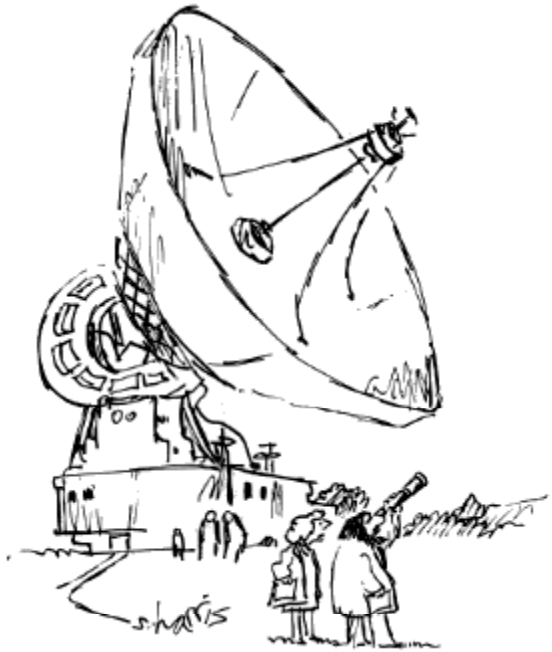
PHYSICS 231
Lecture 6: Relative motion & a monkey



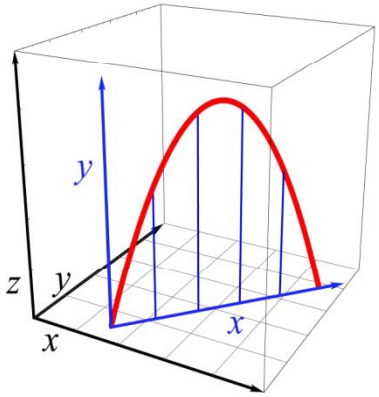
"Just checking."

Review: Trajectory of Projectile

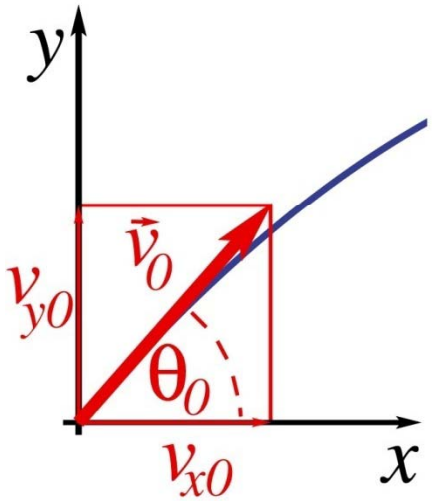
- Horizontal motion: constant velocity

$$(1)\quad x = x_0 + v_{x0}t$$

$$(2)\quad v_x = v_{x0}$$



- Vertical motion: free fall



$$(3)\quad y = y_0 + v_{y0}t - \tfrac{1}{2}gt^2$$

$$(4)\quad v_y = v_{y0} - gt$$

$$(5)\quad y = y_0 + \bar{v}_y t$$

$$(6)\quad \bar{v}_y = \tfrac{1}{2}(v_y + v_{y0})$$

$$(7)\quad v_y^2 = v_{y0}^2 - 2g(y - y_0)$$

- Use notation convention: $v_{x0} \equiv v_x(t=0);\ \ v_{y0} \equiv v_y(t=0)$

Maximum Height of Trajectory

- What characterizes maximum height? $v_y = 0$
- Solve for time to reach maximum height:

$$v_y = v_{y0} - gt \quad \Rightarrow \quad t = \frac{v_{y0}}{g}$$

- The height at this time:

$$\begin{aligned} y &= y_0 + v_{y0}t - \tfrac{1}{2}gt^2 \\ &= y_0 + v_{y0}\left(\frac{v_{y0}}{g}\right) - \tfrac{1}{2}g\left(\frac{v_{y0}}{g}\right)^2 \\ &= y_0 + \frac{v_{y0}^2}{2g} \end{aligned}$$

- Using $v_{y0} = v_0 \sin\theta_0$ gives:

$$H \equiv y - y_0 = \frac{v_0^2 \sin^2\theta_0}{2g}$$

Range of Trajectory

- Time to reach same height again: $t = \dfrac{2\,v_{y0}}{g}$ (2x time to reach peak)
- Horizontal distance traveled at this time:

$$R \equiv x - x_0 = v_{x0} t$$

$$= v_{x0}\left(\frac{2v_{y0}}{g}\right)$$

$$= \frac{2v_0^2 \sin\theta_0 \cos\theta_0}{g} = \frac{v_0^2 \sin 2\theta_0}{g}$$

$$v_{x0} = v_0 \cos\theta_0$$
$$v_{y0} = v_0 \sin\theta_0$$

$$\boxed{R = \frac{v_0^2 \sin 2\theta_0}{g}}$$

QUIZ (graded for extra credit)

2 quizpoints if correct
1 quizpoints if incorrect

$$\text{Extra-credit} = \frac{5*(\text{quizpoints collected during semester})}{0.75*2*(\text{total number of quizzes})}$$

but not exceeding the full 5 points extra-credit.

It's okay to discuss with neighbor.

Absences for in-class quizzes will not be excused as the 75% base allows for more than a reasonable number of absences.

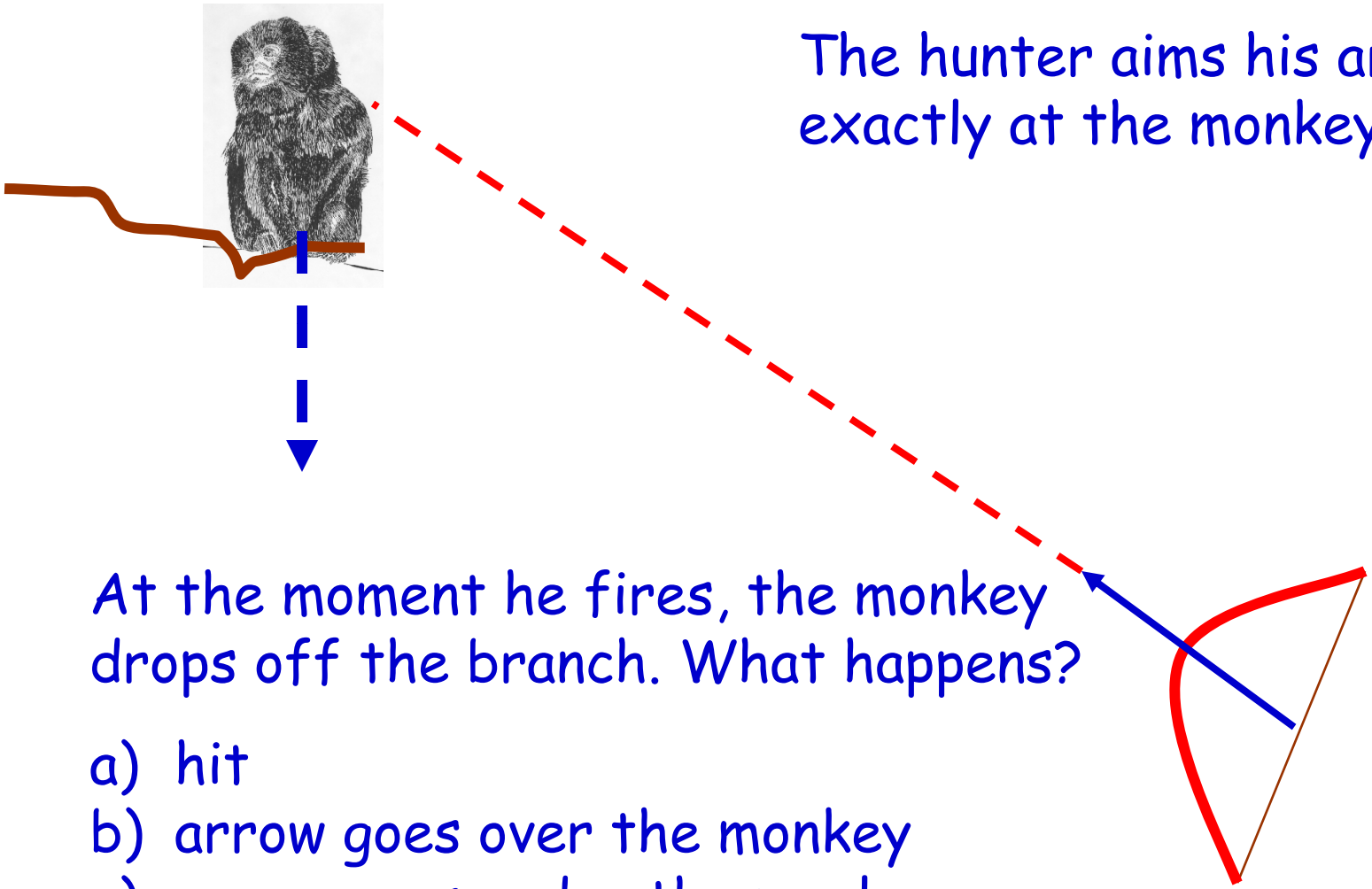
Stop jumping already!

A fish jumps out of the water with a velocity of 4 m/s in the vertical direction and 1 m/s in the horizontal direction. What is his lowest speed while out of the water?

a) 0 m/s
b) 1 m/s
c) 3 m/s
d) 4 m/s
e) just give me 1 point

At the highest point the vertical component of the velocity is 0 but the horizontal velocity is still 1 m/s.

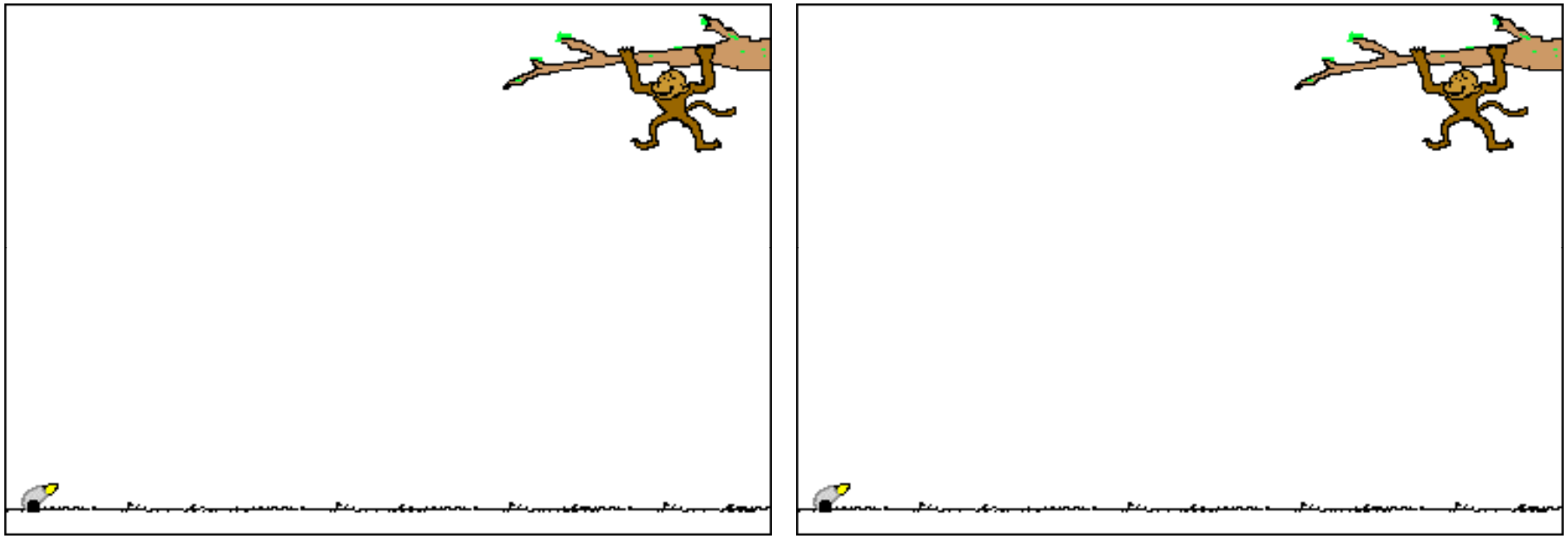
Shoot the monkey



The hunter aims his arrow exactly at the monkey

At the moment he fires, the monkey drops off the branch. What happens?

a) hit
b) arrow goes over the monkey
c) arrow goes under the monley
d) no idea



It doesn't matter what the velocity of the projectile is!!

The hor. Position of the arrow is:

$x(t)=d-v_0\cos(\theta)t$

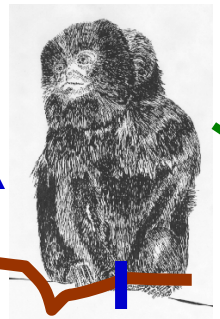
$x(t)=0$ at $t=d/v_0\cos(\theta)=t_{x=0}$

Its vertical position is:

$y(t)=v_0\sin(\theta)t-0.5gt^2$

Let $t=t_{x=0}$, then $v_0\sin(\theta)\,t_{x=0} = d\tan(\theta)$

$y(t_{x=0})=d\tan(\theta)-0.5gt^2=h-0.5gt^2$



h

The vertical position of the monkey is: $y(t)=h-0.5gt^2$

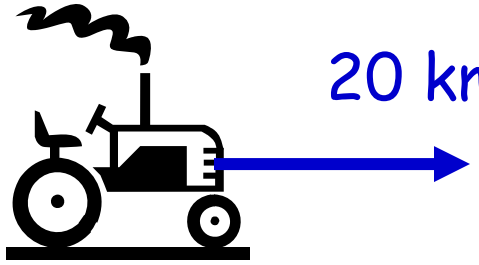
The horizontal position is 0

Always hit!!

d

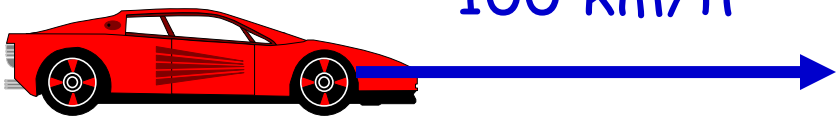
θ

Relative motion of 2 objects



20 km/h

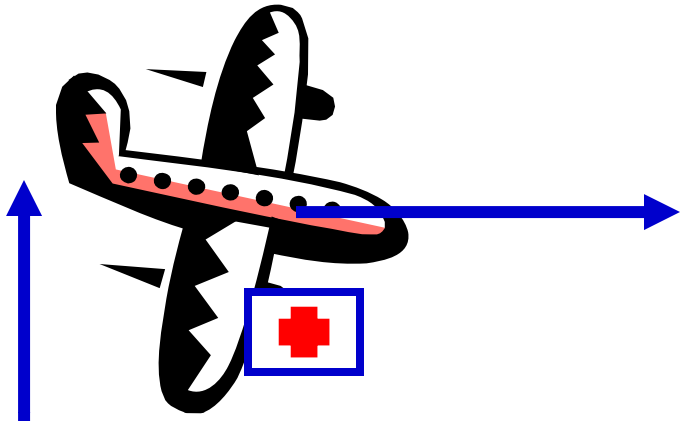
What is the velocity of
the Ferrari relative to the tractor?
And the other way around?



100 km/h

A) 80 km/h & 80 km/h
B) 20 km/h & -80 km/h
C) 80 km/h & -80 km/h
D) 100 km/h & 20 km/h

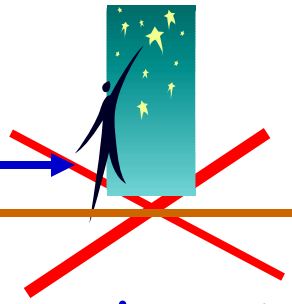
Relative motion of 2 objects II



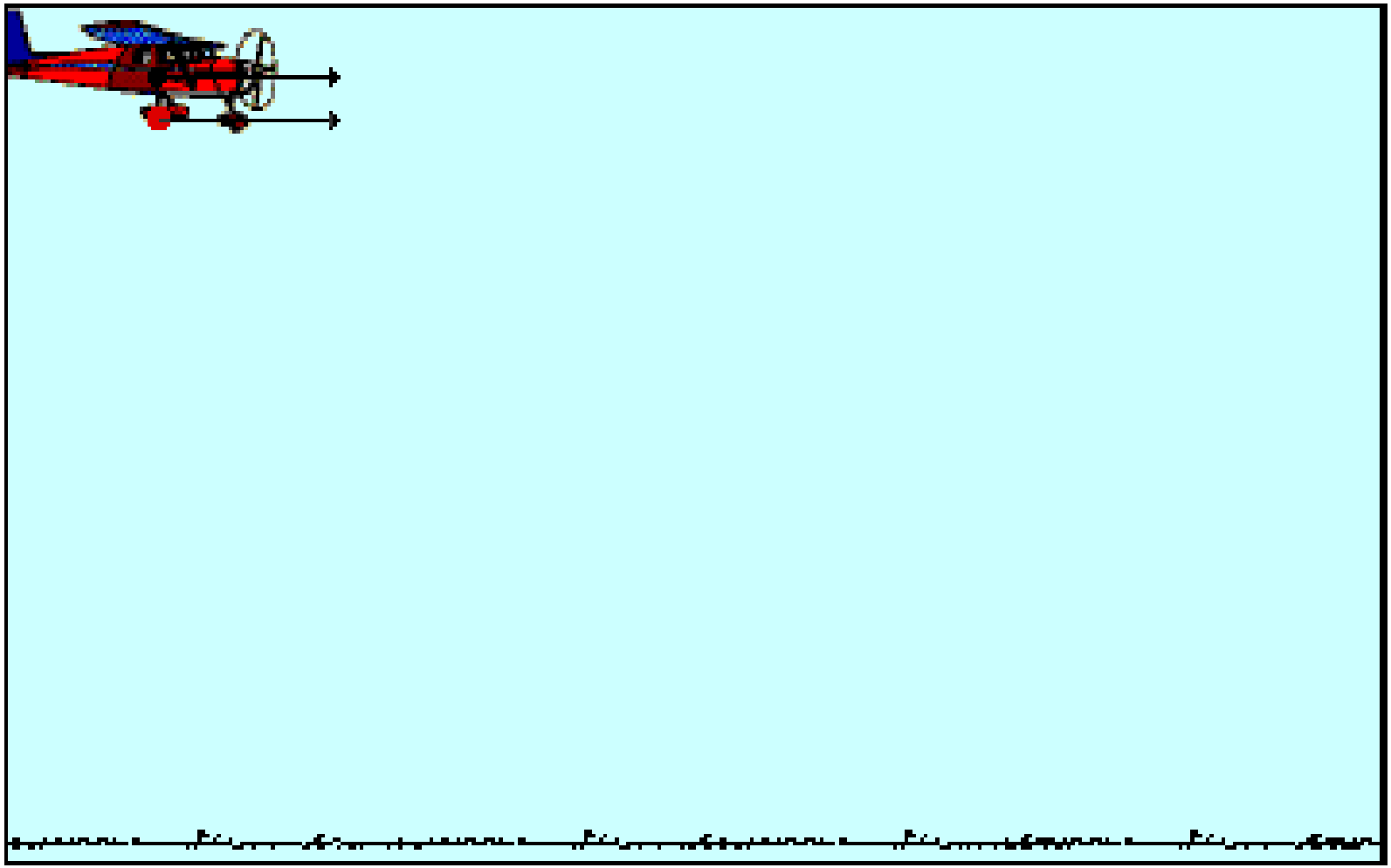
A UN plane drops a food package from a distance of 500 m high aiming at the dropzone X.
What does the motion of the package look like from the point of view of
a) the pilot b) the people at the drop zone

500m

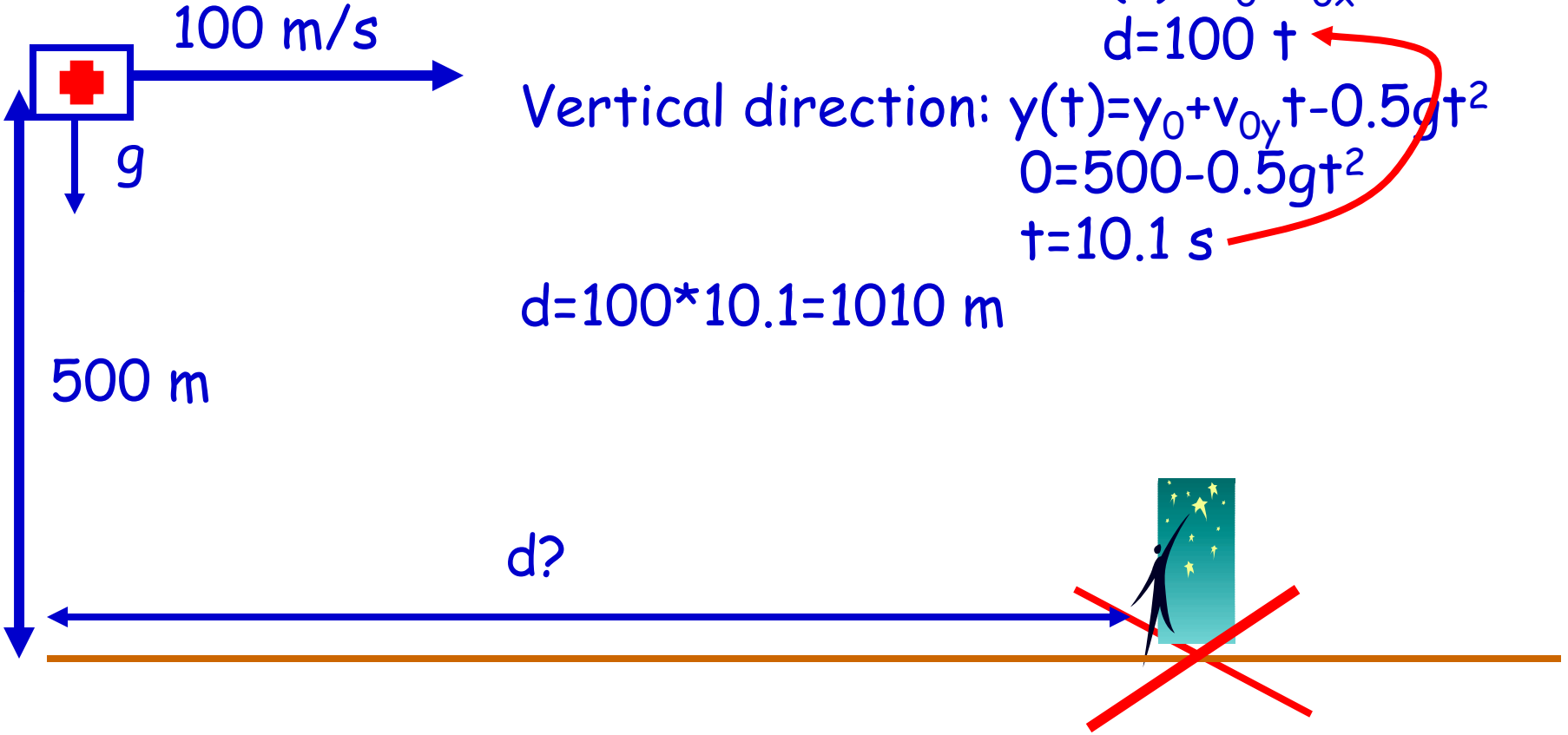
d



Recall of previous Lecture: if the plane is going at 100m/s, at what distance d from X should the plane drop the package?



Answer



Horizontal direction: $x(t)=x_0+v_{0x}t$

$d=100\ t$

Vertical direction: $y(t)=y_0+v_{0y}t-0.5gt^2$

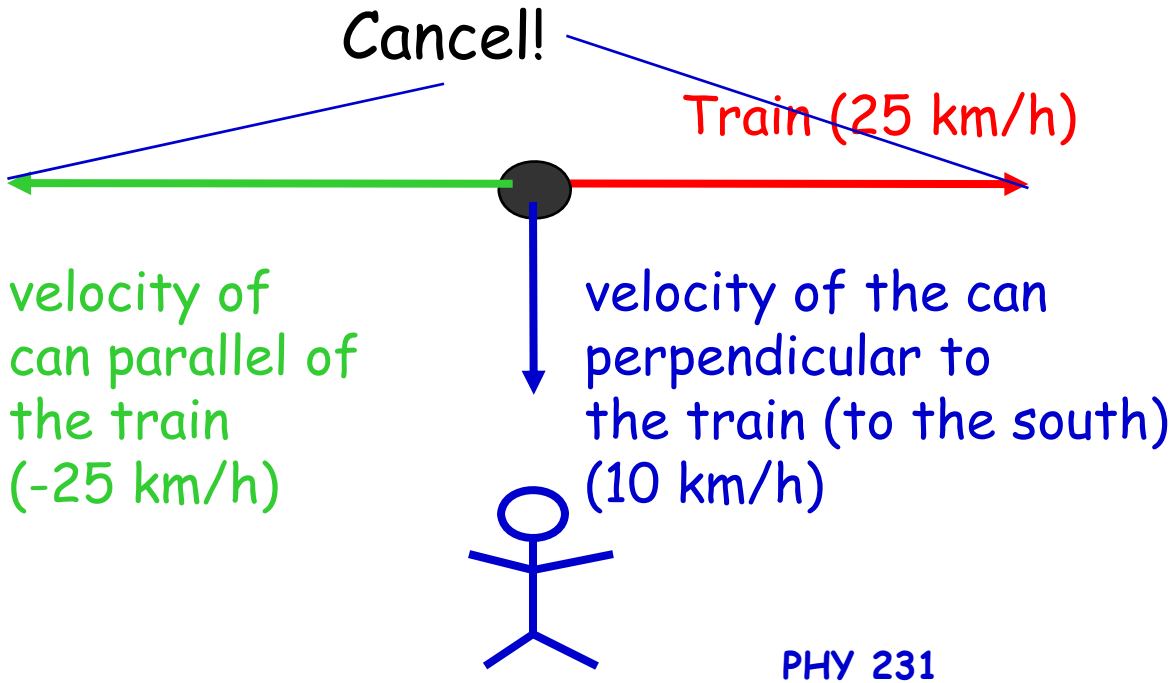
$0=500-0.5gt^2$

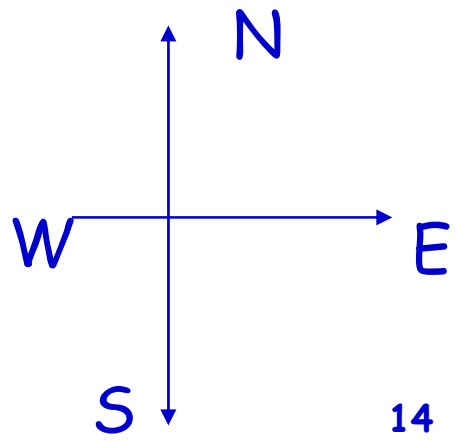
$t=10.1\ s$

$d=100*10.1=1010\ m$

Can out of the train

A train is moving with a speed of 25 km/h to the east. An environment-unfriendly passenger throws a can out of the window. The velocity with which he throws the can **relative to the moving train** is 25 km/h toward the back of the train (west) and 10 km/h away from the train toward the south. To an onlooker standing on the ground (south of the track), what is the observed direction of motion of the can?





vector subtraction

Consider 2 vectors:
r_1=6.3 making an angle of 33° with the x-axis
r_2=8.1 making an angle of 55° with the x-axis
What is the length and direction of $r_3=(r_2-r_1)$?

decompose:

$x_1=r_1\cos\theta_1=6.3\cos(33)=5.4$ $\quad$ $x_2=r_2\cos\theta_2=8.1\cos(55)=4.7$

$y_1=r_1\sin\theta_1=6.3\sin(33)=3.2$ $\quad$ $y_2=r_2\sin\theta_2=8.1\sin(55)=6.6$

$$\begin{pmatrix}x_3\\y_3\end{pmatrix}=\begin{pmatrix}x_2\\y_2\end{pmatrix}-\begin{pmatrix}x_1\\y_1\end{pmatrix}$$

$x_3=4.7-5.4=-0.7$

$y_3=6.6-3.2=3.4$

Make sure to set your calculator to degrees!

length of r_3: $\sqrt{([-0.7]^2+[3.4]^2)} = 3.5$

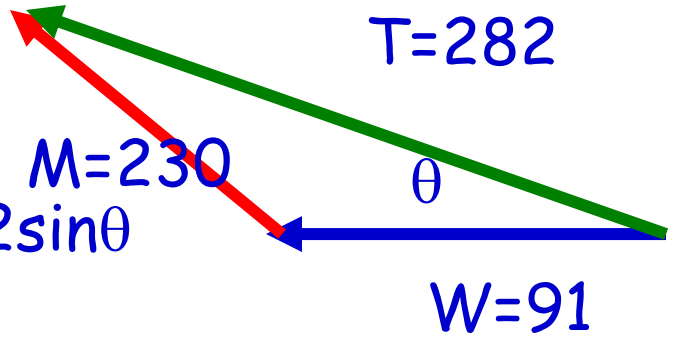
angle with x-axis: $\theta_3=\text{atan}(y_3/x_3)=-78°$!!!!!! add 180°: 102°

(to get in right quadrant)

plane in the wind I

A plane, capable of flying at 230 mph with no wind,
is moving at full speed in a gale blowing 91 mph from the east.
The plane's ground speed is measured at 282 mph.
What direction is the plane moving relative to the ground?
(Give angle in degrees relative to an axis pointing east.)

M=T-W (vectors!)



$T_x=282\cos\theta$ $T_y=282\sin\theta$

$W_x=91$ $W_y=0$

$M_x=T_x-W_x=282\cos\theta-91$ $M_y=T_y-W_y=282\sin\theta$

$(M_x^2+M_y^2)=230^2$ pythagorean theorem

$(282\cos\theta-91)^2+(282\sin\theta)^2=230^2$

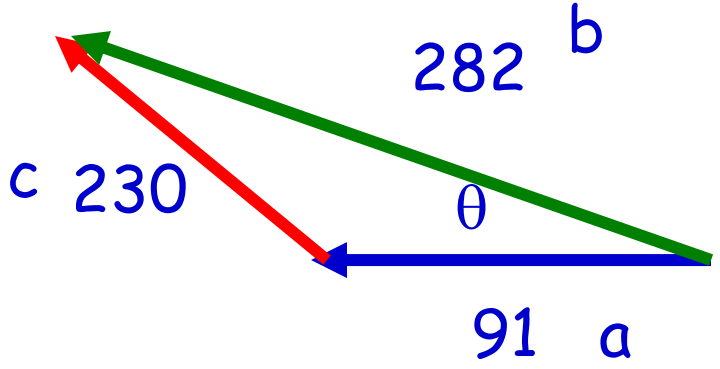
$282^2\cos^2\theta-2*91*282\cos\theta+91^2+282^2\sin^2\theta=230^2$

$\sin^2\theta+\cos^2\theta=1$ so $\cos\theta=(230^2-91^2-282^2)/(-2*91*282)$

$\theta=47.2^o$ so $180-47.2=132.8^o$ relative to axis pointing east

plane in the wind II

A plane, capable of flying at 230 mph with no wind,
is moving at full speed in a gale blowing 91 mph from the east.
The plane's ground speed is measured at 282 mph.
What direction is the plane moving relative to the ground?
(Give angle in degrees relative to an axis pointing east.)



$c^2=a^2+b^2-2ab\cos\theta$ θ is the angle opposite to vector c.

$230^2=91^2+282^2-2*91*282\cos\theta$

$\theta=47.2^0$ so $180-47.2=132.8^0$ relative to axis pointing east