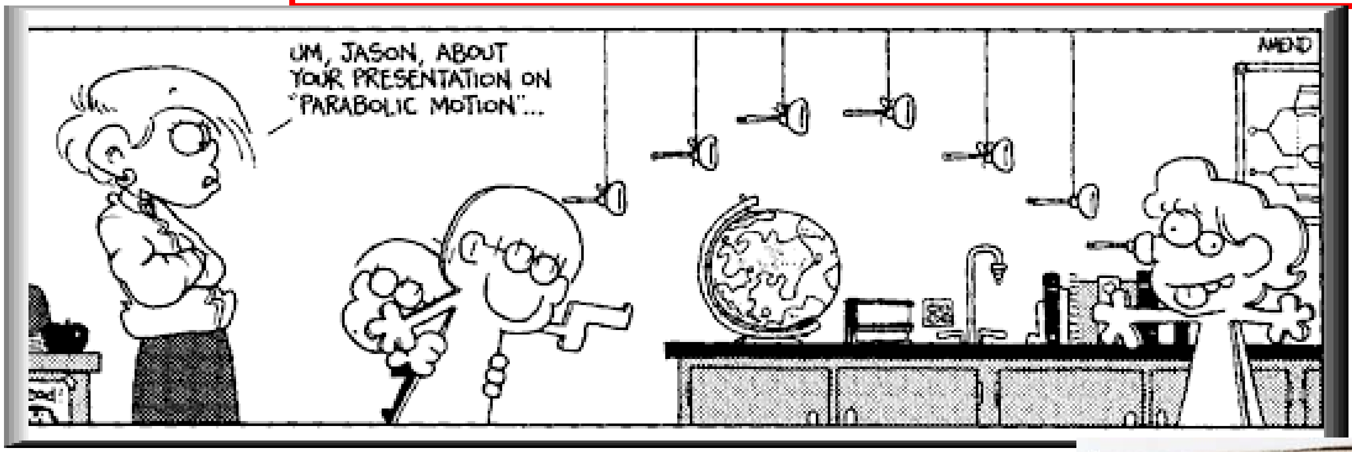
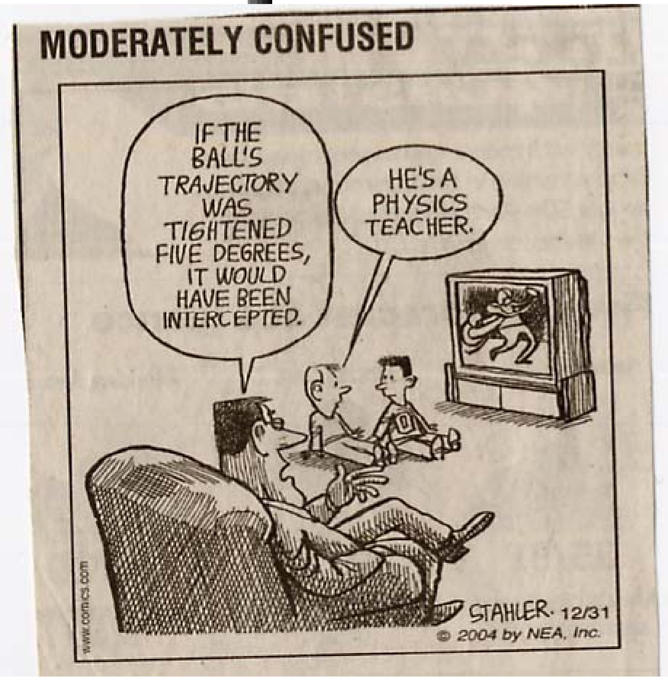
PHYSICS 231
Lecture 5: Motion in 2 dimensions



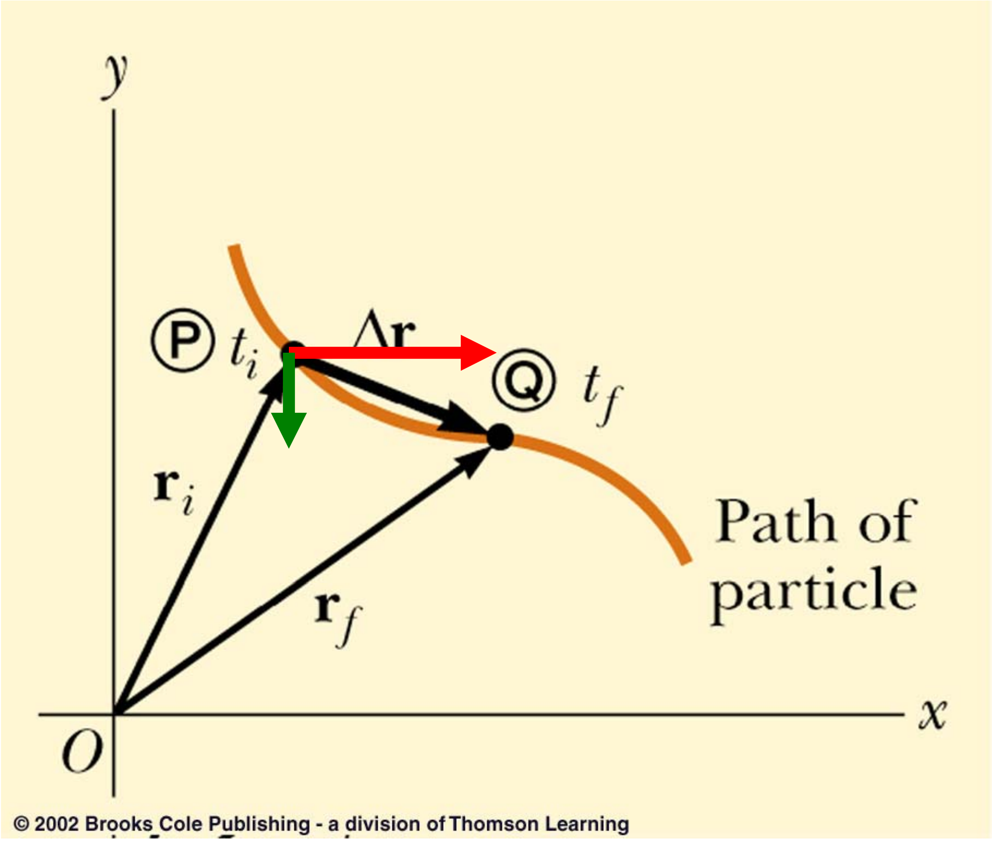


sample quiz I

- A fish jumps out of the water with v=5.0 m/s straight up. At which point is its speed highest (consider only the time after just leaving the water until just before reentering)?

A. just when it leaves the water
B. just before it reenters the water
C. just when it leaves and before it reenters the water
D. at the top

Displacement in 2D



Often, we replace motion in 2D into horizontal and vertical components.

In vector notation:

$\Delta \mathbf{r} = \Delta \mathbf{x} + \Delta \mathbf{y}$

1d motion

$$x(t)=x_0+v_0t+0.5at^2$$
$$v(t)=v_0+at$$

decomposition for 2D

2D motion; decompose into horizontal and vertical components

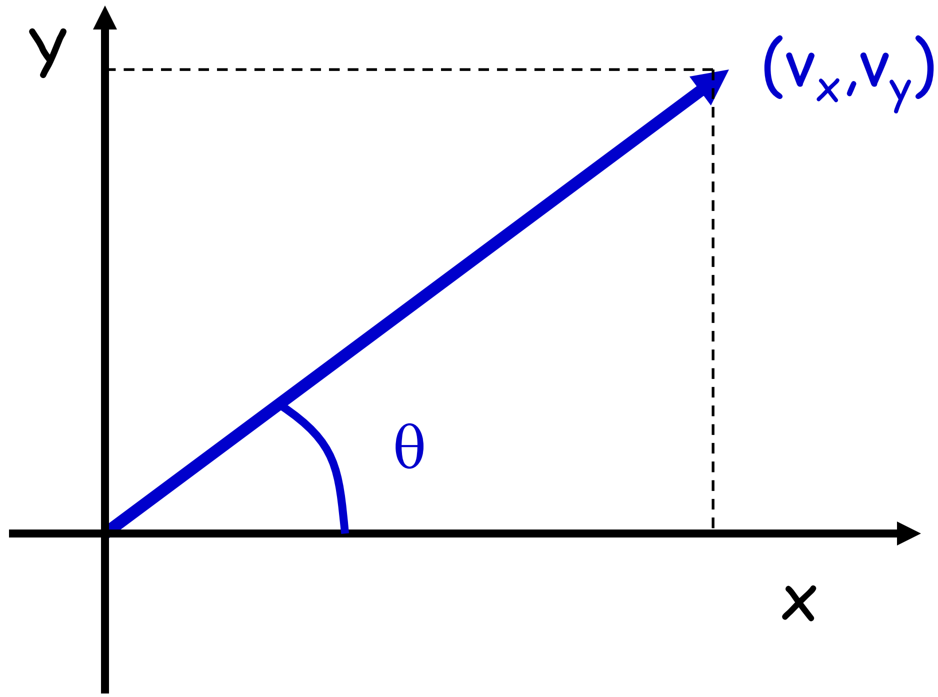
general 2D motion

$$x(t)=x_0+v_{0x}t+0.5a_xt^2$$
$$v_x(t)=v_{0x}+a_xt$$
$$y(t)=y_0+v_{0y}t+0.5a_yt^2$$
$$v_y(t)=v_{0y}+a_yt$$

Parabolic motion:

$$x(t)=x_0+v_{0x}t$$
$$v_x(t)=v_{0x}$$
$$y(t)=y_0+v_{0y}t-0.5gt^2$$
$$v_y(t)=v_{0y}-gt \qquad g=9.81\ m/s^2$$

The length of a vector and its components



Length of vector (use pythagorean theorem): $v = \sqrt{v_x^2 + v_y^2}$

$$v_x = v\cos\theta$$

$$v_y = v\sin\theta$$

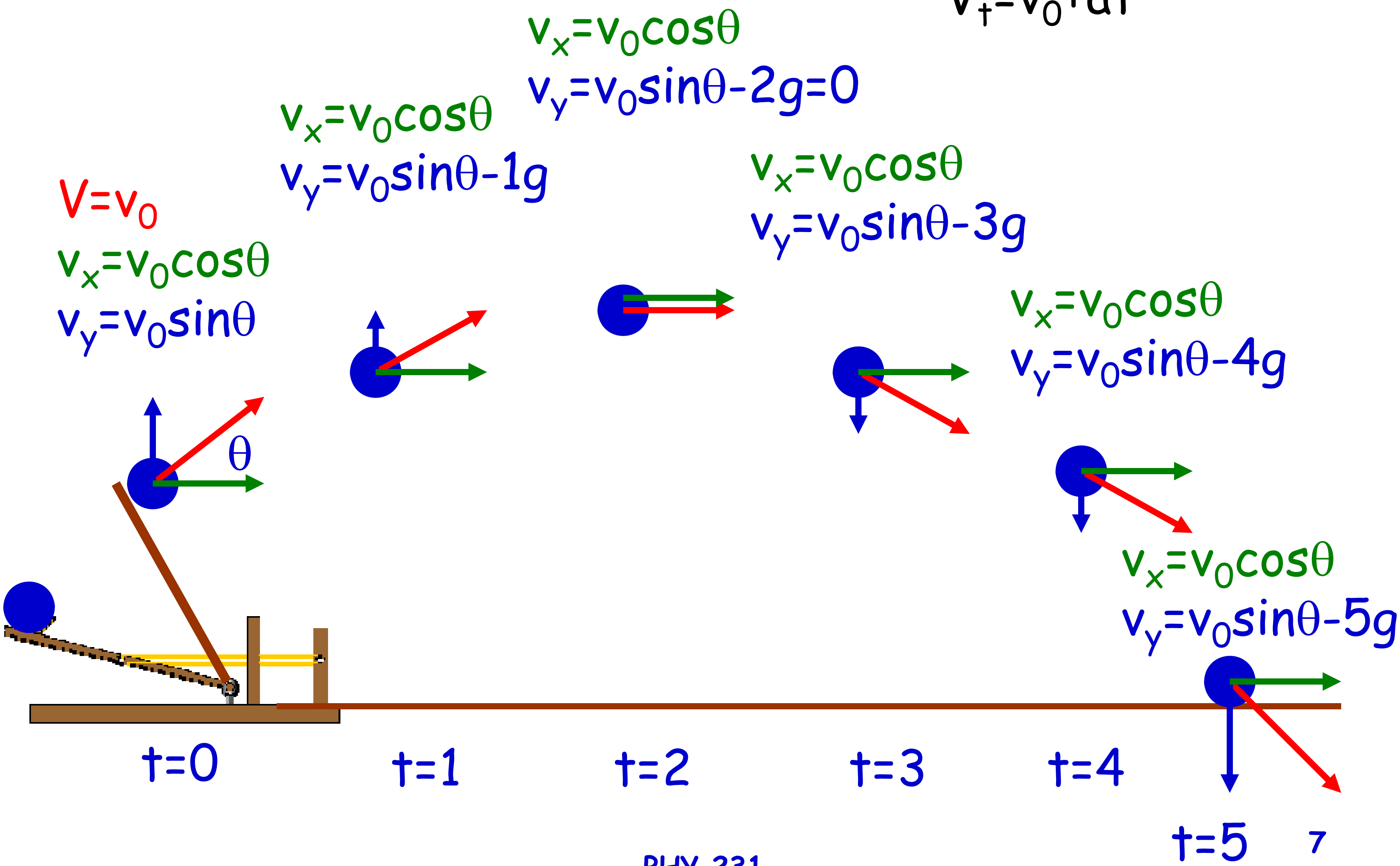
$$\tan\theta = v_y / v_x$$

While studying motion in 2D one almost always makes a decomposition into horizontal and vertical components of the motion, which are both described in 1D

- Remember that the object can accelerate in one direction, but remain at the same speed in the other direction.
- Remember that after decomposition of 2D motion into horizontal and vertical components, you should investigate both components to understand the complete motion of a particle.
- After decomposition into horizontal and vertical directions, treat the two directions independently.

Parabolic motion: a catapult

$V_t = v_0 + at$



Parabolic motion

$X(t)=X_0+V_0\cos\theta t$

$Y(t)=Y_0+V_0\sin\theta t-1/2gt^2$

$X=X_0$

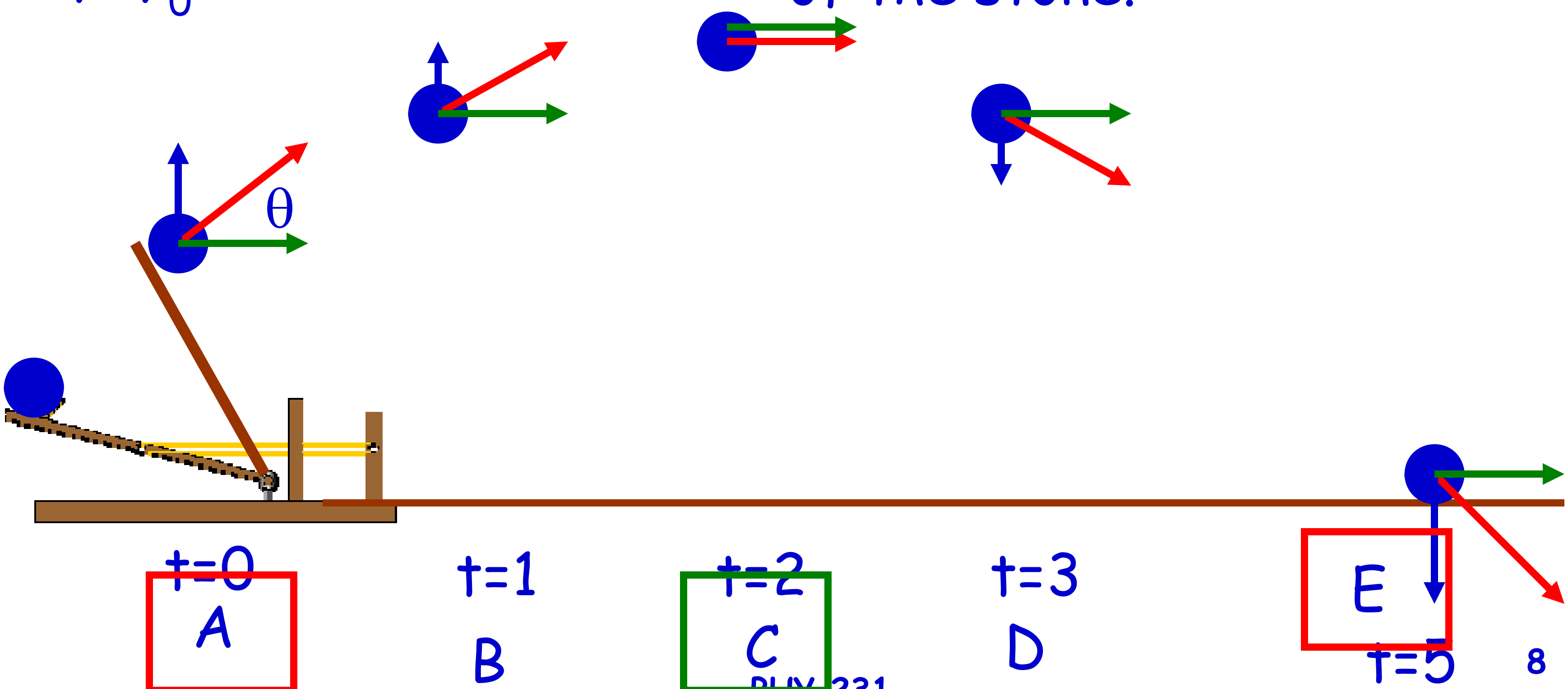
$Y=Y_0$

Where is the speed…

1) highest?

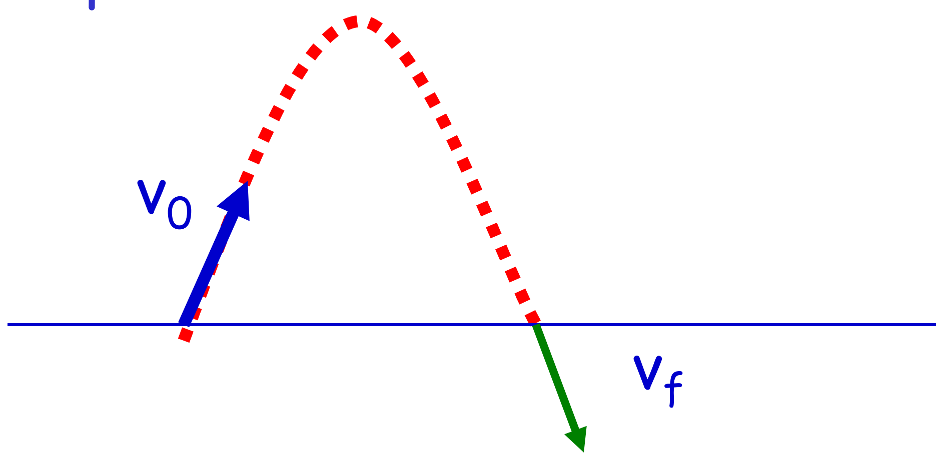
2) lowest ?

Assume height of catapult is negligible to the maximum height of the stone.



Question

- A hunter aims at a bird that is some distance away and flying very high (i.e. consider the vertical position of the hunter to be 0), but he misses. If the bullet leaves the gun with a speed of v_0 and friction by air is negligible, with what speed v_f does the bullet hit the ground after completing its parabolic path?



Answer

- First consider the horizontal direction:

  $V_{0x}=V_0\cos(\theta)$

  Since there is no friction, there is no change in the horizontal component: $V_{fx}=V_0\cos(\theta)=V_{0x}$

- Next the vertical direction:

  $V_{0y}=V_0\sin(\theta)$

  $V_y(t)=V_{0y}-gt$ $\quad$ $x_y(t)=V_{oy}t-0.5gt^2$ ($g=9.81\ m/s^2$)

  Boundary condition: bullet hits the ground:

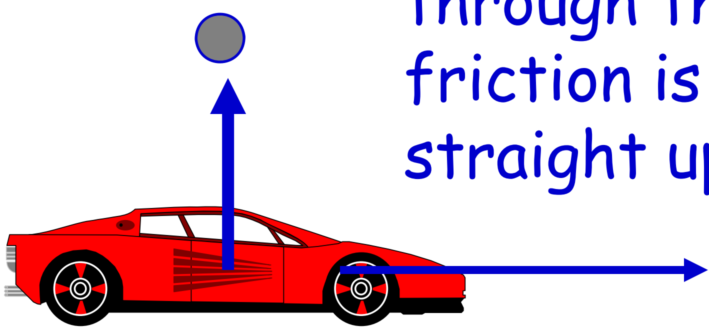
  $0=V_{oy}t-0.5gt^2 \longrightarrow t=0$ or $t=2V_{0y}/g$

  So, $V_{fy}(t)=V_{0y}-(2V_{0y}/g)g=-V_{0y}$

- Total velocity$=\sqrt{(V_{0x}{}^2+(-V_{0y})^2)}=V_0$!!!!

- The speed of the bullet has not changed, but the vertical component of the velocity has changed sign.

A careless driver.



A man driving in his sportscar finishes his drink and throws the can out of his car through the sun roof. Assuming that air friction is negligible and his throw is straight up, what happens?

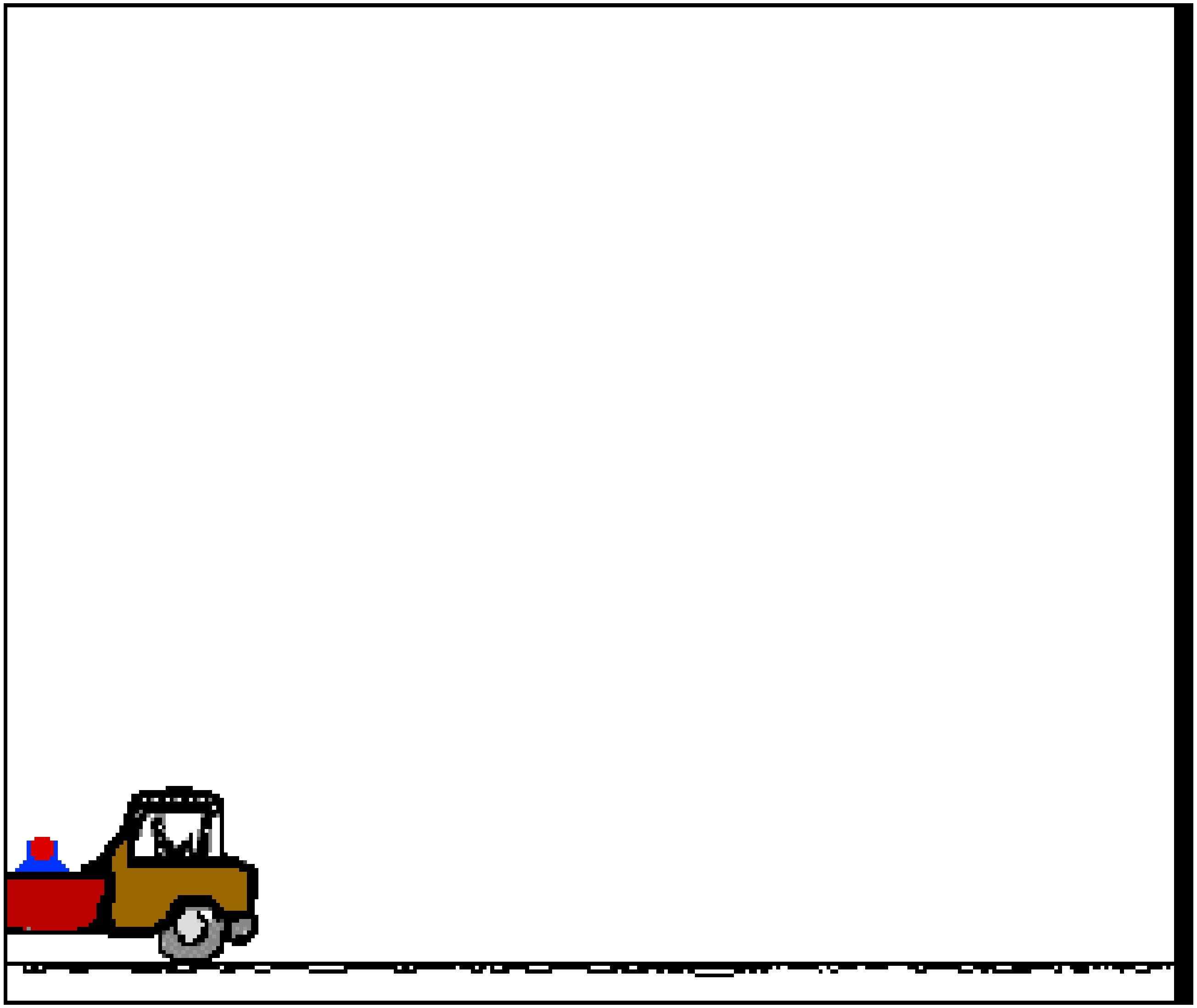
For the can: horizontal direction: $x(t)=v_{car}t$

vertical direction: $y(t)=v_{drink}t-0.5gt^2=0$ if $t=0$ (start) or $t=\sqrt{(2V_{drink}/g)}$

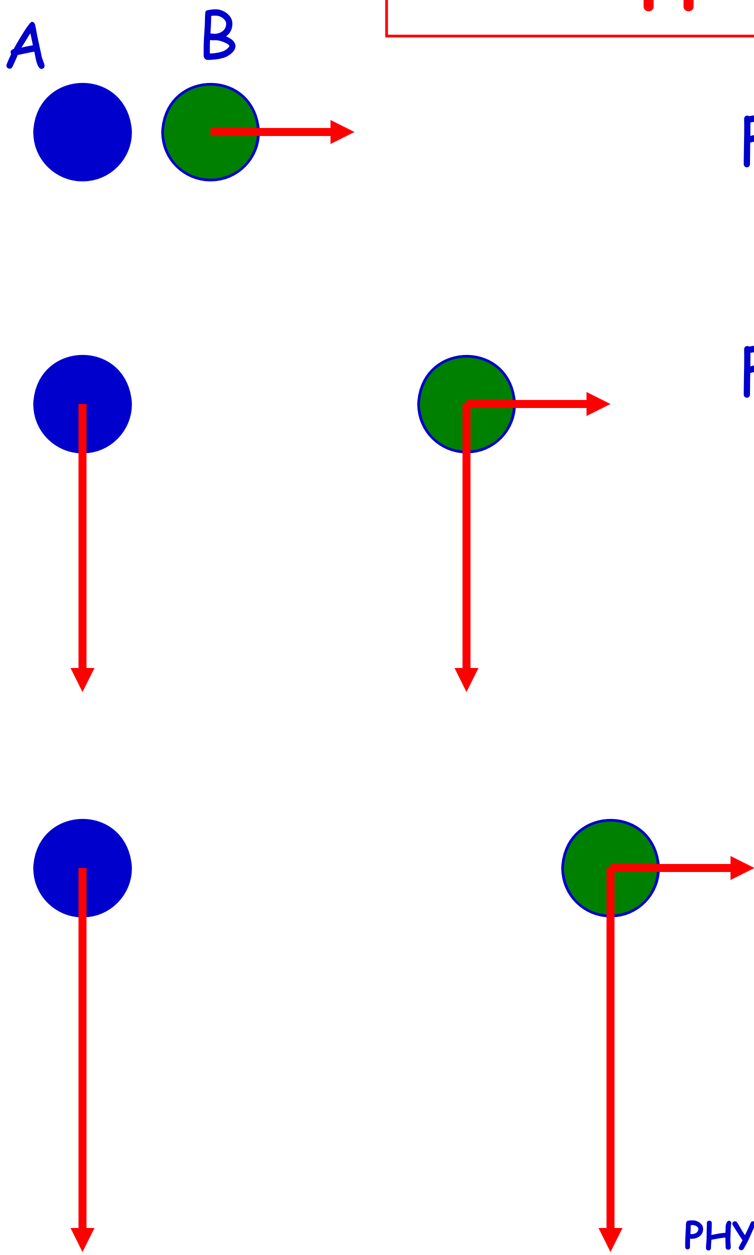
At $t=\sqrt{(2V_{drink}/g)}$

For the car: horizontal direction: $x(t)=v_{car}t$

After $t=\sqrt{(2V_{drink}/g)}$ the can drops back on the drivers head!



Dropping while moving (for B)



For A: $V_y=-0.5gt^2$
$V_x=0$

For B: $V_y=-0.5gt^2$
$V_x=V_0$

For A: $X_y=X_0-0.5gt^2$
$X_x=0$

For B: $X_y=X_0-0.5gt^2$
$X_x=V_0t$

Another example

- A football player throws a ball with initial velocity of 30 m/s at an angle of 30^o degrees w.r.t. the ground. How far will the ball fly before hitting the ground? And what about 60^o? And at what angle is the distance thrown maximum?

$X(t)=30\cos(\theta)t$

$Y(t)=30\sin(\theta)t-0.5gt^2$

$=0$ if $t(30\sin(\theta)-0.5gt)=0$

$t=0$ or $t=30\sin(\theta)/(0.5g)$

$X(t=30\sin(\theta)/(0.5g))=900\cos(\theta)\sin(\theta)/(0.5g)$

$=900\sin(2\theta)/g$

if $\theta=30^o$ $X=79.5$ m

if $\theta=60^o$ $X=79.5$ m !!

Maximum if $\sin(2\theta)$ is maximum, so $\theta=45^o$

$X(\theta=45^o)=91.7$ m

Yet another fish!

A fish jumps straight up out of the water with a velocity of 4 m/s. He first reaches a height of 0.9 m after 0.3 s. After falling back, he jumps again, but not straight up. His velocity in the vertical direction is 4 m/s and in the horizontal direction 1 m/s. The time that he takes to reach a height of 0.9 m is:

a) less that 0.3 s

b) again 0.3 s

c) more than 0.3 s

The velocity in the vertical direction is equal in both cases, so the time it takes is the same!