{object}} = \rho_{\text{water}}V_{\text{displaced}} \quad h = \rho_{\text{object}}V_{\text{object}} / (\rho_{\text{water}}A)$$

$$\rho_{\text{object}} < \rho_{\text{fluid}}$$

PHY 231



Bernoulli's equation

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

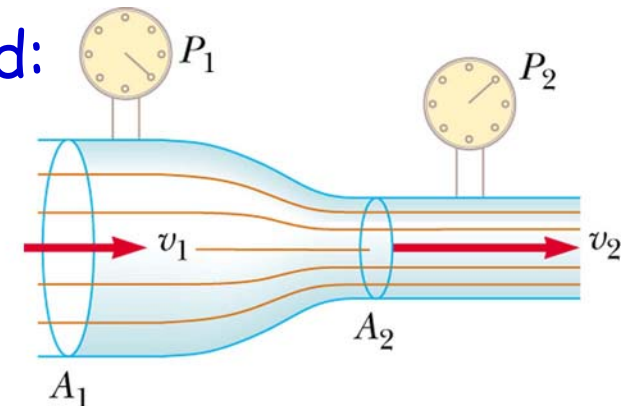
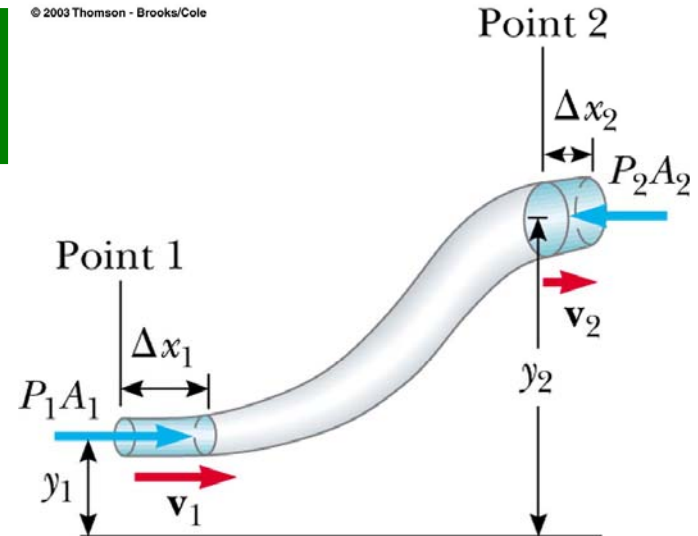
$$P + \frac{1}{2}\rho v^2 + \rho g y = \text{constant}$$

The **sum** of the pressure (P), the kinetic energy per unit volume ($\frac{1}{2}\rho v^2$) and the potential energy per unit volume ($\rho g y$) is constant at all points along a path of flow.

Note that for an incompressible fluid:

$$A_1 v_1 = A_2 v_2$$

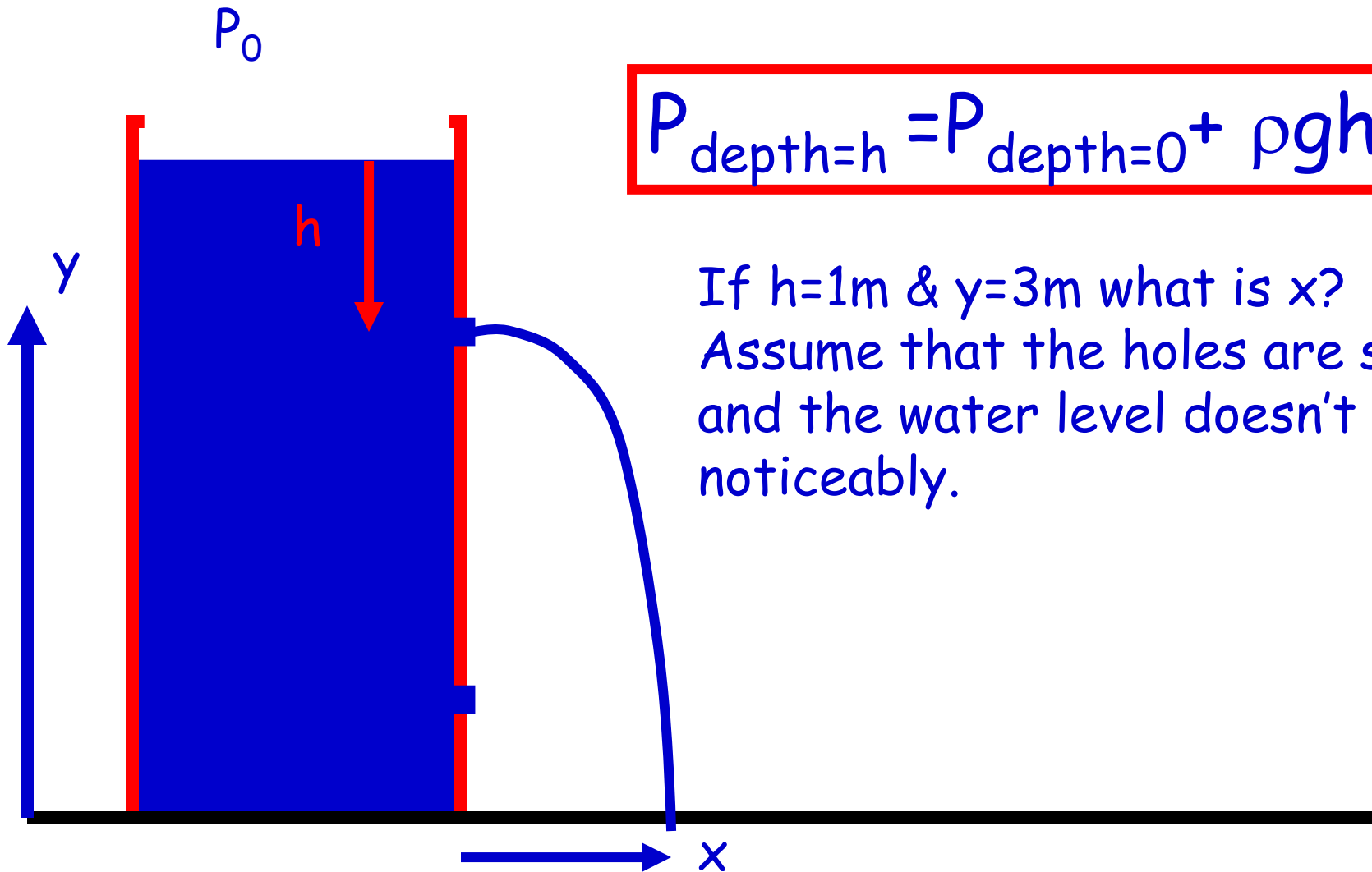
This is called the **equation of continuity**.



© 2003 Thomson - Brooks/Cole

(a)

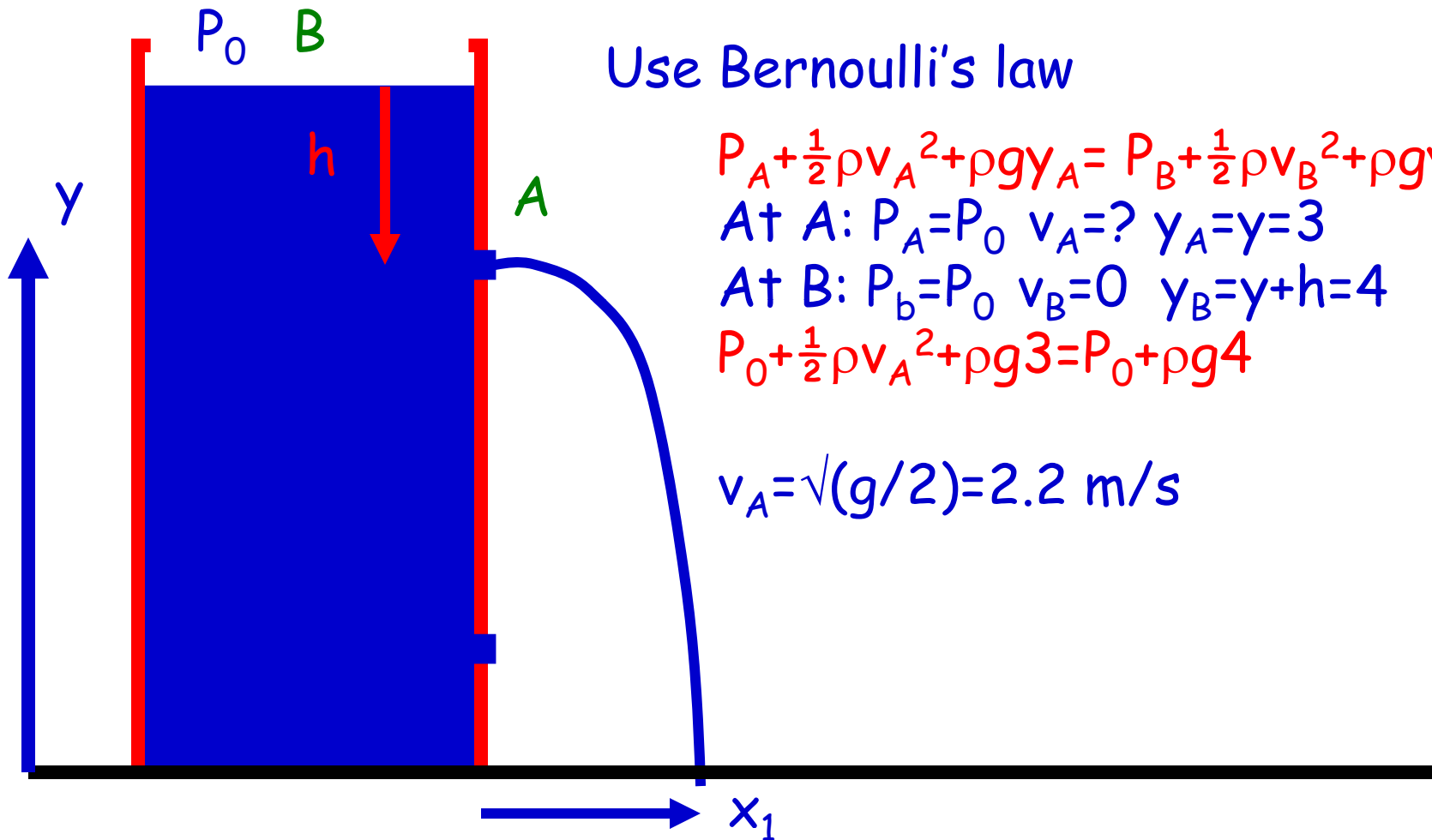
hole in a tank



$$P_{\text{depth}=h} = P_{\text{depth}=0} + \rho gh$$

If $h=1\text{m}$ & $y=3\text{m}$ what is x ?
Assume that the holes are small
and the water level doesn't drop
noticeably.

If $h=1$ m and $y=3$ m what is X ?



Use Bernoulli's law

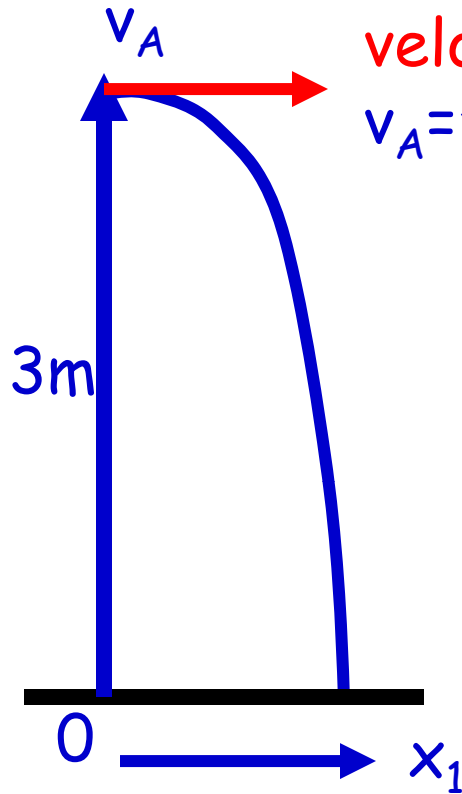
$$P_A + \frac{1}{2}\rho v_A^2 + \rho g y_A = P_B + \frac{1}{2}\rho v_B^2 + \rho g y_B$$

$$\text{At A: } P_A = P_0 \quad v_A = ? \quad y_A = y = 3$$

$$\text{At B: } P_B = P_0 \quad v_B = 0 \quad y_B = y + h = 4$$

$$P_0 + \frac{1}{2}\rho v_A^2 + \rho g 3 = P_0 + \rho g 4$$

$$v_A = \sqrt{(g/2)} = 2.2 \text{ m/s}$$



Each water element of mass m has the same velocity v_A . Let's look at one element m .

$$v_A = \sqrt{g/2} = 2.2 \text{ m/s}$$

In the horizontal direction:

$$x(t) = x_0 + v_{0x}t + \frac{1}{2}at^2 = 2.2t$$

In the vertical direction:

$$y(t) = y_0 + v_{0y}t + \frac{1}{2}at^2 = 3 - 0.5gt^2$$

= 0 when the water hits the ground, so

$$t = 0.78 \text{ s}$$

$$\text{so } x(0.78) = 2.2 * 0.78 = 1.72 \text{ m}$$

Viscosity



Viscosity: stickiness of a fluid.
a measure of **its resistance to flow**.

One layer of fluid feels a large resistive force when sliding along another one or along a surface of for example a tube.

Any real fluid requires a **pressure difference** between the ends of a tube to keep the fluid moving at constant velocity.

Viscosity

Consider a viscous fluid flowing with a constant average speed V_{avg} through a tube of length L and cross-section area A .

$$\Delta p = 8\pi\eta \frac{Lv_{avg}}{A}$$

η =coefficient of viscosity
unit: Ns/m^2
or poise= 0.1 Ns/m^2

The pressure difference causes a volume flow rate

$$Q = v_{avg} A$$

Poiseuille's equation

TABLE 9.5 The Viscosities of Various Fluids

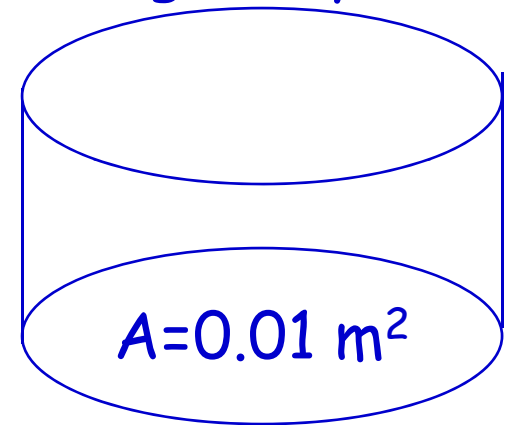
Fluid	$T(^{\circ}\text{C})$	Viscosity $\eta(\text{N} \cdot \text{s}/\text{m}^2)$
Water	20	1.0×10^{-3}
Water	100	0.3×10^{-3}
Whole blood	37	2.7×10^{-3}
Glycerin	20	$1\,500 \times 10^{-3}$
10-wt motor oil	30	250×10^{-3}

© 2003 Thomson - Brooks/Cole

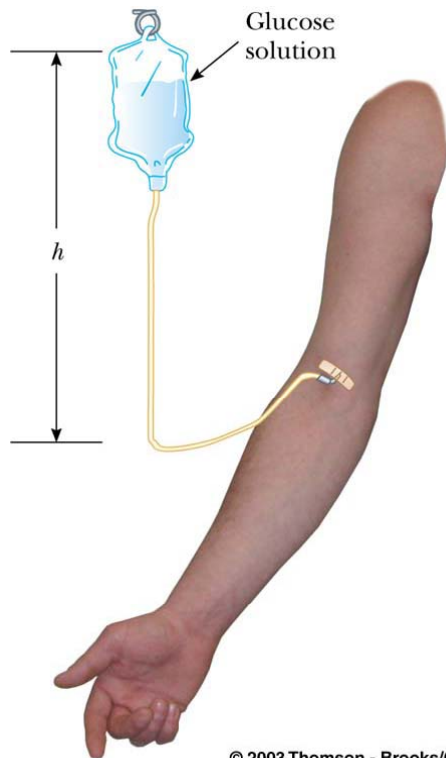
Buoyant forces

When submerged in water an object weighs 1.6N. At the same time, the water level in the water container (with $A=0.01 \text{ m}^2$) rises 0.01 m. What is the specific gravity (sg) of the object? ($\rho_{\text{water}}=1.0 \times 10^3 \text{ kg/m}^3$)

Use the fact that the Buoyant force on a submerged object equals the weight of the displaced water.



$$\begin{aligned} W &= F_g - B \\ &= M_{\text{object}}g - M_{\text{water, displaced}}g \\ &= \rho_{\text{object}} V_{\text{object}}g - \rho_{\text{water}} V_{\text{object}}g \\ &= V_{\text{object}}g (\rho_{\text{object}} - \rho_{\text{water}}) \\ 1.6\text{N} &= 0.01 * 0.01 * g (\rho_{\text{object}} - \rho_{\text{water}}) = 1.0 \times 10^{-4} * 9.8 * \rho_{\text{water}} (\text{sg} - 1) \\ \text{sg} &= 2.63 \end{aligned}$$



© 2003 Thomson - Brooks/Cole

Keep it coming.

A plastic bag contains a glucose solution. The part of the bag that is not filled is under vacuum. If the pressure in a blood vein is 1.33×10^4 Pa, how high must one hang the bag to make sure the solution (specific gravity 1.02) enters the body? ($\rho_w = 1.0 \times 10^3 \text{ kg/m}^3$)

$$P = P_0 + \rho gh$$

$$1.33 \times 10^4 = 0 + 1.02 * 1.0 \times 10^3 * 9.8 * h$$

$$h = 1.33 \text{ m}$$

Titanic: After the Titanic sunk, 10 people manage to seek refuge on a 2x4m wooden raft. It is still 1.0 cm above water. A heavy debate follows when another person (60 kg) wants to board as well. Fortunately, a PHY231 student is among the 10. Can she convince the others that it is safe to pull the person on board without the whole raft sinking? ($\rho_w = 1.0 \times 10^3 \text{ kg/m}^3$)



With 10 people: $F_g = B$

$$(M_{\text{raft}} + M_{10})g = V_{\text{displaced, before}} \rho_w g$$

With 11 people: $F_g = (M_{\text{raft}} + M_{10} + M_1)g$

$$B = (V_{\text{displaced, before}} + V_{\text{extra}}) \rho_w g$$

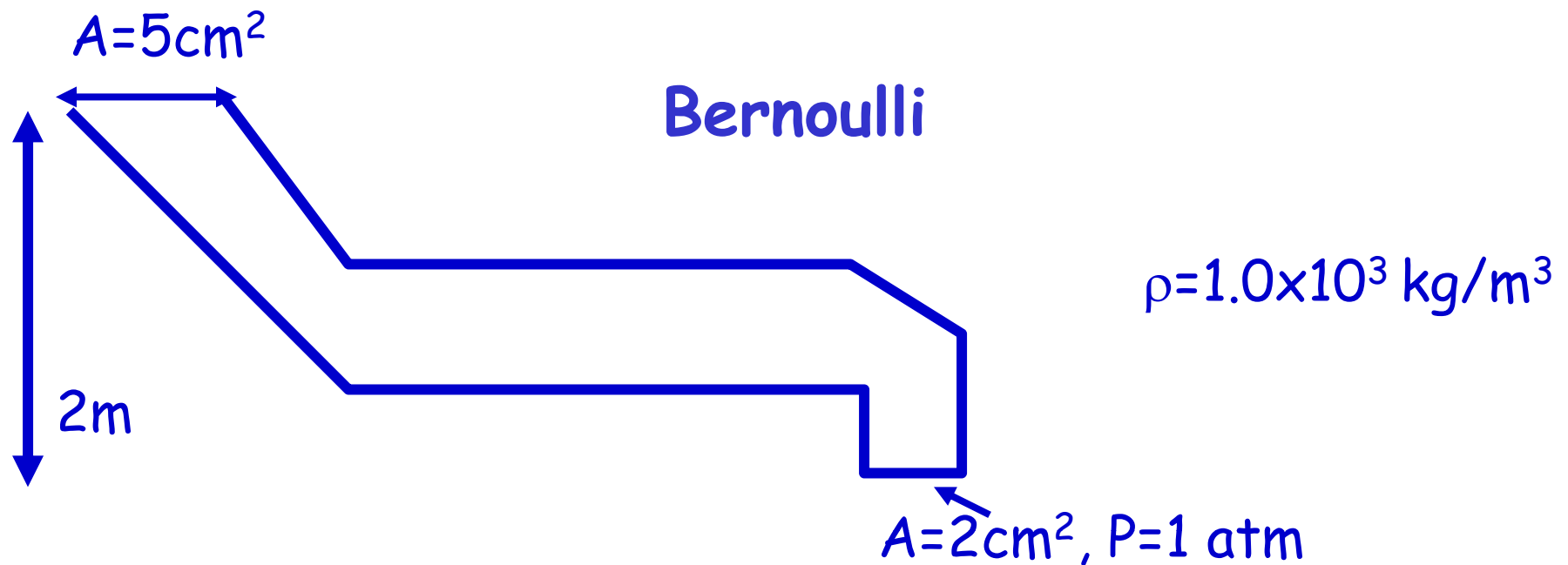
stationary if $F_g = B$ $(M_{\text{raft}} + M_{10})g + M_1 g = (V_{\text{displaced, before}} + V_{\text{extra}}) \rho_w g$

$$M_1 g = V_{\text{extra}} \rho_w g \text{ so } V_{\text{extra}} = (M_1 / \rho_w)$$

$$V_{\text{extra}} = 60 / 1.0 \times 10^3 = 0.06 \text{ m}^3$$

$$V_{\text{extra}} = L \times W \times H = 8H = 0.06 \text{ so } H = 0.75 \text{ cm}$$

so there is still 0.25cm to spare!



Water flows over a height of 2m through an oddly shaped pipe.

A) If the fluid velocity is 1 m/s at the bottom, what is it at the top? B) What is the water pressure at the top?

A) Use the equation of continuity: $A_1v_1=A_2v_2$

$$5*v_{\text{top}}=2*1 \quad v_{\text{top}}=0.4\text{ m/s}$$

B) Use Bernoulli. $P_{\text{top}}+\frac{1}{2}\rho v_{\text{top}}^2+\rho gh_{\text{top}}=P_{\text{bot}}+\frac{1}{2}\rho v_{\text{bot}}^2+\rho gh_{\text{bot}}$

$$P_{\text{top}}+0.5*(1\text{E}+03)*0.4^2+(1\text{E}+3)*9.8*2=(1\text{E}+05)+0.5*(1\text{E}+03)*1^2$$

$$P_{\text{top}}=80820\text{ Pa.}$$

question

Two cups are filled to the same level with water. One of the two cups has ice cubes floating in it. When the ice cubes melt, in which cup is the level of water higher?

- a) The cup without ice cubes
- b) The glass with ice cubes
- c) It is the same in both

Before the ice melts: $B = F_g$ so $\rho_w g V_{\text{disp}} = \rho_{\text{ice}} g V_{\text{ice}}$
 $V_{\text{disp}} = \rho_{\text{ice}} V_{\text{ice}} / \rho_{\text{water}}$

After it melts: $M_{\text{melted water}} = M_{\text{ice}}$
 $\rho_{\text{water}} V_{\text{melted}} = \rho_{\text{ice}} V_{\text{ice}}$
 $V_{\text{melted}} = \rho_{\text{ice}} V_{\text{ice}} / \rho_{\text{water}}$ / the same!