Break Gauge Symmetry to generate Mass

1. Yang-Mills gauge theory

1) 1954, Yang & Mills proposed that the strong nuclear interaction among (p, n, π) be described by a gauge invariant field theory.
The local gauge group was the SU(2) isotopic-spin group.
⇒ non-abelian gauge theory

2) Yang-Mills theory failed in it's original purpose.

Because "gauge invariance" implies massless gauge boson, which induces $V(r) \sim \frac{1}{r}$ (long range force)

Yet, the nuclear force is short range.

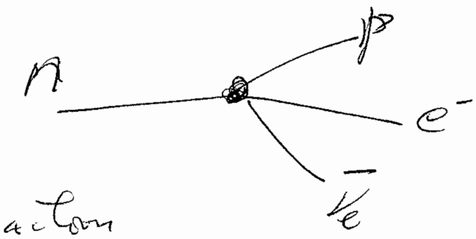
3) However, SU(2) isotopic-spin symmetry determines the fundamental form of the interaction among (p, n, π) ⇒ Chiral Lagrangian

2. Weak interaction

1) Fermi 4-fermion theory, (1934)

$$H = \frac{G_F}{\sqrt{2}} \underbrace{(\bar{p}\gamma_\mu n)}_{\uparrow} \underbrace{(\bar{e}\gamma^\mu \nu_e)}_{\uparrow}$$

like QED interaction
(vector current)



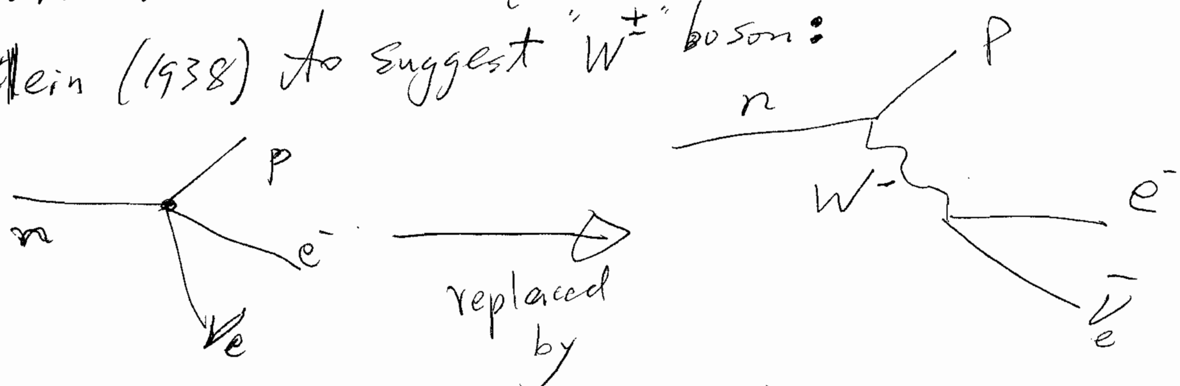
Note: Parity violation was not known yet.

2) Lee & Yang, 1956, suggested "Parity violation".

$\Rightarrow$ "V−A" theory

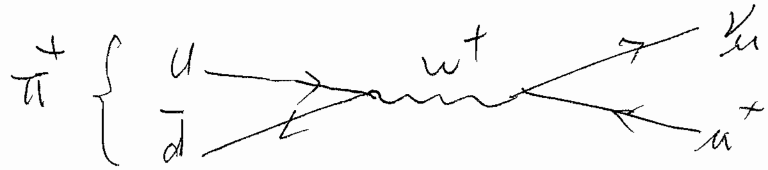
$\Rightarrow$ $(\bar{e}\gamma^\mu(1-\gamma_5)\nu_e)$
$(\bar{p}\gamma_\mu(1-\gamma_5)n)$

3) Yukawa meson theory (1935) motivated Klein (1938) to suggest "$W^\pm$" boson:



4) Schwinger (1957) and Glashow (1961) suggested that leptons must carry (weak) isotopic spin $\Rightarrow$ $\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}$ forms an SU(2) doublet

5) Gell-Mann proposed quark model (1964)
So that $\pi^+ \to \mu^+ \nu_\mu$ can be understood as



$\Rightarrow \begin{pmatrix} u \\ d \end{pmatrix}$ forms a weak doublet.

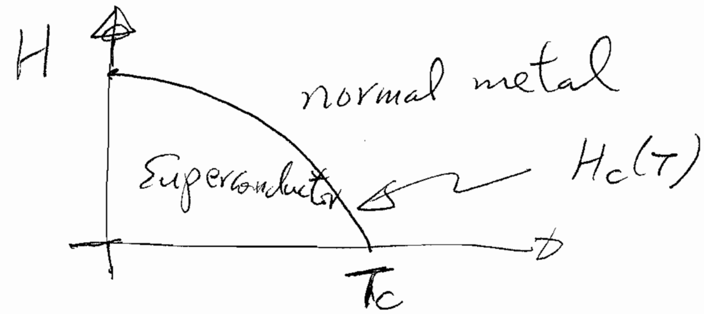
6) The gauge bosons $W^\pm$ have to be massless in Yang-Mills gauge theory, but $G_F \sim \frac{1}{M_W^2} \sim 10^{-5}$
So that M_W has to be large.
If gauge symmetry is broken by adding an explicit interaction term in the Lagrangian, then it can not be renormalized. $\Rightarrow$
Can not predict finite physical observables.

$\Rightarrow$ The big question was:

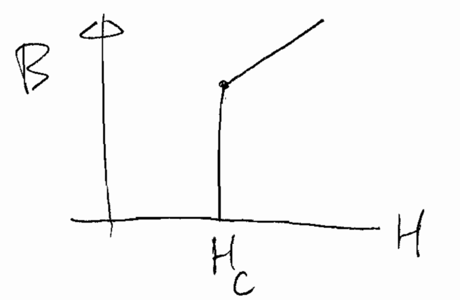
How to generate W-boson mass in a renormalizable Yang-Mills gauge theory?

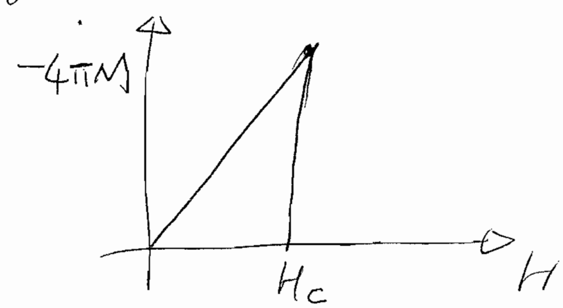
3. Superconductor

1) The phase bondary between the superconducting and normal states of a type I superconductor is



⇒ Below a critical magnetic field $H_c(T)$, there is no penetration of flux.





The Magnetization
$$4\pi M = B - H$$

⇒ Meissner effect

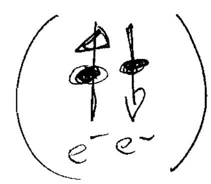
(If a normal metal in a magnetic field is cooled below its superconducting transition temperature, the magnetic flux is abruptly expelled.)

It's resulted from the interaction of an external magnetic field with the "self-coherent" electrons in the superconductor.

2) Bardeen, Cooper and Schrieffer (BCS) theory (1957)

(1) The interaction of the conduction electrons with the atomic lattice of the superconductor produces an "attractive" force between the electrons.

$\Rightarrow$ Cooper pairs



opposite spins

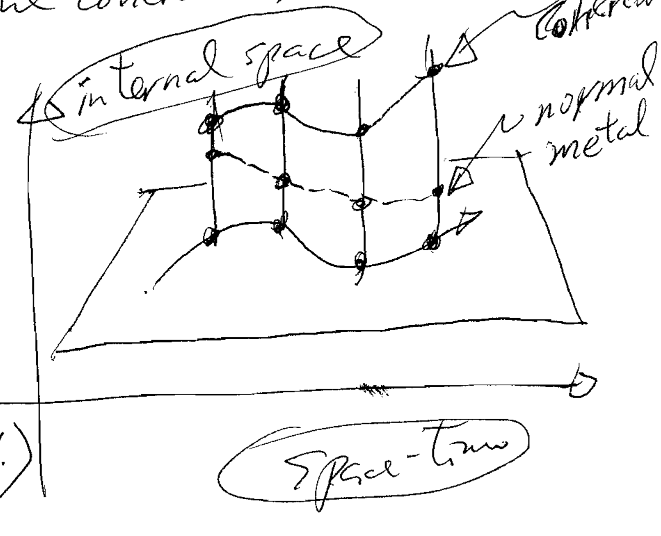
$\equiv$ Scalar (spin-0) with two units of negative charge

(2) Owing to the extremely weak binding, the effective size of a Cooper pair is $\sim 10^{-4}$ cm $= 10^{4}$ Å.

Every Cooper pair overlaps with approximately 10^{6} other Cooper pairs, and this superposition results in a strong correlation which "locks" the phases of the wavefunctions together coherently.

$\Rightarrow$ Cooper pair current (the coherent system) violates gauge invariance.

$\Rightarrow$ generate a short-range field (the external magnetic field penetrates a small distance into superconductor.)



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(3) The interaction of an external magnetic field with the coherent electrons (Cooper pairs) in the superconductor results in Meissner effect.

(1) The coherent system of Cooper pair electrons can be described by the Schrödinger wavefunction

$$\psi(\underline{x},t) = \left(\frac{N}{2}\right)^{\frac{1}{2}} e^{\left[\frac{-2ie\,\phi(\underline{x},t)}{\hbar c}\right]}$$

where $\frac{N}{2}$ is the density of Cooper pairs.

The current density is obtained from

$$\underline{J} = \frac{-e\hbar}{2im}\left(\bar{\psi}\nabla\psi - \psi\nabla\bar{\psi}\right) - \frac{2e^2}{mc}|\psi|^2\underline{A}$$

$$\Rightarrow \quad \underline{J} = \frac{Ne^2}{mc}\left(\nabla\phi - \underline{A}\right)$$

(2) In the Coulomb gauge, $\nabla\cdot\underline{A} = 0$, current conservation requires $\nabla^2\phi = 0$ (for $\nabla\cdot\underline{J} = 0$)

This implies $\nabla\phi$ = constant, which can be chosen to be zero.

$$\Rightarrow \quad \underline{J} = \frac{-Ne^2}{mc}\underline{A} \qquad \text{(London equation)}$$

__Note__: $\underline{J}$ depends on $\underline{A}$. Hence, it violates gauge invariance.

(3) Combine it with Maxwell's eq.

$$\Rightarrow \quad \nabla^2\underline{B} = \frac{-Ne^2}{mc}\underline{B}$$ $\Rightarrow$ Meissner effect.

(Effectively, photon gain a mass. $\Rightarrow$ short range force)

4. Spontaneous symmetry breaking (Higgs Mechanism)

1) The symmetry breaking property of the superconductor was generalized for non-abelian gauge theory by

Anderson (1963),
Englert & Brout (1964)
Higgs (1964)
Kibble (1966)

2) In place of the Cooper pairs, one postulate the existence of a new fundamental spin-0 field ϕ, which is called Higgs boson field.

Its kinetic energy term is $\frac{1}{2}\left|D_\mu\phi\right|^2 \equiv K.E.$

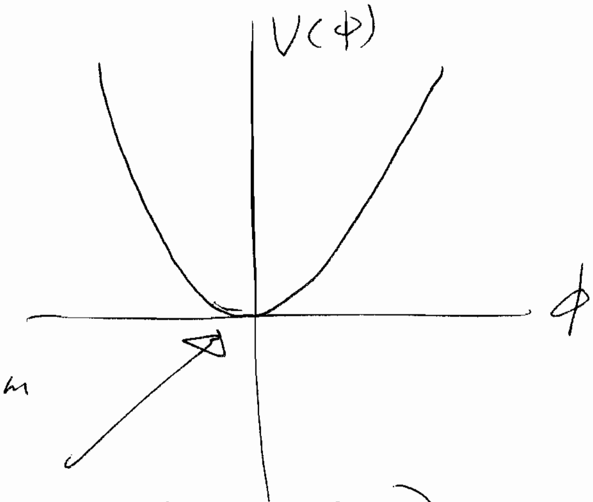
and its potential energy is

$$V(\phi) = \mu^2|\phi|^2 + \lambda\left(|\phi|^2\right)^2$$

<u>Note</u>: The form of $V(\phi)$ was first proposed by Ginzburg & Landau (1950), before BCS theory, to describe the free energy density of the superconductor.

3) For $\mu^2 > 0$,

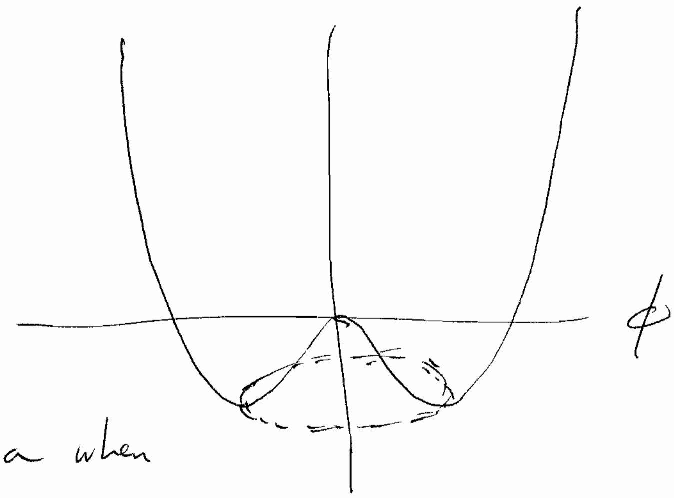
①



Unique minimum when $\phi_0 = 0$

(normal metal conducting state, with $T > T_c$)

② For $\mu^2 < 0$,



An infinite number of degenerate minima when

$$\phi_0 = \sqrt{\frac{-\mu^2}{\lambda}}\, e^{i\theta(x)}$$

(superconducing state, with $T < T_c$.)

The degenerate Higgs ground state ϕ_0 plays the role of the self-coherent field which breaks the gauge symmetry.

$\Longrightarrow$ Vacuum breaks gauge symmetry.

4) How to generate W-boson mass ?

In the broken phase,

$\phi \rightarrow \langle \phi_0 \rangle = v$ which is called Vacuum expectation Value

$D_\mu = \partial_\mu - i g_w W_\mu^a \tau^a$ — Pauli matrices ($a = 1, 2, 3$) for $SU(2)$ symmetry

$\frac{1}{2} |D_\mu \phi|^2 \rightarrow M_W^2 W_\mu^+ W^{-\mu}$

with $\boxed{M_W = \frac{1}{2} g_w \cdot v}$

Note ① Since $G_F = \frac{1}{\sqrt{2} v^2} \sim 10^{-5} (\mathrm{GeV})^{-2}$,

we get $v \simeq 246$ GeV

② The weak coupling constant

$g_w = \frac{e}{\sin\theta_w}$, where $e \equiv \sqrt{4\pi\alpha_{em}}$

③ $\sin\theta_w$ can be measured from ratio of neutrino-nucleon scattering cross sections. (θ_w = weak-mixing angle)

$\Rightarrow$ Predict $M_W \simeq 80$ GeV

(with $\alpha_{em} \simeq \frac{1}{137}$, $\sin^2\theta_w \simeq 0.23$)